

John Milnor – Abel Lecture

Master's mathematical vision embraces humanity

John Milnor, a professor of mathematics at Stony Brook University, U.S.A., is one of the best known mathematicians to Koreans partly because of his sweep of major awards in mathematics. He won the Fields Medal in 1962, the Wolf Prize in 1989 and the Abel Prize in 2011.

The Norwegian Academy of Science and Letters bestowed the Abel Prize on him for his lifetime achievements in topology, geometry and algebra.

The 83-year-old scholar, who is also known for his gift of communicating difficult mathematics to general audiences, gave the Abel Lecture on Friday, Aug. 15, during the SEOUL ICM 2014.

The same day, he sat down for an interview moderated by Jongil Park, a professor of mathematics at Seoul National University.

Q. You won three major awards in mathematics. Which prize are you most proud of?

A. It's very nice to be appreciated; I don't want to compare. I got the prizes at different times. I suppose it was more startling when I was young, but more wonderful to be appreciated when I was elderly.

You proved what is now known as the Fary-Milnor Theorem in knot theory at a young age. When did you start working on complex mathematics?



John Milnor, a professor of mathematics at Stony Brook University in the United States, at an interview on Friday, shortly before his Abel lecture at the SEOUL ICM 2014.

This was a problem that the professor described in class, and it was very intriguing. I had been studying mathematics for a few years at that point and had some idea of work methods. There is always more to learn in mathematics. Like the Red Queen, we have to keep running.

You are currently focused on dynamical systems. Could you explain it in layman's term?

Dynamics is a very general subject,

which can be applied in many different areas. I certainly don't want to make it sound as if I am contributing to all of these areas. I work in a narrow part of the field, just trying to understand very simple mathematical models that give rise to very complex behaviors.

Many physical systems are extremely complicated. The only way to analyze them is by experiments with very fast and large computers, but one can often get ideas of general principles from study-

ing much simpler systems. That is the approach I take, driven by mathematical curiosity and trying to understand some of the simplest systems. Even though they look simple, they can induce very complicated behaviors which are difficult to understand.

You are known as one of the best writers among mathematicians. Many engineering and science students in Korea have difficulty in writing. How did you become a good writer?

Well, most of my writing has been driven by my desire to understand something. In order to understand something, I have to write it down. If I write it down clearly enough that I can understand it, then other people will be able to understand it. But, it takes a great deal of care to translate an idea into words. So, typically I have to write things over and over again.

Why is mathematics important?

I think it was the physicist Eugene Wigner who said that mathematics seems to miraculously describe things in the real world. Often, mathematics is developed before it is needed, and turns out to be useful afterwards. At other times, mathematics has been developed because it was clearly needed for a given purpose.

Achievements

John Milnor spent his undergraduate and graduate student years at Princeton University, studying knot theory under the supervision of Ralph Fox. He has worked in many areas of mathematics, including game theory, differential geometry, algebraic and differential topology, algebraic K-theory and dynamical systems.

His profound ideas in mathematics have had a great influence on the mathematical community from the second half of the 20th century until now. One of his most celebrated works is his proof in 1956 of the existence of 7-dimensional spheres with several nonstandard differentiable structures.

In 1959, he showed that the diffeomorphism classes of oriented exotic spheres of any dimension form the non-trivial elements of an abelian monoid under connected sum, which is a finite abelian group if the dimension is not 4. (An exotic sphere is a differentiable manifold M that is homeomorphic but not diffeomorphic to the standard Euclidean n -sphere.) The classification of exotic spheres by Michel Kervaire and Milnor in 1963 showed that the oriented exotic 7-spheres are the non-trivial elements of a cyclic group of order 28 under the operation of connected sum.

Subsequently, he worked on the topology of isolated singular points of complex hypersurfaces in general,

developing the theory of the Milnor fibration, whose fiber has the homotopy type of a bouquet of μ spheres, where μ is known as the Milnor number. All of his work in topology, geometry and algebra displays features of great research: salient facts, profound insights, vivid imagination, striking surprises and supreme beauty.

Numerous mathematical concepts, results and conjectures are named after him, including the Fary-Milnor theorem, Milnor exact sequence, Hilton-Milnor theorem, Milnor conjecture in algebraic K-theory and also in knot theory, Milnor construction, Milnor K-theory, Milnor fibration, Milnor number and Milnor sphere.

Seven volumes of his collected papers have been published by the American Mathematical Society.

He has also written many influential books, which include "Differential Topology," "Morse Theory," "Lectures on the h-Cobordism Theorem," "Singular Points of Complex Hypersurfaces," "Introduction to Algebraic K-Theory," "Dynamics in One Complex Variable" and "Characteristic Classes" (with J. Stasheff).



Compiled by
Dae-Woong Lee
Chonbuk National
University