Conference Schedule

Wednesday, May 25

6:00pm *Wine and Cheese Reception*

Thursday, May 26

9:30am **Stanislav Smirnov** (Université de Genève & St. Petersburg State University)
CFT and SLE

10:30am *Break (refreshments on 2^{nd} floor)*

11:00am **Ib Madsen** (University of Copenhagen)
Diffeomorphism groups from a homotopical viewpoint I

noon *Lunch break*

2:00pm **Kevin Costello** (Northwestern University)
Supersymmetric, holomorphic and topological field theories in dimensions two and four, I

3:15pm **Moira Chas** (Stony Brook University)
String topology and three-manifolds

4:15pm *Break (refreshments on 2^{nd} floor)*

4:45pm **Jim Simons** (Stony Brook Univeristy)
Dennis saves the day

Friday, May 27

9:30am **Ib Madsen** (University of Copenhagen)
Diffeomorphism groups from a homotopical viewpoint II

10:30am *Break (refreshments on 2^{nd} floor)*

11:00am **Ralph Cohen** (Stanford University)
String topology and the classification of topological field theories

noon *Lunch break*

2:00pm **John Morgan** (Stony Brook University)
Rational homotopy theory I

3:15pm **Kenji Fukaya** (Kyoto University)
Open closed Gromov-Witten theory and its application to symplectic topology

4:15pm *Break (refreshments on 2^{nd} floor)*

4:45pm **Kevin Costello** (Northwestern University)
Supersymmetric, holomorphic and topological field theories in dimensions two and four, II

5:45pm *Dinner break*

8:30pm *Evening Session*

Leonid Chekhov (Steklov Mathematics Institute)
Quantum Riemann surfaces related to solutions of the Schroedinger equation

Charles Tresser (IBM)
Manifold structure OF physics vs. the emergence and decay of geometry IN physics

Saturday, May 28

9:30am **Ib Madsen** (University of Copenhagen)
Diffeomorphism groups from a homotopical viewpoint III

10:30am *Break (refreshments on 2^{nd} floor)*

11:00am **Kevin Costello** (Northwestern University)
Supersymmetric, holomorphic and topological field theories in dimensions two and four, III

noon *Lunch break*

2:00pm **John Morgan** (Stony Brook University)
Rational homotopy theory II

3:15pm **Steve Halperin** (University of Maryland)
The growth and Lie structure of the rational homotopy groups of a finite dimensional complex

4:15pm *Break (refreshments on 2^{nd} floor)*

4:45pm **Jim Stasheff** (University of North Carolina at Chapel Hill)
How Dennis and I intersected

7:00pm *Party at Myong-Hi Kim's house (87 Ridgeway, Setauket)*

Sunday, May 29

9:00am **Mohammed Abouzaid** (MIT)
On the symplectic topology of cotangent bundles

10:00am *Break (refreshments on 2^{nd} floor)*

10:30am **John Morgan** (Stony Brook University)
Rational homotopy theory II

11:45am **Graeme Segal** (Oxford University)
Fields, deformations, and the axiomatization of quantum field theory

Monday, May 30

9:30am **Alexander Shnirelman** (Concordia University)
Fluid dynamics I

10:30am *Break (refreshments on 2^{nd} floor)*

11:00am **Boris Khesin** (University of Toronto)
Symplectic fluids and point vortices

noon *Lunch break*

2:00pm **Gerard Misiolek** (University of Notre Dame)
Geometry of diffeomorphism groups and H^1 optimal transport

3:15pm **Bruce Kleiner** (Courant Institute)
Hyperbolic groups and analysis on metric spaces I

4:15pm *Break (refreshments on 2^{nd} floor)*

4:45pm **Jeff Cheeger** (Courant Institute)
Quantitative differentiation

6:45pm *buffet dinner (on 2^{nd} floor)*

7:30pm *"Sullivan's Circle" evening session*

Xiaojun Chen (University of Michigan)
Some string topology inspired structures in symplectic topology

Somnath Basu (Stony Brook University)
What is transversal string topology?

Michael Sullivan (University of Massachusetts)
Transverse string topology and knots

Scott Wilson (Queens College, CUNY)
Equivariant extensions of holonomy and secondary invariants

Tuesday, May 31

9:30am **Alexander Shnirelman** (Concordia University)
Fluid dynamics II

10:30am *Break (refreshments on* 2^{nd} *floor)*

11:00am **Steve Preston** (University of Colorado)
Fredholmness of Riemannian exponential maps on diffeomorphism groups

noon *Lunch break*

2:00pm **Claude Bardos** (University of Paris)
Boundary effects and turbulence

3:15pm **Bruce Kleiner** (Courant Institute)
Hyperbolic groups and analysis on metric spaces II

4:15pm *Break (refreshments on* 2^{nd} *floor)*

4:45pm **Mario Bonk** (University of California, Los Angeles)
Expanding Thurston maps

