MAT 362 SPRING 05 HOMEWORK I

Due Thursday, Feb. 9

1. Read through as much as you can of Chapters 1-3. The most important material is Ch. 1 and Ch. 2.1, but try to read further also. Read theorems and examples and do some of the problems. Ask me any questions you have.

2. Let $\gamma : [a, b] \to \mathbb{R}^n$ be any smooth parametrized curve, and let $\gamma(a) = p$, $\gamma(b) = q$.
   (a). Show that, for any constant vector v, with $|v| = 1$,

$$(q - p) \cdot v = \int_a^b \dot{\gamma}(t) \cdot v dt \leq \int_a^b |\dot{\gamma}(t)| dt.$$

   (b) Set

$$v = \frac{q - p}{|q - p|}$$

and show that

$$|\gamma(b) - \gamma(a)| \leq \int_a^b |\dot{\gamma}(t)| dt|.$$

Show means that the curve of shortest length from $\gamma(a)$ to $\gamma(b)$ is the straight line joining these points.

3. Consider the curve $\gamma(t) = (3t, 3t^2, 2t^3)$ in $\mathbb{R}^3$.
   (a) Show this curve is regular and find its tangent vector.
   (b) Show the tangent lines to this curve make a constant angle with the line $y = 0$, $z = x$.

4. Find the signed curvature of the parametrized curve

$$\gamma(t) = (t, \cosh t)$$

The image of γ is called the catenary.

5. Consider the curve in $\mathbb{R}^2$ given by

$$\gamma(t) = (e^{-t} \cos t, e^{-t} \sin t),$$

for $t \in [0, \infty]$.
   (a). Show that γ has finite length.
   (b). Is the curvature bounded or unbounded as $t \to \infty$?

6. Is there a simple closed curve in the plane with length equal to 6 feet and bounding an area of 3 square feet? If yes, describe it, if not, show why not.