Scientific Highlight October 2008

Title of the Highlight: Wavelets on Smooth Manifolds and Applications

Keywords: Riemaniann compact manifolds, Laplace-Beltramie operator, wavelet, frame, sphere, needlets.

Central statement:
Using spectral and multiplier theory approach we developed the definition of continuous wavelets on smooth compact oriented Riemaniann compact manifold and study their discretization into a nearly tight frames, which are significant for instance in the study and analyzing of dates.

Text:
Using spectral theory for the Laplace-Beltramie operator for compact manifolds we constructed continuous wavelet as kernel of integral operators. By the same approach we also obtain smooth wavelets with compact support and higher vanishing moments. The prime practical examples for these manifolds are the sphere and the torus.
In order to discretize our wavelets, we imposed a condition on the Schwartz function used on the spectrum of the Laplace-Beltramie, namely the Daubechies condition. The condition assures one that the wavelets can be discretized to be nice wavelet frames. The word nice here means that they are nearly tight; such frames are considered most often for practical reasons, since they are close to being an othonormal basis, in the sense that they allow a simple way to reconstruct a function very closely from its frame coefficients. Using the wavelet frames we constructed, we characterized the inhomogeneous Besov spaces on the manifolds for the entire range of indices, in terms of the size of coefficients in frame decompositions.

With specializing our study to the case where the manifold is the sphere, we constructed mexican needlets. Mexican needlets are wavelets with good localization property. Applying this property we prove that the random needlet coefficients are asymptotically uncorrelated for any fixed angle on the sphere. The Mexican needlets can be utilized in the analysis of Cosmic Microwave Background radiation.