6:30pm *Banquet at Lombardi's on the Sound*

Wednesday, June 1

9:00am **Alexander Shnirelman** (Concordia University)
Fluid dynamics III

10:00am *Break (refreshments on* 2^{nd} *floor)*

10:30am **Bruce Kleiner** (Courant Institute)
Hyperbolic groups and analysis on metric spaces III

11:45am **Richard Canary** (University of Michigan)
Sullivan's dictionary I

Thursday, June 2

9:30am **Peter Jones** (Yale University)
Product formulas for measures and applications to analysis and geometry

10:30am *Break (refreshments on 2^{nd} floor)*

11:00am **Misha Lyubich** (Stony Brook University)
Renormalization I

noon *Lunch break*

2:00pm **Michael Yampolsky** (University of Toronto)
Renormalization of critical circle mappings and related topics

3:15pm **Richard Canary** (University of Michigan)
Sullivan's dictionary II

4:15pm *Break (refreshments on 2^{nd} floor)*

4:45pm **David Gabai** (Princeton University)
Shrinkwrapping and the taming of hyperbolic 3-manifolds

5:45pm *Dinner break*

8:00pm *Evening Session*

Qian Yin (University of Michigan)
Lattes maps and combinatorial expansion

Daniel Smania (ICMC-USP)
Renormalization operator for multimodal maps

Mike Shub (Universidad de Buenos Aires and CUNY)
On the convexity of the condition number in the condition metric

Friday, June 3

9:30am **Ken Bromberg** (University of Utah)
The Bers-Thurston-Sullivan density conjecture

10:30am *Break (refreshments on 2^{nd} floor)*

11:00am **Misha Lyubich** (Stony Brook University)
Renormalization II

noon *Lunch break*

2:00pm **Jeremy Kahn** (Stony Brook University)
Renormalization: Past, Present and Future

3:15pm **Richard Canary** (University of Michigan)
Sullivan's dictionary III

4:15pm *Break (refreshments on 2^{nd} floor)*

4:45pm **Curt McMullen** (Harvard University)
Algebraic patterns for dynamical systems

7:00pm *Party at Jim Simons' house*

Saturday, June 4

9:00am **Yair Minsky** (Yale University)
Ending laminations, classification and models for hyperbolic 3-manifolds

10:00am *Break (refreshments on 2^{nd} floor)*

10:30am **Marco Martens** (Stony Brook University)
Universality and rigidity in low dimensional dynamics

11:45am **Mitsuhiro Shishikura** (Kyoto University)
Renormalizations and the Teichmüller theory

Abstracts of Minicourses

listed alphabetically by speaker

Misha Lyubich: *Renormalization*

Thursday, June 2 11:00am. Friday, June 3 11:00am. .

In the introductory lecture a general overview of the idea of renormalization and its various incarnations in low-dimensional dynamics will be given.

One of the main themes in low-dimensional dynamics is the investigation of the interplay between order (periodic or KAM behavior) and chaos (nonuniform hyperbolicity). In the best understood cases, the analysis involves the description of the dynamics of a renormalization operator acting on parameter space and presenting an attractor.

We will discuss two incarnations of this general idea. The first concerns unimodal dynamics, and we will focus on the proof of the existence of a global renormalization attractor (its applications, such as the "regular or stochastic dichotomy", being discussed in the first talk). The second concerns perhaps the simplest class of dynamical systems compatible with both KAM behavior and nonuniform hyperbolicity: one-frequency cocycles. We will explain the emerging global picture for the parameter space and its application to certain Schrödinger operators (the "spectral dichotomy"), and then describe the role played by a (non-longer global) renormalization attractor in the measure-theoretical analysis of the phase-transition.

Richard Canary: *Sullivan's dictionary*

Wednesday, June 1 11:45am. Thursday, June 2 3:15pm. Friday, June 3 3:15pm.

Talk 1: Kleinian groups and the Sullivan Dictionary I

The Sullivan dictionary provides a conceptual framework for understanding the connections between the dynamics of rational functions and Kleinian groups. We will survey the basic theory of Kleinian groups with an emphasis on the quasiconformal deformation theory where the analogies between the two theories are closest.

We then discuss the three major conjectures which drove many of the major developments in Kleinian groups since Thurston revolutionized the field in the 1970s. Marden's Tameness Conjecture predicts that every hyperbolic 3-manifold with finitely generated fundamental group is topologically tame, i.e. homeomorphic to the interior of a compact 3-manifold. Thurston's Ending Lamination Conjecture proposed a classification of all hyperbolic 3-manifolds with finitely generated fundamental group. The Bers-Sullivan-Thurston Density Conjecture asserted that every finitely generated Kleinian group is a limit of geometrically finite Kleinian groups. Each of these conjectures has been resolved in the last decade.

