MAT 203 SPRING 2008
PRACTICE FINAL EXAM

1. Consider the vector field $\vec{F}(x, y, z) = xy\vec{i} + z\ln x\vec{j} + xz^2\vec{k}$

(a) Compute Div $\vec{F}$.

(b) Compute Curl $\vec{F}$.

(c) Can you find a function f such that $\vec{\nabla} f = \vec{F}$?

2. Suppose a particle's postition in space is given by the vector valued function $\vec{r}(t) = 6t\vec{i} + 3\sqrt{2}t^2\vec{j} + 2t^3\vec{k}$. Assume the particle begins moving at time $t = 0$ and finishes at time $t = 2$.

(a) Compute the velocity, acceleration and speed of the particle at time t.

(b) Compute the distance the particle traveled.

(c) Suppose the particle travels through a force field given by $\vec{F}(x, y, z) = x\vec{i} + (1/\sqrt{2})\vec{j} + (x^2/z)\vec{k}$. Compute the work done by this field.

(d) Suppose the particle travels through a force field given by $\vec{G}(x, y, z) = y\sin z\vec{i} + x\sin z\vec{j} + xy\cos z\vec{k}$. Compute the work done by this field.

3. Find the points on the surface $z^2 = xy + 1$ which are closest to the origin

(a) without using lagrange mutlipliers.

(b) using the method of lagrange multipliers.

4. Evaluate $\iiint_R x\,dV$, where R is the region bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.

5. Evaluate $\iiint_R x^2\,dV$, where R is the region bounded by the spheres $\rho = 1$, $\rho = 3$ and above the cone $\phi = \pi/4$.

6. Find the volume of the region bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

7. Compute the surface area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

8. Evaluate $\iint_R xy\,dA$, where R is the region bounded by the lines $2x - y = 1$, $2x - y = -3$, $3x + y = 1$, and $3x + y = -2$.

9. Find the tangent plane to the surface $z = 3x^2 - 4y^2$ at the point $(1, 1, -1)$.

10. Consider the line integral $\int_C x\,dx - x^2y^2\,dy$, where C is the triangle with vertices (0,0), (1,1), and (0,1).

(a) Evaluate the integral directly.

(b) Evaluate the integral using Green's Theorem.