MAT 211, Linear Algebra, Summer 1, 2008

MANDATORY READING ASSIGNMENTS

1. *DUE by the Wednesday lecture - June 18*
   -concepts: the change of basis matrix, the matrix of a linear transformation;
   - the rank-nullity theorem;
   References: textbook, page 164 (def.4.2.1); page 176 (fact 4.3.3); pages 176-177(example 5); page 172 (fact 4.3.1);page 174 (example 4).
   Requirements: Please try to prove the rank-nullity theorem by yourself. However, this proof will also be presented in class.

2. *DUE by the Thursday lecture - June 19*
   During our first meeting we asked the following question: "For the system of linear equations $Ax = b$ only one of the following situations may occur:
   (A) The system has no solutions.
   (B) The system has a unique solution.
   (C) The system has infinitely many solutions.
   Why is that indeed? In other words, why isn't it possible for a system of linear equations to have 104 solutions, for instance?"
   This was our guiding question so far; this question motivated us to introduce the concept of linear space/subspace (if $b = 0$, the space of solutions of the linear homogeneous system of equations $Ax = 0$ is a linear subspace). We intend to give a complete answer on June 19. In other to be able to answer the question, we will first answer the following question: "If A is an $(n \times m)$ matrix with real entries and $rank A = r$, what is the dimension of the space of solutions of the linear homogeneous system $Ax = 0$ in terms of m- the number of unknowns and r? ". The answer to this question is, in itself, a very important theorem which gives as a consequence, the answer to our initial question.
   Please try to solve this second problem for some particular cases of n, m and r and make sure that you understand and are able to work with all concepts involved in this paragraph.

3. *DUE by the Thursday lecture - June 26*
   -Concepts: orthogonal matrix, orthogonal transformation;
   -The Pythagorean theorem: Let $(V, <,>)$be an inner product space and $v, w \in V$. Then

   $$\|v + w\|^2 = \|v\|^2 + \|w\|^2 \Leftrightarrow < v, w >= 0.$$

   Please prove this theorem by yourself - this is an easy and short exercise that only requires the definition of an inner product space.

- Orthogonal transformations preserve orthogonality: if $T : \mathbb{R}^n \to \mathbb{R}^n$ is an orthogonal transformation, and $v, w \in \mathbb{R}^n$are orthogonal (which is the same as perpendicular), then so are $T(v)$ and $T(w)$.
Please prove this easy exercise by yourself and then generalise this to arbitrary inner product spaces.
References: textbook, page 209, definition 5.3.1; page 210, Fact 5.3.2 and its proof; page 214, Fact 5.3.8.

4. *DUE by the Monday lecture - June 30*
   Basic knowledge of complex numbers will be assumed throughout the remaining lectures and on the final exam. If you are not confident in your background in complex numbers, please ask for help on monday BEFORE the lecture. Alternatively, you may use the *textbook pages 342 - 346.*

5. *DUE by the Wednesday lecture - July 2*
   -Concepts: eigenvectors and eigenvalues of a matrix and of a linear operator (of course, the former is just a particular case of the latter); eigenspaces; algebraic/geometric multiplicity of an eigenvalue, characteristic polynomial.
   -Facts: the possible eigenvalues of an orthogonal transformation are $1, -1$. Please prove this fact by yourself - it is an immediate application of the definitions. This will not be proved in class, but will be assumed on the final exam. If you are unable to complete this easy proof by yourself, please use the textbook, page 299.
   -the number of eigenvalues of an $(n \times n)$ matrix is at most n.
   References: textbook, page 297, definition 7.1.1; page 307, Fact 7.2.2, Fact 7.2.4; page 310, definition 7.2.6, Fact 7.2.7; page 317, definition 7.3.1; page 319, definition 7.3.2; page 335, definition 7.4.6.

6. *DUE by the Thursday lecture - July 3*
   -Fundamental theorem about the orthogonal complement (available in the Homework Asssignments section of the course web page). The proof of this theorem should only be considered by the students who would like to approach the last subject on the final exam.
   -Concepts: diagonalizable matrix (*page 330, definition 7.4.2);*
   -The orthogonal projection is a linear transformation. (*page 189; The proof is NOT required, but you are expected to know the statement. The proof is not required just to allow you more time to study for the final exam; however, this is an important result, with a fairly easy proof.)*