Talk 2: Kleinian groups and the Sullivan dictionary II

We will continue our discussion of the three major conjectures. We will then discuss applications of Marden's Tameness Conjecture. We will focus on dynamical applications, the most prominent of which is the resolution of Ahlfors' Measure Conjecture. We will also discuss applications to the dynamics of geodesic flows of hyperbolic 3-manifolds and to limit sets of Kleinian groups. Marden's Tameness conjectures also has topological and group-theoretic applications which we will discuss if time permits.

Talk 3: Kleinian groups and the Sullivan dictionary III

We discuss the space $AH(M)$ of (marked) hyperbolic 3-manifolds homotopy equivalent to a fixed compact 3-manifold M. One may view this as a natural generalization of Teichmueller space in the 3-dimensional setting. The resolution of Thurston's Ending Lamination Conjecture gives a classification of the manifolds in $AH(M)$, but the topology of $AH(M)$ remains elusive since the invariants in this classification do not vary continuously over $AH(M)$. We will survey recent work which shows that the topology of $AH(M)$ is actually quite pathological.

$AH(M)$ naturally sits inside the character variety $X(M)$ of conjugacy classes of representation of the fundamental group of M into $PSL(2, \mathbb{C})$. The outer automorphism group of the fundamental group of M acts naturally on both $AH(M)$ and $X(M)$. If time permits, we will discuss recent work on the dynamics of this action.

Kevin Costello: *Supersymmetric, holomorphic and topological field theories in dimensions 2 and 4*
Thursday, May 26 11:45am. Friday, May 27 3:15pm. Saturday, May 28 11am.

Applications of quantum field theory to mathematics often involve topological twists of supersymmetric field theories. However, for a mathematician, the physics presentation of supersymmetric field theories and their twists can be difficult to understand.

In this series of talks, I will explain how many of the twisted supersymmetric field theories of relevance in mathematics have very natural interpretations in terms of derived geometry. I will, in particular, explain how to construct the A and B models of mirror symmetry, and the P^1 of twisted $N = 4$ gauge theories which appear in Kapustin and Witten's work on the Langlands program.

Bruce Kleiner: *Hyperbolic groups and analysis on metric spaces*
Monday, May 30 3:15pm. Tuesday, May 31 3:15pm. Wednesday, June 1 10:30am.

These lectures will survey some developments connecting analysis on metric spaces with the asympototic geometry of Gromov hyperbolic spaces. The roots of this topic go back to Mostow rigidity on the group theory side, and the classical theory of quasiconformal homeomorphisms on the analytical side. Seminal work by Heinonen-Koskela and Cheeger in the late 90's created the possibility of extending the classical framework for quasiconformal/hyperbolic geometry to the much broader setting of metric measure spaces, with potential applications to group theory and rigidity. After reviewing the relevant background, the lectures will cover the subsequent progress along these lines.

Ib Madsen: *Diffeomorphism groups from a homotopical viewpoint*
Thursday, May 26 11:00am. Friday, May 27 9:30am. Saturday, May 28 9:30am.

The first lecture will concentrate on surgery with special emphasis on Sullivan's contributions: the surgery exact sequence, calculation of normal invariants, and the Adams conjecture. The next lectures will outline the solution of the generalized Mumford conjecture about the stable homology of the mapping class group and a recent generalization thereof, namely the stable homological structure of diffeomorphism groups of $(d-1)$-connected $2d$-dimensional manifolds for large d. The latter uses the techniques from the surface case, surgery and Waldhausen's algebraic K-theory of spaces.

John Morgan: *Rational homotopy theory*
Friday, May 27 2:00pm. Saturday, May 28 2:00pm. Sunday, May 29 9:00am.

Dennis' work on rational homotopy theory had several motivations. One was to give a fairly simply algebraic category that is equivalent to the rational homotopy category. It was known that there were models for rational homotopy theory that were elegant algebraic categories, for example Quillen's category of differential graded Lie algebras. Dennis, for reasons having to do with geometric applications, wanted a model closer to differential forms. This turned out to be the category of (connected) differential graded algebras over $\mathbb{Q}$, up to quasi-isomorphism. A central ingredient of his construction was the notion of a minimal, free object in every equivalence class, an object he called the minimal model. It turns out that the minimal model faithfully reflects the (rational) Postnikov tower of the space, so for example it is easy to read off the homotopy groups and even the k-invariants from the minimal model.

Another part of the program was to connect this construction of a category equivalent to the homotopy category to the differential forms on a manifold, so that one could use the differential forms to extract not just cohomological information but also homotopy theoretic information. For this one needed to construct differential forms over $\mathbb{Q}$ for any simplicial complex and show their minimal model was the one associated to the Postnikov tower of the space. These are the so-called piecewise polynomial forms, which can be defined over $\mathbb{Q}$. The last step was to connect smooth differential forms on a smooth manifold to the piecewise polynomial forms on some smooth triangulation. Once this bridge had been established one could apply results about differential forms to establish homotopy theoretic results. Maybe the most striking of these is the applications to compact Kahler manifolds, e.g. non-singular projective varieties and more generally to open non-singular varieties.

