MAT 211, Introduction to Linear Algebra, Summer 1, 2008
PREPARING FOR THE STANDARD SUBJECTS ON THE FINAL EXAM

The following are meant to help you in your preparation for the final exam.

(I) FIRST make sure you are able to solve both 1 and 2 below.

(II) Then please make sure that you know how to solve ANY of the standard problems in the homework assignments (since the standard problems on the final will be similar to some of the standard problems in the homework assignments).

(III) After you are able to solve any of the standard problems in the homework assignments, you may start preparing for the non-standard subject. In order to approach this last subject succesfully, one has to have in depth understanding of all concepts and results presented in class, to be able to solve any non-standard problem in the homework, to prove any result proved in class, to solve any exercise assigned in class, to prove claims other than those mentioned in class and having a maximum level of difficulty equal to the most difficult results given as homework or presented in class.

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1. You should be able give precise definitions, several examples and counterexamples for the following fundamental linear algebra concepts: matrix multiplication, real/complex linear space, span, linear dependence/independence, basis, dimension of a finitely generated linear space, linear transformation, kernel, image, isomorphism, isomorphic/non-isomorphic linear spaces, inner product and norm, orthogonal transformation and matrix, orthonormal basis, the matrix of a linear transformation.
   Note: You should be able to give these definitions in their full generality. For instance, if you define basis for $\mathbb{R}^3$ instead of definig the concept of a basis in an arbitary finitely generated linear space, you will receive very little or no credit.

2. You should be able to solve the following standard types of exercises:
   - compute determinants of (1×1),(2×2), (3×3),(4×4),(5×5) or (6×6) matrices with real entries;
   - decide whether a matrix is invertible and compute the inverse of a matrix for (1×1),(2×2), (3×3),(4×4) matrices with real entries;
   - state the principle of mathematical induction and apply it for matrix computations;
   - prove that a given set of elements of a linear space is a basis;
   - express a given element of a linear space as a linear combination in terms of the elements of a given basis;
   - decide with proof whether a transformation between two linear spaces is a linear transformation;
   - compute the kernel of a linear transformation and the dimension of the kernel;

- decide with proof whether a linear transformation between two linear spaces is an isomorphism;
- compute the product between two matrices, compute the powers of a matrix, compute the composition of linear transformations;
- compute the rank of a matrix;
- apply Cramer's rule;
- apply the Gram-Schmidt process and compute the change of basis matrix between the orthonormal basis and the initial basis;
- find the eigenvalues (both in the real and complex case) and the eigenvectors of a matrix, find a basis of eigenspaces;
- decide whether matrix A is diagonalizable; if it is, find an invertible S and a diagonal D such that $S^{-1}AS = D$.