MAT 211, Linear Algebra, Summer 1, 2008

APPENDIX TO Homework 1 (due Wednesday, June 11) - HINTS for problem 8, the computation of d_{n}

1. At part (a) of problem 56 on page 260, you proved that

$$d_{\mathrm{n}} = (-1)^{\mathrm{n}+1} d_{\mathrm{n\text{-}1}} \forall n \geq 2.$$

2. Using this, prove that

$$d_{\mathrm{n}+2} = -d_{\mathrm{n}} \forall n \geq 1.$$

3. Using (2) prove that

$$d_{\mathrm{n}+4} = d_{\mathrm{n}} \forall n \geq 1.$$

4. Using (3) prove that

$$d_{1+4\mathrm{k}} = d_1; d_{2+4\mathrm{k}} = d_2; d_{3+4\mathrm{k}} = d_3; d_4 = d_{4+4\mathrm{k}} \forall k \geq 0, k\, integer.$$

(Of course, you have computed d_1, d_2, d_3, d_4 at part (b) so you can replace these by real numbers.)
Although this is almost obvious, a rigorous proof uses the following theorem:
Theorem (The Principle of Mathematical Induction):
Let $P(k)$be a statement that depends on a non-negative integer k. Assume that the following two conditions hold:
(i) $P(0)$ is true.
(ii) If $P(m)$is true, then $P(m+1)$is true as well.
Then $P(k)$ holds for all non-negative integers k.

Thus, the proof of the fact that $d_{1+4\mathrm{k}} = d_1 \forall k \geq 0$, goes as follows:
Proof. We denote by $P(k)$the claim $d_{1+4\mathrm{k}} = d_1$.

We will do the proof by induction on k. (This means that we will prove tha claim called $P(k)$using the above mentioned Principle of Mathematical Induction.)
We verify (i). $P(0)$says that $d_1 = d_1$which is trivially true, so we did verify (i) in the above theorem.
We verify (ii). This step of the proof is usually called THE INDUCTION STEP. Let m be a non-negative integer. Assume that $P(m)$is true. We will prove that, under this assumption, $P(m+1)$ is true as well. Since $P(m)$is assumed to be true, it means that

$$d_{1+4\mathrm{m}} = d_1.$$

We will prove that

$$d_{1+4(\mathrm{m}+1)} = d_1.$$

$d_{1+4(\mathrm{m}+1)} = d_{4\mathrm{m}+5} =^{\text{BY (3)}} d_{4\mathrm{m}+1} =^{\text{BY THE ASSUMPTION THAT P(m) IS TRUE}}$
d_1 so we are done.

5. After you have completed the proofs at (4), write a general formula for d_{n}. This formula depends on the remainder obtained by dividing n by 4. Now the proof for a general formula of d_{n} is finished using (4).
IMPORTANT: At problem 5 in homework 1, another "Principle of Mathematical Induction" was stated. Actually, the so called "Principle of Mathematical Induction" is a collection of theorems. One of them is the theorem stated at problem 5; another theorem in this collection is the one stated above. There are also other thorems that are included in the so called "Principle of Mathematical Induction". Though djfferent, they are all used to prove statements that depend on non-negative integers.