Corrections to
Basic Algebra*, *Digital Second Edition

Page 32, statement of Problem 23. Change "positive integer n" to "positive integer k".

Page 42, line 10. Change "$\dim(\{v_1,\dots,v_r\})$" to "$\dim(\{v_1,\dots,v_i\})$", and change "for $i\ge 0$" to "for $i\ge 1$".

Page 53, statement of Proposition 2.21, display. The matrix on the right side should have a transpose symbol, superscript t, on it.

Page 53, lines 2 and 3 of the proof of Proposition 2.21. Change "Write B and A for the respective matrices in the formula in question" to "Write $B=\begin{pmatrix} L^t \\ \Gamma'\Delta' \end{pmatrix}$ and $A=\begin{pmatrix} L \\ \Delta\Gamma \end{pmatrix}$".

Page 73, line 7. The two equations on this line are reversed from what they should be. The first should be "$A^{\rm adj}A=(\det A)I$", and the second should be "$AA^{\rm adj}=(\det A)I$".

Page 77, line -15. Change this line from "$=(\det C^{-1})\det(\lambda I-A)(\det C^{-1})$" to "$=(\det C^{-1})\det(\lambda I-A)(\det C)$".

Page 77, line -14. Change "$(\det C^{-1})(\det C^{-1})$" to "$(\det C^{-1})(\det C)$".

Page 83, display in Problem 10. Change "max" to "min".

Page 92, line 3 of the proof of Proposition 3.1. Change the left member of the equation from "$\big|u-\|v\|^{-2}(u,v)v\big|^2$" to "$\big\|u-\|v\|^{-2}(u,v)v\big\|^2$".

Page 97, line 7. Change "in S" to "in V", and change "$v=\sum_{j=1}^n u_j$" to "$v=\sum_{j=1}^n (v,u_j)u_j$".

Page 102, line -6. Change "$L(u,v)$" to "$(L(u),v)$".

Page 107, line 1 of statement of Corollary 3.22. Change "positive semidefinite" to "self-adjoint".

Page 109, line 3 of the proof of Corollary 3.23. Change "For each j, the commutativity of the linear maps L_i forces" to "The linearity and the commutativity of L_1 with each L_i force".

Page 116, line -7. Change "make use of Problem 26" to "make use of Problem 29".

Page 123, line 7 of proof of Proposition 4.1. Change "$-c_j^{-1}c_kx^{k-j-1}$" to "$-\cdots-c_j^{-1}c_kx^{k-j-1}$".

Page 145, line 2 after end of the proof of Proposition 4.19. Change "just of subring" to "just a subring".

Page 157, line 2. Change "$T=R\times\cdots\times R$" to "$T=R$".

Page 157, line 2. Change "Example 3" to "Example 2".

Page 157, line 3 of the proof of Proposition 4.30. Change "$a_{j_1,\dots,j_n}t_1^{j_1}\dots t_n^{j_n}$" to "$\varphi(a_{j_1,\dots,j_n})t_1^{j_1}\dots t_n^{j_n}$".

Page 164, line 4 of the proof of Proposition 4.34. Change "$g_1p=\varphi(g_1p)=\varphi(g_2p)=g_2p$" to "$g_1p=\varphi(g_1)=\varphi(g_2)=g_2p$".

Page 165, line 12. Change "$xg^{-1}=g^{-1}(gx)g^{-1}=g^{-1}(xg)g^{-1}=g^{-1}x$" to "$gx^{-1}=x^{-1}(xg)x^{-1}=x^{-1}(gx)x^{-1}=x^{-1}g$".

Page 166, line 2 of proof of Corollary 4.39. Change "If fact" to "In fact".

Page 172, line 13. Change "fewer" to "more".

Page 175, display $(*)$ in the proof of Theorem 4.49. Change the display from

$$\begin{aligned} G_m \supseteq G_{m-1} \supseteq \cdots \supseteq G_1 \supseteq G_0 \\ H_n \supseteq H_{n-1} \supseteq \cdots \supseteq H_1 \supseteq H_0 \end{aligned} \qquad (*)$$

to

$$\begin{aligned} G_0 \supseteq G_1 \supseteq \cdots \supseteq G_{m-1} \supseteq G_m \\ H_0 \supseteq H_1 \supseteq \cdots \supseteq H_{n-1} \supseteq H_n \end{aligned} \qquad (*)$$

Page 184, line -6. Change "$\mathbb{Z}/p^{l_{M'-j}}\mathbb{Z}$" to "$\mathbb{Z}/p^{l_{M'-j-1}}\mathbb{Z}$".