In these three lectures, we will give some of the homotopy theoretic background, describing the difference between rational and integral homotopy theory, explaining the significance of the Steenrod squaring operation and the closely related fact that the Whitney cochain cup product is not commutative. We will also talk about A_∞ structures and some of the recent developments in integral homotopy theory. We will introduce the minimal model and show how it is related to the Postnikov tower of a space. We will introduce the $\mathbb{Q}$ piecewise polynomial forms on a simplicial complex and show their relationship with the rational homotopy type of the space. Then we will turn to geometric applications especially the results on Kahler manifolds, arising from the Hodge decomposition on cohomology and the so-called d-d bar lemma.

Alexander Shnirelman: *Fluid dynamics*
Monday, May 30 9:30am. Tuesday, May 31 9:30am. Wednesday, June 1 9:00am.

Lecture 1: General notions

Group D of volume preserving diffeomorphisms; D as a Riemannian manifold; the Least Action Principle; geodesics on D, Lagrange and Euler equations; conservation laws and vorticity equations; local existence theorem for the Euler equations; singularity problem; known results.

Lecture 2: Long-time behavior of 2-d flows

Global solvability in the 2-d case; partial analyticity of solutions; stability of steady flows; Arnold stable and minimal flows; irreversibility of 2-d fluid dynamics; Liapunov functions and wandering domains; Generalized Minimal Flows as final states.

Lecture 3: Weak solutions of the Euler Equations

Formal definition; paradoxical weak solutions; energy dissipation due to the irregularity of weak solutions; construction of energy dissipating weak solutions; existence problem of physically meaningful weak solutions.

Abstracts of Lectures

listed alphabetically by speaker

Mohammed Abouzaid: *On the symplectic topology of cotangent bundles*

Sunday, May 29. 10:30am

I will discuss progress in our understanding of the symplectic topology of cotangent bundles. Some of the techniques involve unravelling the connections between Floer theory in the cotangent bundle and string topology on the base.

Claude Bardos: *Boundary effects and turbulence*

Tuesday, May 31. 2:00pm

The high Reynolds number limit of solutions of Navier-Stokes equation is, in the presence of boundary, a challenging open problem. Very few results do exist and one of the more mathematical is a remark of Kato who shows that anomalous dissipation of energy is in this situation closely related to the apparition of turbulence.

I will elaborate on these ideas both at the level of Navier-Stokes and Boltzmann limit and argue that boundary effects may be the most natural explanation for turbulence.

Somnath Basu: *What is transversal string topology?*

Monday, May 30. 8:10pm

We consider smooth paths in $M \times M$ that start and end on the diagonal and only intersect the diagonal transversally, including the end points. Such strings can be naturally split at the intersection points giving rise to a differential graded coalgebra. We'll analyze where this coalgebra lives and discuss further algebraic structures in this setting. Time permitting, we'll apply this to probe the homotopy type of the complement of the diagonal in $M \times M$ which is known not to be an invariant of the homotopy type of M.

Mario Bonk: *Expanding Thurston maps*

Tuesday, May 31. 4:45pm

A Thurston map f is a branched cover of a 2-sphere for which the forward orbit of each critical point under iteration is finite. If the inverse branches of the iterates f^n shrink distances at an exponential rate as $n \to \infty$, then we call f expanding. The study of expanding Thurston maps is connected to areas such as dynamical systems, classical conformal analysis, hyperbolic geometry, geometric group theory, and analysis on metric spaces. In my talk I will give a survey on this subject.

Ken Bromberg: *The Bers-Thurston-Sullivan density conjecture*

Friday, June 3. 9:30am

When studying a family of dynamical systems a basic question is if the structurally stable systems are dense. While in general this is not true, one entry of the Sullivan dictionary is that this density holds both for Kleinian groups and rational maps. For rational maps this is still an open conjecture but on the Kleinian groups side there are now two proofs of the density conjecture, one via the ending lamination theorem and another via hyperbolic cone-manifolds. Our discussion will concentrate on this second proof with an emphasis on the role played played by 3-dimensional hyperbolic geometry.

Moira Chas: *String topology and three manifolds (joint with Siddhartha Gadgil)*
Thursday, May 26. 3:15pm

In the late nineties, we found in joint work with Dennis Sullivan that for an oriented manifold M there is a Lie algebra structure on the equivariant homology of the mapping space of the circle into M, equivariant with respect to the rotation of the domain circle. When M is a surface this reduces to the Goldman bracket on the free abelian group generated by free homotopy classes of oriented closed curves on M.

