IMSXXV Conference Schedule

All talks in Simons Center Auditorium, room 103

Friday, May 8

12:00pm *Registration in math common room, Math 4-125*

1:00pm **Mitsuhiro Shishikura** (Kyoto University)
Toward arithmetic surgery of rational maps

2:00pm *Tea in math common room, Math 4-125*

2:45pm **Vadim Kaloshin** (University of Maryland, College Park)
On quasi-ergodic hypothesis and renormalization for nearly integrable systems

4:00pm *Wine & Cheese reception in SCGP lobby*

Saturday, May 9

8:30am *Coffee and bagels in math common room, Math 4-125*

9:00am **Xavier Buff** (Université Paul Sabatier, Toulouse)
Wandering domains for polynomial endomorphisms of $\mathbb{C}^2$

10:00am *Coffee in Simons Center Café*

10:30am **Barak Weiss** (Tel Aviv University)
Dynamics of the horocycle flow on the eigenform locus

11:45am **Araceli Bonifant** (University of Rhode Island)
Points visible from zero and infinity for antipode preserving maps

1:00pm *Lunch in Simons Center Garden*

1:45pm **Poster Session**, *SCGP lobby*

2:30pm **Dierk Schleicher** (Jacobs University Bremen)
Newton's method as an unexpectedly efficient root finder

3:30pm *Tea in Simons Center Café*

4:00pm **Laura DeMarco** (Northwestern University)
Complex dynamics and elliptic curves

5:30pm *Picnic at Avalon Park*

Sunday, May 10

8:30am *Coffee and bagels in math common room, Math 4-125*

9:00am **Yair Minsky** (Yale University)
Windows, cores and skinning maps

10:00am *Coffee in Simons Center Café*

10:30am **Jeremy Kahn** (CUNY Graduate Center)
Elephants and Renormalization

11:40am **Kevin Pilgrim** (Indiana University Bloomington)
Mapping class semigroups and new characterizations of rational functions

1:00pm *Afternoon free*

Monday, May 11

8:30am *Coffee and bagels in math common room, Math 4-125*

9:00am **John Hubbard** (Cornell University)
When is the deformation space $\mathrm{Def}(A, B)$ ***nonempty? In that case, it is contractible.***

10:00am *Coffee in Simons Center Café*

10:30am **Romain Dujardin** (Université Paris-Est Marne-la-Vallée)
Holomorphic motions in two dimensions

11:45am **Adam Epstein** (University of Warwick)
Magic Formulas and Mating Discontinuities

12:45pm *Break for lunch*

1:45pm **Poster Session**, *SCGP lobby*

2:30pm **Welington de Melo** (IMPA)
Rigidity of critical circle maps

3:30pm *Tea in Simons Center Café*

4:00pm **Michael Yampolsky** (University of Toronto)
Renormalization and rotational attractors of two-dimensional dissipative dynamical systems

6:30pm *Banquet at Lombardi's on the Sound*

Tuesday, May 12

8:30am *Coffee and bagels in math common room, Math 4-125*

9:00am **John Smillie** (University of Warwick)
XXV years of the complex Hénon map

10:00am *Coffee in Simons Center Café*

10:30am **Carsten Lunde Petersen** (Roskilde Universitet)
On quasi-conformal (in-)compatibility of satellite copies of the Mandelbrot set

11:40am **Sebastian van Strien** (Imperial College London)
Quasi-symmetric rigidity for real maps which are not real analytic

Abstracts of Lectures

listed alphabetically by speaker

Xavier Buff: *Wandering domains for polynomial endomorphisms of $\mathbb{C}^2$*

Saturday, May 9. 9:00am

We prove the existence of polynomial endomorphisms of $\mathbb{C}^2$ having a wandering Fatou component. The proof relies on techniques of parabolic implosion and is based on an original idea of Lyubich.
Joint work with Astorg, Dujardin, Peters and Raissy.