Page 184, line -4. Change

$$(\mathbb{Z}/p^{l_{t'-j-1}}\mathbb{Z})/(\mathbb{Z}/p^{l_{t'-j}}\mathbb{Z}) \oplus \cdots \oplus (\mathbb{Z}/p^{l_{M'-j-1}}\mathbb{Z})/(\mathbb{Z}/p^{l_{M'-j}}\mathbb{Z})$$

to

$$(\mathbb{Z}/p^{l_{t'-j}}\mathbb{Z})/(\mathbb{Z}/p^{l_{t'-j-1}}\mathbb{Z}) \oplus \cdots \oplus (\mathbb{Z}/p^{l_{M'-j}}\mathbb{Z})/(\mathbb{Z}/p^{l_{M'-j-1}}\mathbb{Z})$$

Page 188, line 1. Change "subgroup of H" to "subgroup H".

Page 188, line 2 of the proof of Lemma 4.62. Change "H_pH_q in the proof of Proposition 4.60" to "H_1H_2 in the second paragraph of the proof of Theorem 4.14".

Page 193, line 8. Change "$= g \circ F(f) = F(g)F(f)$" to "$= g \circ F(f)(L) = F(g)F(f)(L)$".

Page 218, line -16. Change "has thus be redone" to "has thus been redone".

Page 436, statement of Proposition 8.52. Change "commutative ring" to "integral domain".

Pages 619–620, solution of Problem 14. Replace the existing text with the following:

"Let $\{v_n\}_{n=1}^{\infty}$ be a countably infinite basis of the vector space V. Write $\mathcal{A}$ for the set of all subsets of the set of positive integers. Certainly $\mathcal{A}$ is uncountable. For each such subset S, define v'_S to be the member of the dual space V' such that $v'_S(v_n)$ is 1 if n is in S and is 0 if not. Let W be the linear span in V' of all the linear functionals v'_S with S in $\mathcal{A}$. Choose by Theorem 2.42a a subset $\mathcal{B}$ of $\mathcal{A}$ such that $\{v'_S \mid S \in \mathcal{B}\}$ is a basis of the linear span W of all v'_S for $S \in \mathcal{A}$. Then the set $\{v'_S \mid S \in \mathcal{B}\}$ is linearly independent, and we shall prove that it is uncountable.

"Arguing by contradiction, suppose that $\mathcal{B}$ is countable. Number the sets S in $\{v'_S \mid S \in \mathcal{B}\}$ as $S_1, S_2, \ldots$. Since each v'_S with S in $\mathcal{A}$ lies in the span of $S_1, S_2, \ldots$, each member S of $\mathcal{A}$ has an associated (nonunique) integer $k \geq 1$ such that v'_S has a unique expansion as $v'_S = c_1 v'_{S_1} + \cdots + c_k v'_{S_k}$ with coefficients in $\mathbb{F}$. In this case let us say that v'_S is expandable for this k.

"Fix k, and fix a set S for which v'_S is expandable for this k. Then fix a subset E of $\{1, \ldots, k\}$. If m and n are two integers from 1 to k with the property that both are in S_j for each j in E and both are not in S_j for each j in $\{1, \ldots, k\} - E$, then $v'_{S_j}(v_m) = v'_{S_j}(v_m)$ and hence $v'_S(v_m) = v'_S(v_n)$. Thus with k fixed, the number of members S of $\mathcal{A}$ for which v_S is expandable for this k is $\leq 2^k$, the number of subsets of $\{1, \ldots, k\}$. In particular it is finite. Consequently the set of S in $\mathcal{A}$ for which v'_S is expandable for some k is contained in the countable union of finite sets and is a countable set. This conclusion contradicts the known fact that $\mathcal{A}$ is uncountable and allows us to conclude that $\mathcal{B}$ is uncountable."

Page 638, solution to Problem 67. Change "has weight 3" to "has weight 3 or 4".

Page 639, line 13. Change "occur is positions" to "occur in positions".

Page 639, solution of Problem 71, line 1. Change "$\frac{1}{2}((X+Y)^n + \frac{1}{2}(X-Y)^n)$" to "$\frac{1}{2}(X+Y)^n + \frac{1}{2}(X-Y)^n$".

Page 639, solution of Problem 71, line 2. Change "$X^6 + 7X^3Y^3$" to "$X^6 + 4X^3Y^3 + 3X^2Y^4$".

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