Suppose now that M is an oriented three manifold with contractible universal covering space. One knows such a manifold decomposes into pieces along incompressible two dimensional tori and that these pieces are either hyperbolic or Seifert fibrations. We will discuss how that the graded Lie algebra structure of String Topology determines the combinatorial structure of the torus decomposition.

To do this, we have to extend the Goldman bracket for surfaces to two dimensional orbifolds. The key result in proving our theorem is that the String Topology bracket, as well as the Goldman bracket for orbifolds ”counts” mutual intersections and self-intersections of curves and surfaces in the three manifold.

Jeff Cheeger: *Qualitative differentiation*
Monday, May 30. 4:45pm

There is a natural measure $\mathcal{C}$ on the collection of all subintervals I of the the interval $[0,1]$. The mass of $\mathcal{C}$ is infinite. Let $f : [0,1] \to \mathbb{R}$ satisfy $|f'| \leq 1$. For any $I \subset [0,1]$, there is a scale invariant measure $\alpha(f,I) \geq 0$ of the deviation of $f \,|\, I$ from being linear.

In this simplest case, ”quantitative differentiation” asserts that for all $\epsilon > 0$, the measure of the collection of intervals I for which $\alpha(f,I) > \epsilon$ is $\leq 5|\log_2 \epsilon|\epsilon^{-2}$. We will explain the sense in which this is actually a particular instance of a much more general phenomenon, of which we will give a number of recent examples.

Leonid Chekhov: *Quantum Riemann surfaces related to solutions of the Schroedinger equation*
Friday, May 27. 8:30pm

Asymptotic methods for constructing solutions for correlation functions and free energy of various matrix models turned out to be closely related to structures of algebraic geometry. The methods (the so-called topological recursion) developed for solving one-matrix (L.Ch., B.Eynard ’05) and two-matrix (L.Ch., B.Eynard, N.Orantin ’06) Hermitian models in $1/N$ expansion has recently found applications in such different fields of mathematics and mathematical physics as intersection indices on moduli spaces, Hurwitz numbers, plane partitions, etc.

I describe this method and the appearing Seiberg–Witten and Whitham–Krichever equations and the generalization of this method (together with generalizations of algebraic geometry notions such as holomorphic and meromorphic differentials, A- and B-periods, period matrix, Bergmann kernel, and recursion kernel) to the case of ”quantum” Riemann surfaces related to solutions of the Schroedinger equation, which emerge as nonperturbative solutions of the asymptotic distribution (the ”loop equation”) for the Wigner beta-ensembles (L.Ch., B.Eynard, O.Marchal ’10). These ensembles and the corresponding solutions play an instrumental role in the hypothesis recently advanced by Alday, Gaiotto, and Tachikawa on the correspondence between Nekrasov–Shatashvili instantonic functions and conformal blocks of the Liouville theory.

Xiaojun Chen: *Some string topology inspired structures in symplectic topology*
Monday, May 30. 7:30pm

In this talk I show that there is a Lie bialgebra structure on the cyclic homology of the Fukaya category in an exact symplectic manifold. Such a Lie bialgebra is deeply related to the ones discovered in string topology and symplectic field theory. Examples of this Lie bialgebra on cotangent bundles will be given.

Ralph Cohen: *String topology and the classification of topological field theories*

Friday, May 27. 11:00am

In this talk I will describe how string topology fits into recent work on the classification of topological field theories by Costello and by Lurie. In particular I will describe the String Topology ”Fukaya-category” of a given manifold M. The objects are are submanifolds of M and the morphisms are equivalent to chains of spaces of paths connecting the submanifolds. We describe Lurie’s notion of a Calabi-Yau object in a symmetric monoidal (infinity) 2-category, and show that the string topology category fits this definition. In so doing, this leads to the question of the role of Koszul duality in topological field theories, and I’ll state some conjectures in this regard. This is joint work with A. Blumberg and C. Teleman.

Kenji Fukaya: *Open closed Gromov-Witten theory and its application to symplectic topology*

Friday, May 27. 3:15pm

Open closed Gromov Witten theory may be regarded as the study of modul space of pseudo-holomorphic curve with boundary condition on Lagrangian submanifold, with marked points both on boundary and interior of the curve. Besides meaning in String theory and Mirror symmetry it now has various applications in symplectic topology. In this talk I would like to explain some of them.

David Gabai: *Shrinkwrapping and the taming of hyperbolic 3-manifolds*

Thursday, June 2. 4:45pm

Marden’s tameness conjecture asserts that any complete hyperbolic 3-manifold with finitely generated fundamental group is topologically tame. We will outline a proof of Marden’s conjecture (that simultaneously shows geometric tameness) along the lines of joint work with Danny Calegari. Marden’s conjecture was independently proven by Ian Agol.

