

**Corrections to  
Advanced Real Analysis, Digital Second Edition**

CORRECTIONS TO CHAPTER III

**Note:** The items below point to corrections in both the digital second edition and the printed first edition.

Page 72, Proposition 3.15, statement up to the formula. Change to read “Let  $H$  be the subset of  $\mathbb{R}^N = \{(x', x_N) \mid x' \in \mathbb{R}^{N-1} \text{ and } x_N \in \mathbb{R}\}$  with  $x_N > 0$ . Suppose that  $u$  and  $v$  are  $C^2$  functions on an open subset of  $\mathbb{R}^N$  containing the closure  $\overline{H}$  and that at least one of  $u$  and  $v$  is compactly supported. Then”.

Page 72, Proposition 3.15, display in statement. Change “ $R^+$ ” to “ $H$ ”, and change “ $x_n$ ” to “ $x_N$ ” in two places.

Page 72, Proposition 3.15, line 2 of proof. Change “ $\mathbb{R}^{N-1}$ ” to “ $\mathbb{R}^N$ ”, and change “ $(R^+)^{\text{cl}}$ ” to “ $\overline{H}$ ”.

Page 73, end of top paragraph. Change “ $R^+$ ” to “ $H$ ”.

Page 99, Problem 3. In the first line, change “open unit disk in  $\mathbb{R}^2$ ” to “open disk in  $\mathbb{R}^2$  with  $x^2 + y^2 < \frac{1}{2}$ .”

Page 100, line 6. Change “ $T \in \mathcal{S}(\mathbb{R}^N)$ ” to “ $T \in \mathcal{S}'(\mathbb{R}^N)$ ”.

CORRECTIONS TO CHAPTER X

**Note:** The corrections below were kindly pointed out by Ehssan Khanmohammadi. They affect only the digital second edition, since the printed first edition does not contain Chapter X.

Page 436, line –13. In the definition of  $\omega_0$ , change “ $\omega|\mathcal{F}f(t\omega)|^2$ ” to “ $\omega|\mathcal{F}f(\omega)|^2$ ”.

Page 437, line –4. Change “ $\sigma_{g,t}$ ” to “ $\sigma_{g,t}^2$ ”.

Page 439, line –6. Insert “ $s_N$ ” after the second limit sign.

Page 440, lines –11 and –10. Delete “, so that  $c_{-k} = \int_{-1/2}^{1/2} (\mathcal{F}f)(\omega)e^{2\pi ik\omega} d\omega$ ”.

Page 440, line –8. Change “ $\mathcal{F}f(\omega) =$ ” to “ $\mathcal{F}f(\omega) = \sum_{k=-\infty}^{\infty} c_k e^{2\pi ik\omega} =$ ”.

Page 443, line 2 of Section 2. Change “ $\psi(2^j - k)$ ” to “ $\psi(2^j x - k)$ ”.

Page 453, line (\*). Change “ $|(P_m g)(x_n)$ ” to “ $|(P_m g)(x_n)|$ ”.

Page 453, line after (\*). Change “ $(f - Jg) + g$ ” to “ $(f - Jg) + Jg$ ”.

Page 456, line 5. Change “ $\nu(\frac{1}{2}y)$ ” to “ $\nu(y)$ ”.

Page 456, line 13. Change “is in  $V_0$ , which is contained in  $V_1$ , and” to “is in  $V_1$  and”.

Page 457, line 5. Change “ $b_0 = 1\sqrt{2}$ ” to “ $b_0 = 1/\sqrt{2}$ ”.

Page 457, line 12. Change “period  $\frac{1}{2}$ ” to “period 1”.

Page 458, line –4. Change “*Basic*,” to “*Basic*,”.

Page 459, line 6 of Lemma 10.12. Change “ $\alpha(\delta_{\cdot k}) = h_{0,k}$ ” to “ $\alpha(\delta_{\cdot k}) = \varphi_{0,k}$ ”.