Araceli Bonifant: *Points visible from zero and infinity for antipode preserving maps*

Saturday, May 9. 11:45am

This talk will describe some of the tools used for studying antipode preserving cubic rational maps with a fixed critical point at zero. One feature of such maps is that there are often points in the Julia set which are "visible" both from zero and infinity. Such points give rise to a concept of "rotation number" which helps to organise the parameter plane. They are also a basic tool for constructing the prominent "fjords" which cut the parameter plane into uncountably many connected components.

Laura DeMarco: *Complex dynamics and elliptic curves*

Saturday, May 9. 4:00pm

In this talk, I will explain some connections between recent research in complex dynamics and the classical theory of elliptic curves and rational points. I will begin with a theorem of Mordell and Weil from the 1920s, presented from a dynamical point of view. I hope to explain how we have exploited this connection to prove new results about elliptic curves.

Romain Dujardin: *Holomorphic motions in two dimensions*

Monday, May 11. 10:30am

The classical concept of a holomorphic motion underlies the notion of dynamical stability in one-dimensional holomorphic dynamics. It is well known that this idea does not extend readily to higher dimensions.

In a joint work with Misha Lyubich, we have defined a notion of weak stability for a family of polynomial diffeomorphisms of $\mathbb{C}^2$. In such a weakly stable family, the Julia set(s) move under a branched holomorphic motion, that is, a holomorphic motion with possible collisions. We have shown that this motion is unbranched at certain special points such as periodic or heteroclinic points, and it remains an open question whether the whole Julia set moves holomorphically (i.e. without collisions).

In collaboration with Pierre Berger, we introduce a notion of regular point inspired by Pesins theory and show that in a weakly stable family, the set of regular points moves holomorphically. It follows that a weakly stable family is structurally stable in a measure theoretic sense. Another consequence of these techniques is that weak stability preserves uniform hyperbolicity.

In the talk I will review these topics and related questions.

Adam Epstein: *Magic formulas and mating discontinuities*

Monday, May 11. 11:45am

John Hubbard: *When is the deformation space* $\mathrm{Def}(A, B)$ *nonempty? In that case, it is contractible.*

Saturday, May 9. 10:15am

Jeremy Kahn: *Elephants and Renormalization*

Sunday, May 10. 10:30am

I will consider the problem of finding bounds for renormalization where the parameter comes a sequence of small Mandelbrot sets that occupy the same place in the "elephants" that are marching out of the cusp of the central cardioid. I will describe my work with Mikhail Lyubich on the case where the period of the renormalization is a constant plus the period of the limb, and then briefly outline our proposed approach to the more general case.

Vadim Kaloshin: *On quasi-ergodic hypothesis and renormalization for nearly integrable systems*

Friday, May 8. 2:45pm

The famous ergodic hypothesis claims that a typical Hamiltonian dynamics on a typical energy surface is ergodic, however, KAM theory disproves this. It establishes a persistent set of positive measure of invariant KAM tori. The (weaker) quasi-ergodic hypothesis, proposed by Ehrenfest and Birkhoff, says that a typical Hamiltonian dynamics on a typical energy surface has a dense orbit. This question is wide open.

During the talk we present a progress toward the quasi-ergodic hypothesis. One of the key elements of our analysis isrenormalisation near diophantine frequencies. This is a joint work with M. Guardia and essentially based on a joint work with K. Zhang and P. Bernard.

Welington de Melo: *Rigidity of critical circle maps*

Monday, May 11. 2:30pm

Two critical circle maps of class C^3 with the same irrational rotation number and the same odd criticality are C^1 conjugate. This is a joint work with Pablo Guarino and Marco Martens.

Yair Minsky: *Windows, cores and skinning maps*

Sunday, May 10. 9:00am

Thurston's skinning map played an important role in his original hyperbolization theorem. His Bounded Image Theorem, in the acylindrical case, gave control of the fixed-point problem on which the proof depends. We generalize this theorem to a relative version which holds for manifolds with cylinders, and along the way refine our understanding of how one constructs compact cores with uniform topological and geometric control in degenerating sequences of hyperbolic structures.