Steve Halperin: *The growth and Lie structure of the rational homotopy groups of a finite dimensional complex*

Saturday, May 28. 3:15pm

I will present what is now fairly complete information about the possibilities for the ranks of the homotopy groups of a finite dimensional complex, X. An old theorem of Milnor-Moore identifies the rational homotopy groups of X, desuspended by one degree, as the primitive Lie algebra of the loop space homology, and I will also describe a structure theorem for the set of ideals. The results (with Y. Felix and J.C. Thomas) while recent, culminate a 35 year joint research program, in which Sullivan’s minimal models have played a central role throughout.

Peter Jones: *Product formulas for measures and applications to analysis and geometry*

Thursday, June 2. 9:30am

We will discuss geometry of Lebesgue measurable sets and differentiablity of Lipschitz functions. The starting point is elementary product formalisms for positive measures, due to R. Fefferman, C. Kenig, and J. Pipher. We will give some background where there have been previous applications to analysis and geometry. Most of the talk will be devoted to joint work with Marianna Csörnyei.

The new result concerns Lebesgue measurable sets E of small Lebesgue measure (in any dimension). The set E can be decomposed into a bounded number of sets with the property that each (sub)set has a nice ”tangent cone”. Roughly speaking each subset has very small intersection with any Lipschitz curve whose tangent vector (to that curve) always lies inside a fixed cone. This had been proven in dimension two by Alberti, Csörnyei, and Preiss by using special, two dimensional combinatorial arguments. The main technical result needed in our work is a d dimensional, measure theoretic version of (a geometric form of) the Erdös-Szekeres theorem. (The discrete form of E-S is known only in $d = 2$.)

In what is perhaps a small surprise, certain ideas from random measures can be used effectively in the deterministic setting. Our result yields strong results on Lipschitz functions: For any Lebesgue null set E in d dimensions, there is a Lipshitz mapping of Euclidean d space to itself, that is nowhere differentiable on E. (The Rademacher theorem is sharp.) This result is best possible.

Jeremy Kahn: *Renormalization: Past, Present and Future*

Friday, June 3. 2:00pm

Renormalization for real folding maps generalizes and extends to renormalization of complex analytic quadratic-like maps. In both cases, finding bounds for the geometry of the renormalization means finding ”space” around the focus of the renormalization. We will first review Dennis's work on bounds for the real renormalization, and then we will present the work of the speaker and M. Lyubich on complex renormalization. In both cases, we obtain the space around the focus of renormalization by erasing preimages of the orbit of the focus, thereby obtaining a buffer region free from all other elements of the orbit of the focus of renormalization.

Boris Khesin: *Symplectic fluids and point vortices*

Monday, May 30. 11:00am

We describe the motion of symplectic fluids as an Euler-Arnold equation for the group of symplectic diffeomorphisms. We relate it to the Lagrangian study of symplectic fluids by D. Ebin, describe symplectic analog of vorticity and the finite-dimensional Hamiltonian systems of symplectic point vortices.

Marco Martens: *Universality and rigidity in low dimensional dynamics*

Saturday, June 4. 10:30

The transition from dynamical systems with zero entropy to systems with positive entropy often occurs along a period doubling cascade. At the transition point, the system has a Cantor set as attractor. In one-dimensional dynamics, the geometry of the Cantor attractor is universal on asymptotic small scale, it is independent of the system. The Cantor attractor of two such systems are smoothly conjugated: rigidity.

Experiments, numerical and real world, indicate that the transitions along period doubling cascades in higher dimensional situations and the real world, have exactly the same small scale geometry as observed in the one-dimensional context. The geometry of one-dimensional dynamics seems to play a definite role in higher dimensional dynamics.

Unfortunately/Surprisingly the phenomena of universality and rigidity turn out to be more delicate in higher dimensional systems. Even in the case of Henon maps, which are perturbations of one-dimensional maps. The Cantor attractor of such a Henon map has arbitrarily small parts whose geometry differs substantially from the corresponding part in the one-dimensional context: no universality, no rigidity (There are even no a priori bounds on the geometry). However, these bad spots become very rare on small scales. They are very difficult to detect, which explains the universality observed in experiments. This phenomenon is called Probabilistic Universality.

The theory of universality and rigidity in one-dimensional dynamics became a probabilistic geometric theory for two-dimensional dynamics.

Curt McMullen: *Algebraic patterns for dynamical systems*

Friday, June 3. 4:45pm

Yair Minsky: *Ending laminations, classification and models for hyperbolic 3-manifolds*

Saturday, June 4. 9:00am

I will outline the proof of Thurston's Ending Lamination Conjecture, joint with Brock and Canary, and what it tells us and fails to tell us about the geometry of hyperbolic 3-manifolds. If time permits I will also discuss recent work with Brock, Namazi and Souto on constructing bilipschitz models for various classes of 3-manifolds.