Page 459, line –7. Change “ $\beta^{-1}\alpha^{-1}(\varphi_{0,k})$ ” to “ $\beta^{-1}\alpha^{-1}(\varphi_{0,k})(\mathcal{F}\varphi)(y)$ ”.

Page 459, line –3. Delete “(a) gives”.

Page 462, line 3. Change second “ $(\mathcal{F}f)$ ” to “ $(\mathcal{F}\varphi)$ ”.

Page 462, line 5. Change “ $(\mathcal{F}f)(y + \frac{1}{2}l)\overline{(\mathcal{F}\varphi)(y + \frac{1}{2}l)}$ ” to “ $|(\mathcal{F}\varphi)(y + \frac{1}{2}l)|^2$ ”.

Page 463, line 3. Change “ $m_f(\frac{1}{2}y)(\mathcal{F}f)(\frac{1}{2}y)$ ” to “ $m_f(\frac{1}{2}y)(\mathcal{F}\varphi)(\frac{1}{2}y)$ ”.

Page 463, line 7 of proof of Theorem 10.10b. Change “ $\sum_l |m_0(\frac{1}{2}y + \frac{1}{2})|^2$ ” to “ $\sum_l |m_0(\frac{1}{2}y + \frac{1}{2}l + \frac{1}{2})|^2$ ”.

Page 464, line 4 of top display. Change “ $|(\mathcal{F}\psi)(y)|^2 dy$ ” to “ $|(\mathcal{F}\psi)(y)|^2 dy$ ”.

Page 467, lines –5 to –3. Change “ $\nu(\frac{1}{2}y)$ ” to “ $\nu(y)$ ” in three places.

Page 468, line 6. Change “ $\sin(2\pi(x - \frac{1}{2})) - \sin(\pi(x - \frac{1}{2}))$ ” to “ $\sin(2\pi(x - \frac{1}{2})) - \sin(\pi(x - \frac{1}{2}))$ ”.

Page 471, line 4 of Proposition 10.17. Delete “Suppose that  $V_0 \subseteq V_1$ , so that  $\{V_j\}$  is an increasing sequence.”

Page 471, line 6 of Proposition 10.17. Change “ $\bigcup_{j=-\infty}^{\infty} V_j$ ” to “ $\sum_{j=-\infty}^{\infty} V_j$  generated by all the  $V_j$ ”.

Page 471, after statement of Proposition 10.17. Insert  
“REMARK. We use this result only when  $V_0 \subseteq V_1$ , in which case  $\{V_j\}$  is an increasing sequence and  $\sum_{j=-\infty}^{\infty} V_j = \bigcup_{j=-\infty}^{\infty} V_j$ .”

Page 480, line –2. Change “ $|\sin \pi z|^2$ ” to “ $|\sin z|^2$ ”.

Page 482, Theorem 10.25e. Change “period  $\frac{1}{2}$ ” to “period 2”.

Page 483, line 6 of the proof. Change “ $\beta^{-1}\alpha^{-1}(c)$ ” to “ $\beta^{-1}\alpha^{-1}(f)$ ”.

Page 484, line –11. Change “ $-e^{\pi iy}\nu(y/2)$ ” to “ $e^{\pi iy}\nu(y)$ ”.

Page 484, line –10. Change “ $e^{\pi iy}\nu(y/2)$ ” to “ $-e^{2\pi iy}\nu(y)$ ”.

Page 484, line –9. Change “ $e^{\pi iy}\nu(y/2)$ ” to “ $-e^{2\pi iy}\nu(y)$ ”.

Page 484, line –8. Change “ $\nu(y) = e^{-2\pi iy}$  and  $\nu(y/2) = e^{-\pi iy}$ ” to “ $\nu(y) = -e^{-2\pi iy}$ ”.

Page 485, formula (††). Change “ ${}^1\gamma_{1,0}$ ” to “ ${}^1\gamma_{1,k}$ ”.

Page 485, formula (‡). Change “ $u_k\gamma(2x - k)$ ” to “ $u_k{}^1\gamma(2x - k)$ ”.

Page 495, Theorem 10.33e. Change “period  $\frac{1}{2}$ ” to “period 2”.