This is joint work with J. Brock, K. Bromberg and R. Canary.

Carsten Lunde Petersen: *On quasi-conformal (in-)compatibility of satellite copies of the Mandelbrot set.*

Tuesday, May 12. 10:30am

In the paper "*On the dynamics of polynomial-like mappings*" Douady and Hubbard introduced the notion of polynomial-like maps. They used it to identify copies M' of the Mandelbrot set inside the Mandelbrot set M. They conjectured that the primitive copies, which are characterized by having a cusp and a root for which the parabolic multiplier is equal to 1, are quasi-conformally homeomorphic to M. This is now a theorem due to Lyubich. The satellite copies M', which are characterized by having a smooth round main component and a root for which the parabolic multiplier is a q-th root of unity for some $q > 1$, are clearly not q-c homeomorphic to M. But are they mutually q-c homeomorphic? Or even q-c homeomorphic to half of the logistic Mandelbrot set?

In this talk I will present a proof that two satellite copies M' and M'' are not q-c homeomorphic, if the root multipliers are q- and q'-roots of unity with $q \neq q'$ and in particular no satellite copy is qc-homeomorphic to half of the logistic Mandelbrot set.

Joint work with Luna Lomonaco Univ. of São Paulo.

Kevin Pilgrim: *Mapping class semigroups and new characterizations of rational functions*

Sunday, May 10. 11:40am

I will give a panoramic picture of new developments related to fundamental combinatorial classification questions related to the dynamics of iterated rational maps.

Mitsuhiro Shishikura: *Toward arithmetic surgery of rational maps*

Friday, May 8. 1:00pm

Consider a family of rational maps f_t parametrized by a small parameter t. Suppose that f_t degenerates to a lower degree map (or a constant map) as $t \to 0$. For example, a stretching deformation of non-simply connected Fatou components gives rise to such a family, and one can associate a tree and piecewise linear map on it. Conversely, given such a tree and a map together with "local dynamics around the vertices, one can reconstruct a rational map via surgery. In this talk, we discuss an alternative method for such a construction using power series expansion in t. We hope that this method gives a construction for J. Kiwi's work on the dynamics over the Puiseux series field and M. Arfeux's work on the dynamical compactification of the moduli space.

Dierk Schleicher: *Newton's method as an unexpectedly efficient root finder*
Friday, May 8. 2:30pm

Newton's method is well known as a root finder locally near the roots. It is often "not recommended" as a global root finder because of its chaotic properties. We give a very efficient theoretical upper bound on its speed of con vergence: all roots of a degree d polynomial can be found with accuracy ε in $O(d^2 \log^4 d + d \log\log|\varepsilon|$ Newton iterations in the expected case very close to the theoretical upper bound of $O(d^2)$ for this method. In practice, all roots of polynomials of degree more than a million are found routinely and in $(4 \ln 2)d^2$ iterations in practice, this means less than a day for degree a million on standard laptops. A modified method, for which we do not have a complete theory, brings this down do $3d \log^2 d$ iterations, which in practice is much faster than optimized software packages for specific polynomials, and all roots of degree a million are found routinely in a matter of minutes.

John Smillie: *XXV years of the complex Hénon family*
Tuesday, May 12. 9:00am

A few more than twenty five years ago John Hubbard suggested that the next frontier in complex dynamics was the study of a certain family of polynomial diffeomorphisms of $\mathbb{C}^2$ which he called the complex Hénon map. He made a good case and a number of mathematicians were convinced to start working on this family. In this talk I would like to give an update on the health of the field and describe some recent joint work with Eric Bedford.

Sebastian van Strien: *Quasi-symmetric rigidity for real maps which are not real analytic*
Tuesday, May 12. 11:40am

In this talk I will discuss some of the issues which need to be overcome when proving quasi-symmetirc rigidity for maps which are not real analytic (joint work with Trevor Clark).