Gerard Misiolek: *Geometry of diffeomorphism groups and H^1 optimal transport*

Monday, May 30. 2:00pm

The quotient space $\mathrm{Diff}(M)/\mathrm{Diff}_\mu(M)$ of the diffeomorphism group by its subgroup of volumorphisms carries a natural Sobolev-type metric of constant curvature. I will describe its geometry as well as the connection to optimal transport and mathematical statistics.

Steve Preston: *Fredholmness of Riemannian exponential maps on diffeomorphism groups*

Tuesday, May 31. 11:00am

We discuss the global properties of the Riemannian exponential map for the group of volume-preserving diffeomorphisms of a two- or three-dimensional manifold, which tells us about the solution operator of the Euler equations for ideal fluids in Lagrangian coordinates. We also describe generalizations for other partial differential equations arising as geodesics on diffeomorphism groups.

Graeme Segal: *Fields, deformations, and the axiomatization of quantum field theory*
Sunday, May 29. 11:45am

I shall discuss the definition of local field operators in a quantum field theory, and how they relate to deformations of the theory and to the multi-tier structure of the theory. I shall focus on some simple non-topological examples, especially on two-dimensional theories, and shall explain what regularity assumptions seem to be needed to prove the theorems one would like.

Mike Shub: *On the convexity of the condition number in the condition metric*
Thursday, June 2. 9:15pm

The condition number is a useful complexity invariant in the study of the numerical solution of systems of polynomial equations. If we let V, contained in the product of the projective spaces of systems of homogeneous polynomials of fixed degrees in $n+1$ variables cross $\mathbb{CP}^{n+1}$, be the solution variety $\{(f,x)|f(x)=0\}$, then the condition number $\mu(f,x)$ is essentially the norm of the inverse of the derivative of f at x restricted to the Hermitian complement of x (i.e., the tangent space to projective space).

We define a new Hermitian structure on V by multiplying the restriction of the usual Fubini-Study Hermitian structure to V by the condition number squared. The length of a path in the condition Hermitian structure gives an upper bound on the number of steps of Newton's method to numerically approximate the path. Geodesics in the condition metric give an efficient method to continue the roots of one system to another. So the geodesics are interesting to understand.

Now the condition number is comparable to the inverse of the distance to the subvariety of V of (f,x) where x is a degenerate root of f. Having sat in Dennis' seminar in the eighties, it was almost impossible not to think of hyperbolic geometry as an analogue of our situation. In this case, $\ln(1/y)$ is convex on the hyperbolic geodesics. Is the same true for the condition metric? In joint work with Carlos Beltran, Jean-Pierre Dedieu and Gregorio Malajovich we prove this for linear systems. The norm is only a Lipschitz function so we are in the realm of Lipschitz Riemannian geometry. This complicates some of the proofs. The conclusion is not true for the usual smooth norms on the space of linear operators.

Jim Simons: *Dennis saves the day*
Thursday, May 26. 3:15pm

A proof of the uniqueness theorem for ordinary differential cohomology, in which deep results in geometric topology play a crucial role.

Daniel Smania: *Renormalization operator for multimodal maps*

Thursday, June 2. 8:40pm

Renormalization theory in one-dimensional dynamics has been a hot topic along the years, specially after the seminal work of Douady-Hubbard and Sullivan. Perhaps one of the most striking developments is that a fine understanding of the renormalization operator can lead us a better knowledge of the behavior of "most" of one-dimensional dynamical systems. For instance, the work of Avila, Lyubich and de Melo on families of real analytic unimodal maps relays deeply on renormalization theory.

A similar approach for multimodal maps (many critical points) pose new difficulties. Mainly the parameter space is not one-dimensional. The parapuzzles, developed by Branner-Hubbard and applied successfully by Yoccoz and many others for unicritical maps, provided a very precise description of the parameter space of the quadratic family. The miraculous properties of codimension one holomorphic laminations were also a crucial tool to understand the space of quadratic-like maps. Both tools are no longer available in the multimodal case. In this work in progress our main result is as follows:

Main Theorem: *Let f_λ be a finite-dimensional family of real analytic multimodal maps and let Λ_b be the subset of parameters λ such that f_λ is infinitely renormalizable with bounded combinatorics (not all the critical points need to be involved in the renormalization). Then for a generic finite-dimensional family the set Λ_b has zero Lebesgue measure.*

One of the main steps of the proof is to show that the action of the renormalization operator on infinitely renormalizable multimodal maps with bounded combinatorics is hyperbolic. The contraction on the hybrid classes of infinitely renormalizable maps can be obtained using available methods (Sullivan, McMullen, Lyubich, S., Lyubich and Avila). To show the expansion in the transversal direction we developed a new approach, based on the study of the derivative cocycle of the renormalization operator instead of the operator itself.

