

MAT118 Final Exam Review

CHAPTER 1: VOTING METHODS

- preference ballots & schedules
- fairness criteria
- voting methods

CHAPTER 2: WEIGHTED VOTING

- weighted voting systems
- quota, dictators, veto power
- Banzhaf power
 - winning coalitions
 - critical players
 - Banzhaf power index

CHAPTER 3: FAIR DIVISION

- fair share
- lone chooser method
- lone divider method
- last diminisher method
- method of sealed bids
- method of markers

CHAPTER 5: EULER CIRCUITS

- graphs, modelling
- Euler paths and Euler circuits
- Euler's path and circuit theorems
- Euler's sum of degrees theorem

CHAPTER 6: TRAVELLING SALESMAN PROBLEMS

- Hamilton paths and circuits
- weighted graphs
- exhaustive search (brute force) method
- nearest neighbor/repetative nearest neighbor methods
- cheapest link method

CHAPTER 10: MATHEMATICS OF MONEY

- percentages
- simple and compound interest
- fixed deferred annuities

MINI-EXCURSION 3: POPULATION GROWTH

- linear growth, common difference
 - arithmetic sum formula
- exponential growth, common ratio
 - geometric sum formula
- logistic growth
 - relative population
 - complimentary seeds

- logistic equation
- attracting points, 2- and 4-cycle patterns

CHAPTER 11: SYMMETRY

- rigid motions
- proper vs improper rigid motions
- finding translation vectors, rotocenters, reflection axes
- symmetry types
 - for finite shapes
 - for border patterns
 - for wallpaper patterns

CHAPTER 15: PROBABILITY

- counting: factorials, permutations, combinations
- sample spaces
- probability
- tallying in equiprobable spaces
- multiplicative property of independent events
- complimentary events

1. Chapter 1, #41, question 1 on exam 1.
2. Consider the weighted voting system $[10: x, 3, 2, 1, 1, 1]$.
 - (a) For what values of x , does the first player have veto power?
 - (b) For what values of x , is the first player a dictator?
3. Consider the weighted voting system $[7: 5, 4, 2, 2]$.
 - (a) Write down all of the winning coalitions.
 - (b) Circle the critical players in each of the winning coalitions you just wrote down.
 - (c) Write the Banzhaf power distribution for this voting system.
- $3\frac{1}{2}$. Chapter 3: 23, 35, 45, 59, 65
4. #34 in Chapter 5.
5. #37 in Chapter 5.
6. In a graph with x vertices of even degree and y vertices of odd degree,
 - (a) for what values of x and y does the graph admit an Euler circuit? Where must the circuit begin and end?
 - (b) for what values of x and y does the graph admit an Euler path? Where must the path begin and end?
7. #37 and #43 in Chapter 6.
8. Let the figure F be a square with sides of length 5 inches. What are the dimensions of the L shaped piece(s) which are gnomons to F ?
9. Express the sum $F_N + F_{N+1} + F_{N+2} + F_{N+3}$ of four consecutive terms in the Fibonacci sequence in terms of the first two.
10. Let ϕ denote the golden ratio. Simplify

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \phi}}}$$
11.
 - (a) Find c and n so that

$$1.11111 = \frac{c^n - 1}{c - 1}$$
 - (b) Add the numbers 23 through 547.
12. Suppose that you invest \$5000 in an account which earns 6% annual interest compounded quarterly. What is the amount in your account after 10 years?
13. Suppose that you invest \$50 at the beginning of every month in an account which earns 6% annual interest compounded monthly. What is the amount in your account at the end of 20 years?
14. Suppose a population of 50 termites grows according to a linear growth model. Suppose that after one month, the population is 75. What will the population be after one year?
15. Suppose the number of people infected by a cold virus increases exponentially. If the number of people infected increased from 3000 to 4000 in one month, how many people will be infected at the end of the year?

16. A population of rabbits is measured on a monthly basis and is found to obey a logistic growth model with growth parameter $r = 3$.
- (a) If the initial population is at 40% capacity, then how many rabbits will there be after 3 months?
 - (b) After 20 months the population is at approximately 63.63% of capacity? What would you expect the population to be after 20 months if the initial population were instead at 60% capacity?

17. Chapter 11: 7,15,25,31,35,49,75-80

You may express your answers to the following questions using factorials and exponents.

18. Carlos brings with him on a trip two pairs of shoes, three pairs of pants, and five shirts. How many outfits can carlos create?

19.

- (a) How many ways can 6 people line up in a queue?
- (b) How many ways can 6 people arrange themselves evenly around a round table?

20. Kermit wants to pick up a half-dozen donuts from the local bakery, each of a different variety. Assuming the bakery is well stocked with 15 different varieties, how many different donut selections can Kermit make? What if his selection need not consist of distinct varieties?

21. A child has a collection of 20 building blocks, each with a distinct color. How many different towers can he form a tower by stacking 7 blocks, one on top of the other.

22. Suppose a slice of bread with jelly on one side is twice as likely to land jelly side down than jelly side up.

- (a) What is the probability that it lands jelly side up?
- (b) If 10 jelly sided slices are dropped simultaneously, what is the probability that half of them land jelly side up?
- (b) If 10 jelly sided slices are dropped simultaneously, what is the probability that at least 2 of them land jelly side up?