Barak Weiss: *Dynamics of the horocycle flow on the eigenform locus*

Saturday, May 9. 10:30am

Let $G = SL(2, \mathbb{R})$ and let U be its upper triangular unipotent subgroup. Motivated by an analogy with the theory of homogeneous flows, in 2005 McMullen classified the G-invariant measures and orbit-closures on strata of translation surfaces in genus 2, and his results have recently been extended to all genera in a breakthrough result of Eskin, Mirzakhani and Mohammadi. In McMullen's genus 2 result, countably many unexpected G-invariant loci appear – the eigenform loci. These were described by Calta and by McMullen independently and by different methods. A central problem in this domain is to extend the classification results above to the restriction to U (the horocycle flow). In joint work with Bainbridge and Smillie, we study the restriction of the U-action to the eigenform loci. We complete the classification of measures initiated by Calta and Wortman, classify the orbit-closures and prove several equidistribution results.

Michael Yampolsky: *Renormalization and rotational attractors of two-dimensional dissipative dynamical systems*

Monday, May 11. 4:00pm

We study dissipative rotational attractors in two settings: Siegel disks of Hénon maps and minimal attractors of diffeomorphisms of the annulus. Jointly with D. Gaydashev, we extend renormalization of Siegel maps and critical circle maps to small 2D perturbations, and use renormalization tools to study the geometry of the attractors. In the Siegel case, jointly with D. Gaydashev and R. Radu we prove that for sufficiently dissipative Hnon maps with semi-Siegel points with golden-mean rotation angles, Siegel disks are bounded by (quasi)circles. In the annulus case, jointly with D. Gaydashev, we prove that for bounded type rotation number, critical annulus maps have a minimal attractor which is a C^0, but not smooth, circle – answering a question of E. Pujals.

Abstracts of Posters

listed alphabetically by author

Jonny Ardila: *On formal first integrals for singularities of complex vector fields*

In a previous work, it was shown that for germs of foliations of dimension 1 in $(\mathbb{C}^3, 0)$ (under the certain conditions), the existence of a formal first integral implies the existence of a holomorphic one, something already known in $\mathbb{C}^2$. In this poster we show the same result in a more general context which, was obtained thanks to the study of groups of diffeomorphism in $(\mathbb{C}^n, 0)$ finitely generated (continuing with the ideas of F. E. Brochero Martínez) where, some interesting results were also found.

Stefanie Hittmeyer: *Generalised Mandelbrot and Julia sets in a nonanalytic perturbation of the quadratic family*

We consider a two-dimensional noninvertible map that was introduced by Bamon, Kiwi and Rivera in 2006 as a model of wild Lorenz-like chaos. The perturbation opens up the critical value to a disk; the bounding critical circle and its images, together with the critical point and its preimages, form the critical set. As the perturbation increases, saddle points and their stable and unstable sets appear. As parameters are varied the stable, unstable and critical sets interact with (a generalised notion of) the Julia set. This leads to the (dis)appearance of saddle points and chaotic attractors and to dramatic changes in the topology of the Julia set. In particular, we find generalised Julia sets in the form of Cantor bouquets, Cantor tangles and Cantor cheeses. Continuation of these bifurcations in two parameters reveals a self-similar bifurcation structure near the period-doubling route to chaos of the complex quadratic family. In addition, we present a generalised version of the Mandelbrot set that encodes the conjectured trichotomy in the nonanalytic setting.

Mikhail Hlushchanka: *Computation and Characterization of Iterated Monodromy Groups of Expanding Thurston Maps*

Nekrashevich's approach to computation and understanding of iterated monodromy groups (IMGs) of polynomials is based on existence of an invariant star-like tree centered at infinity (called an invariant spider). Nekrashevych managed to describe a special class of automata, the kneading automata, such that the IMG of every post-critically finite polynomial is generated by an automaton in the class.