Stanislav Smirnov: *CFT and SLE*

Thursday, May 26. 9:30am

We will give an expository talk comparing two approaches to problems of 2D statistical phasics.

Developed two decades ago, Conformal Field Theory led to spetacular predictions for 2D lattice models: e.g., critical percolation cluster a.s. has dimension $91/48$ or the number of self-avoiding length N walks on the hexagonal lattice is $\approx \left(\sqrt{2+\sqrt{2}}\right)^N N^{11/32}$. While the algebraic framework of CFT is rather solid, rigorous arguments relating it to lattice models were lacking.

More recently, a geometric approach involving random SLE curves was proposed by Oded Schramm, and developed by him, Greg Lawler, Wendelin Werner, Steffen Rohde and others. Not only this approach is completely rigorous, it also constructs new objects of physical interest.

Jim Stasheff: *How Dennis and I intersected*

Saturday, May 28. 4:45pm

After some reminiscences about how things were in the good old days, including how Dennis' work and mine were somewhat transverse ;–) I will review and update my ancient, unpublished paper with Mike Schlessinger on the deformation theory of rational homotopy types. Then I will return briefly to another transverse intersection of my work with Dennis.

Michael Sullivan: *Transverse string topology and knots*

Monday, May 30. 8:50pm

I will describe some joint work in progress with Dennis Sullivan on applying transverse string topology to detecting knot invariants.

Charles Tresser: *Manifold structure OF physics vs. the emergence and decay of geometry IN physics*

Friday, May 27. 9:15pm

In his famous Thesis, Bernhard Riemann pointed out the possibility that there would not be geometry at small enough scale, leaving the responsibility to the physicists to decide on this crucial issue. While many physicists would concede the decay of geometry at the so-called Planck length $\ell_P = \sqrt{\hbar G/c^3} \approx 1.616252(81) \times 10^{-35}$ meters.

I will explain that such decay starts much higher, in fact way above the electron radius that is often taken as $r_{\mathrm{e}} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e c^2} = 2.8179402894(58) \times 10^{-15}\mathrm{m}$, so 10^{20} times bigger. In fact, some problems should happen already at the size of the nucleus of an atom, i.e., a diameter of about 10^{-14} meter, whereas the atomic diameter is about 10^{-10} meter and that might be indeed the beginning of the problems, as I will also explain. So it was good to not try dynamics as we know it at the quantum mechanics size.

Before that some great challenge exist in which it seems obvious that mathematicians, and at least mathematics or something of that type, have to play a central role. I will leave that ball, i.e., the answer to the question in the title, to others but before I will explain some crucial issue that may help paving the first steps of the way ahead. At least I will show the lucidity of Riemann (and Napoleon) on the issue of tee limits of the maths that he has created: these are deep questions, although the maths are not that hard... for that part. I will also indicate another global way along which topology and/or geometry may help in describing theories for the physical world.

Scott Wilson: *Equivariant extensions of holonomy and secondary invariants*

Monday, May 30. 9:30pm

I'll describe how classical bundle invariants such as holonomy, Chern classes, and the Chern-Simons form can be lifted to the free loopspace of the base of a bundle with connection. Analogous constructions will be described where the bundle is replaced by an abelian gerbe, and the free loopspace is replaced by the space of maps of a torus into the base.

Michael Yampolsky: *Renormalization of critical circle mappings and related topics*
Thursday, June 2. 2:00pm

Critical circle mappings are the second main example (after unimodal mappings) of universality in one-dimensional dynamics. The conjectural renormalization picture which explains the universality was formulated in full generality by O. Lanford in early 1980s, and is known as Lanford's Program. The road to the proof of Lanford's Program exhibits many deep similarities to the proof of Feigenbaum-Coullet-Tresser universality, started by the groundbreaking work of Sullivan. However, there are also some fundamental differences. I completed the proof of Lanford's Program in the early 2000's. In my talk I will outline the main steps in the proof, involving work of Sullivan, de Faria, McMullen, de Melo, and others.

There is an important connection between renormalization of critical circle maps and renormalization of maps with Siegel disks. A key tool in my proof of hyperbolicity of renormalization is a renormalization operator known as cylinder renormalization, which bridges the gap between these two subjects. I will describe some applications of cylinder renormalization which go beyond critical circle maps, such as a renormalization-based view of Douady's Program and Buff-Cheritat's theorem on the existence of quadratics with Julia sets of positive measure; as well as some open problems and conjectures.

Qian Yin: *Lattes maps and combinatorial expansion*
Thursday, June 2. 8:00pm

We characterize Lattes maps by their combinatorial expansion behavior, and deduce new necessary and sufficient conditions for a Thurston map to be topologically conjugate to a Lattes map. In the Sullivan dictionary, this characterization corresponds to Hamenstadt's entropy rigidity theorem.