Page 501, two lines before “REMARKS.” Change “ $0 \leq y \leq 1$ ” to “ $0 \leq w \leq 1$ ”.

Page 501, Lemma 10.35, line 1. Change “ $\sum_{k=0}^{N-1} \binom{2N-1}{k} w^k (1-w)^{N-k}$ ” to “ $\sum_{k=0}^{N-1} \binom{2N-1}{k} w^k (1-w)^{N-k-1}$ ”.

Page 502, line -2. Change “ $w^k$ ” to “ $y^k$ ”.

Page 503, line -6. Delete “ $0 =$ ” at left side.

Page 503, line -2. Change “ $R(\frac{1}{2} - \frac{1}{2}(1-w))$ ” to “ $R(\frac{1}{2} - (1-w))$ ”.

Page 503, line -1. Change “ $R(\frac{1}{2}w) = -R(\frac{1}{2}w)$ ” to “ $R(-w) = -R(w)$ ”.

Page 505, two lines after (\*\*). Change “with  $\bar{z}_0$ ” to “with  $1/\bar{z}_0$ ”.

Page 507, line -5. Change “ $h(1)$ ” to “ $h(0)$ ”.

Page 508, first display. Change “ $a_j$ ” on the left side to “ $a_k$ ”.

Page 509, line -5. Change “ $\|T_k(f - f_n)\|_2$ ” to “ $\|T_k(f - f_n)\|_2$ ”.

Page 510, line 12. Change “ $(\frac{1}{2}y + l)$ ” to “ $(\frac{1}{2}y + l)$ ”.

Page 510, line 13. Change “ $m_0(\frac{1}{2}y + \frac{1}{2})$ ” to “ $m_0(\frac{1}{2}y + \frac{1}{2})$ ”.

Page 510, line 14. Change “ $m_0(\frac{1}{2}y + \frac{1}{2})$ ” to “ $m_0(\frac{1}{2}y + \frac{1}{2})$ ”.

Page 511, line 7. Change “ $C_1|\mathcal{F}\varphi(y)|$ ” to “ $C_1|(\mathcal{F}\varphi)(y)|$ ”.

Page 511, line 15. Change “ $\inf_{|y|\leq 1/2} |\mathcal{F}\varphi(y)|$ ” to “ $\inf_{|y|\leq 1/2} |(\mathcal{F}\varphi)(y)|$ ”.

Page 511, line 17. Change “ $|y| \leq 2^{n-1}$ ” to “ $|y| \leq 2^{n-1}$ ”.

Page 513, first displayed line. Change “ $\sin(2^{J-1})$ ” to “ $\sin(2^{-J+1}\pi y)$ ”. The result no longer fits in the available space. Thus run the equation into the text, put a period at the end of the displayed part, and change “and therefore” to “Therefore”.

Page 513, footnote, line 2. Change “ $\nu(y/2)$ ” to “ $\nu(y)$ ”.

Page 516, line 9. Change “proof the” to “proof of the”.

Page 516, three lines before “PROOF OF THEOREM 10.39”. Change “ $e^{-2\pi i 2N}$ ” to “ $e^{-2\pi i(2N-1)x}$ ”. The result no longer fits in the available space, but it fits if one changes “the ones” to “those”.

Page 516, line 3 of “PROOF OF THEOREM 10.39”. Change “ $m_0(y) = 1$ ” to “ $m_0(0) = 1$ ”.

Page 517, line 1. Change “ $h(1)$ ” to “ $h(0)$ ”.

Page 517, line 5. Change “ $\nu(\frac{1}{2}y)$ ” to “ $\nu(y)$ ”.

Page 517, line 6. Change “ $\nu(\frac{1}{2}y)$ ” to “ $\nu(y)$ ”.

Page 517, line 7. Change “ $e^{-4\pi i y}$ ” to “ $e^{-2\pi i y}$ ”.

Page 517, line 2 of Lemma 10.40. Change “in a open” to “in an open”.