There is no known class of such automata for the rational case. Bonk and Meyer (and also Haïssinsky-Pilgrim) studied dynamical properties of a certain subclass of post-critically finite branched covering maps called expanding Thurston maps. We managed to prove existence of an invariant star-like tree for an iterate of an expanding Thurston map which has a periodic critical point. This tree let us to compute and characterize automata generating IMGs of expanding Thurston maps.

Matthew Jacques : *Continued fractions and semigroups of Mbius transformations.*

Motivated by the theory of Kleinian groups and recent work of Fried, Marotta and Stankewitz, we show how semigroups of Möbius transformations can be used to answer natural questions about the convergence of continued fractions. In particular, for any compact set X that does not contain 0, we find necessary and sufficient conditions for the convergence of every continued fraction with coefficients chosen from X.

Mario Martinez-Rios: *Boundary of Siegel disks in cubic and quartic polynomials*

We show arithmetic conditions on the rotation number in a polynomial family that guarantee the existence of a Siegel disk with critical points on the boundary and that the Julia set is locally connected.

Anca Radulescu: *Julia sets for networks and templates*

Behavior under iterations of quadratic maps has been one of the earliest and most studied topics in discrete dynamics, in both the real and complex case. However, many subtler aspects of discrete dynamics centered around the behavior of logistic maps remain largly unexplored.

For example, while iterations of a single map have been exhaustively studied, less effort has been directed towards addressing what happens when the map itself evolves in time according to a symbolic template and (2) when the maps are organized as nodes in a network, and interact in a time-dependent fashion. We investigate how the traditional theory changes in these cases, illustrating how the hardwired structure (e.g., symbolic template, or respectively adjacency graph) can affect dynamics (behavior of orbits, topology of Julia set, etc.)

This is of potential interest to a variety of applications (including genetic and neural coding), since (1) it investigates how an occasional or a reoccurring error in a replication or learning algorithm may affect the outcome and (2) it relates to algorithms of synaptic restructuring and neural dynamics in brain networks.

Hiroki Sumi: *Multifractal Analysis for the Complex Analogues of the Devil's Staircase and the Takagi Function in Random Complex Dynamics*

We consider random dynamical systems of rational maps on the Riemann sphere. We investigate the function T of probability of tending to one minimal set and its partial derivative C with respect to the probability parameter. It turns out that under certain conditions, the function T is a complex analogue of the devil's staircase or Lebesgue's singular functions and the function C is a complex analogue of the Takagi function. Those functions T and C are continuous on the Riemann sphere and vary precisely on the Julia set of the associated semigroup, which is a thin fractal set. We investigate the multifractal analysis for the pointwise Holder exponents of these functions. This is a joint work with Johannes Jaerisch (Shimane University, Japan).

Hiroki Takahasi: *Large deviation principle for one-dimensional maps under weak hyperbolicity assumptions*

We study the dynamics of a smooth multimodal interval map f with non-flat critical points and all periodic points hyperbolic repelling. Assuming that $|Df^n(f(c))| \to \infty$ as $n \to \infty$ holds for each critical point c, and additional minor conditions, we establish the large deviation principle for empirical distributions.

Joint work with Yong Moo Chung (Hiroshima University) and Juan Rivera-Letelier (PUC-Chile).

Garima Tomar: *Escaping sets of composition of transcendental entire functions*

In dynamical systems, the escaping set is an interesting concept as we deal with the points which go to infinity after a number of iterations. It is denoted by $I(f)$. Mathematically,

$$I(f) = \Big\{ z \in \mathbb{C} : f^n(z) \to \infty \text{ as } n \to \infty \Big\}.$$

Moreover, we also discuss fast escaping sets which includes the points which go to infinity as fast as possible. Mathematically,

$$A(f) = \Big\{ z \in I(f) : \text{there exits } L \in N \text{ such that } |f^{n+L}(z)| \geq M^n(R, f) \Big\}.$$

Here, the boundary of $I(f)$ as well as $A(f)$ is the Julia set. Various types of escaping sets and an intricate structure formation which occurs with some imposed conditions on these sets namely Spiders Web are discussed.