Page 518, line 6 of the proof. Change “ $-\xi < -M$ ” to “ $\xi < -M$ ”.

Page 518, line 8 of the proof. Change “ $\text{Re } z$ ” to “ $\text{Im } z$ ”.

Page 518, line 12 of the proof. Change “an arbitrary” to “arbitrary”.

Page 519, line 8. Change “=” to “≤”.

Page 521, line 4. Change “ $\sum_{k=-2n}^{2n}$ ” to “ $\sum_{k=0}^{2n}$ ”.

Page 521, line 2 of Proposition 10.43. Change “wavelet of wavelet of” to “wavelet of”.

Page 521, line –2. Change “10.42” to “10.39”.

Page 522, line 1. Change “ $e^{\pi i N y}$ ” to “ $e^{-\pi i N y}$ ”.

Page 523, line 5. Change “10.42” to “10.43”.

Page 523, line 14. Change “10.41” to “10.43”.

Page 524, line 3 of big display. Change “=” to “≤”.

Page 524, line 6 of big display. Change “ $j(h)$ ” to “ $(j(h) + 1)$ ”.

Page 524, line –1. Change “ $2\pi C(2^{1-\alpha} - 1)^{-1}|h|^\alpha$ ” to “ $2\pi C 2^{1-\alpha}(2^{1-\alpha} - 1)^{-1}|h|^\alpha$ ”.

Page 525, line 4 of top display. Change “ $2^{-\alpha j(h)}$ ” to “ $2^{-\alpha(j(h)+1)}$ ”.

Page 525, line 5 of top display. Delete the line.

Page 525, line 6 of top display. Change “ $2^\alpha|h|^\alpha$ ” to “ $|h|^\alpha$ ”.

Page 525, two lines before (\*\*). Change “ $|y|(\mathcal{F}f)(y)$ ” to “ $|y||(\mathcal{F}f)(y)|$ ”.

Page 525, line before (\*\*). Move footnote mark from the word “inequality” to the word “prove”.

Page 525, line (\*\*). Change “ $4\pi$ ” to “ $c$ ”.

Page 525, line after (\*\*). Change “ $|h| \leq 1$ ” to “ $|h| \leq 1$ ; this inequality follows with  $c = 4\pi$  by applying the Mean Value Theorem to the real and imaginary parts separately”.

Page 525, line –5. Change “ $\int$ ” to “ $\int_{\mathbb{R}}$ ”.

Page 525, line –4. Change “ $\leq 4\pi|y|$ ” to “ $\leq c|y|$ ”.

Page 526, Proposition 10.47, line 2. Change “ $\{2^{j/2}\psi(2^j x - j)\}_{j,k \in \mathbb{Z}}$ ” to “ $\{2^{j/2}\psi(2^j x - k)\}_{j,k \in \mathbb{Z}}$ ”.

Page 527, line –9. Change “ $\{2^{j/2}\psi(2^j x - j)\}_{j,k \in \mathbb{Z}}$ ” to “ $\{2^{j/2}\psi(2^j x - k)\}_{j,k \in \mathbb{Z}}$ ”.

Page 527, line –7. Change “ $\{2^{j/2}\psi(2^j x - j)\}_{j,k \in \mathbb{Z}}$ ” to “ $\{2^{j/2}\psi(2^j x - k)\}_{j,k \in \mathbb{Z}}$ ”.

Page 532, line 13. Change “ $L^2(\mathbb{R})$ ” to “ $V_j$ ”.

Page 534, line –9. Change “ $c_{j,k}$ ” to “ $c_{j,l}$ ”.

Page 538, line –1. Change “ $(F(u, v)/(\alpha Q(u, v)))$ ” to “ $(F(u, v)/(\alpha Q(u, v)))$ ”.

Page 541, line –7. Change “ $2^{j/2}\varphi(2^j - k)$ ” to “ $2^{j/2}\varphi(2^j x - k)$ ”.

Page 542, line 5. Change “ $2^{j/2}\psi(2^j - k)$ ” to “ $2^{j/2}\psi(2^j x - k)$ ”.