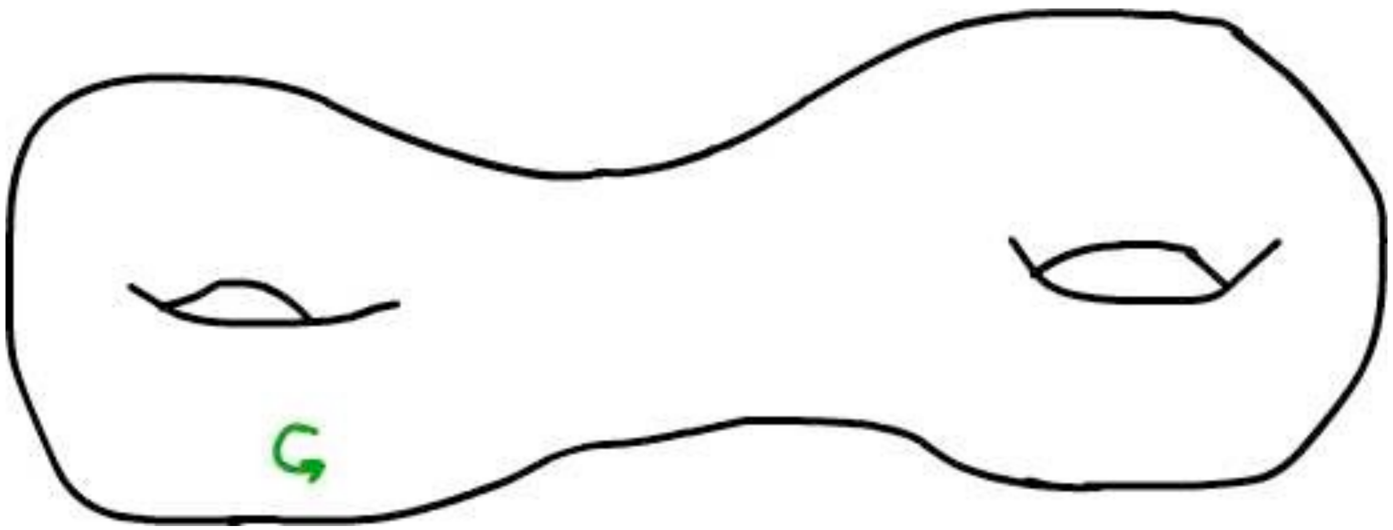
String topology and three manifolds

Stony Brook
May 26th 2011

The Goldman bracket



S an orientable surface

Monday, May 30, 2011

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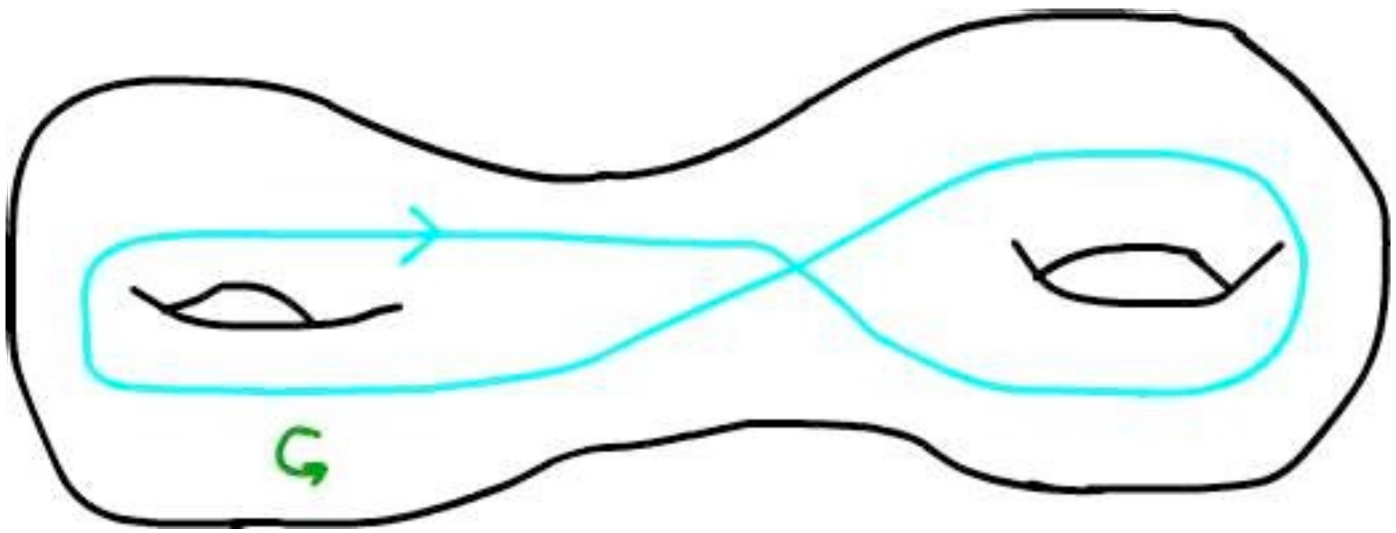

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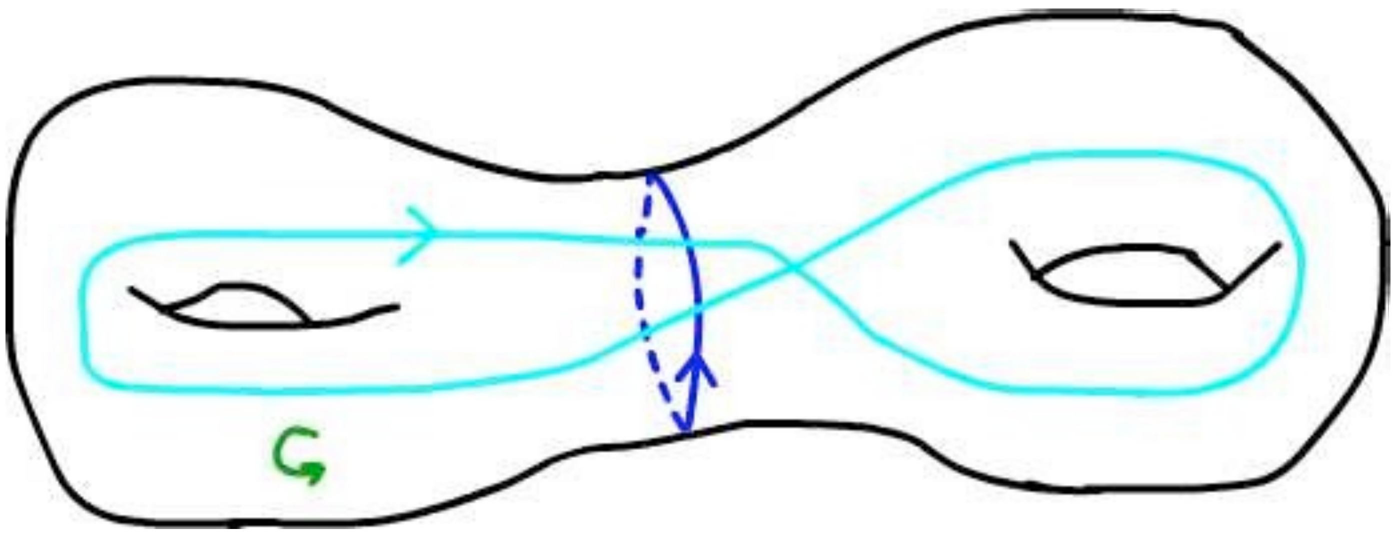

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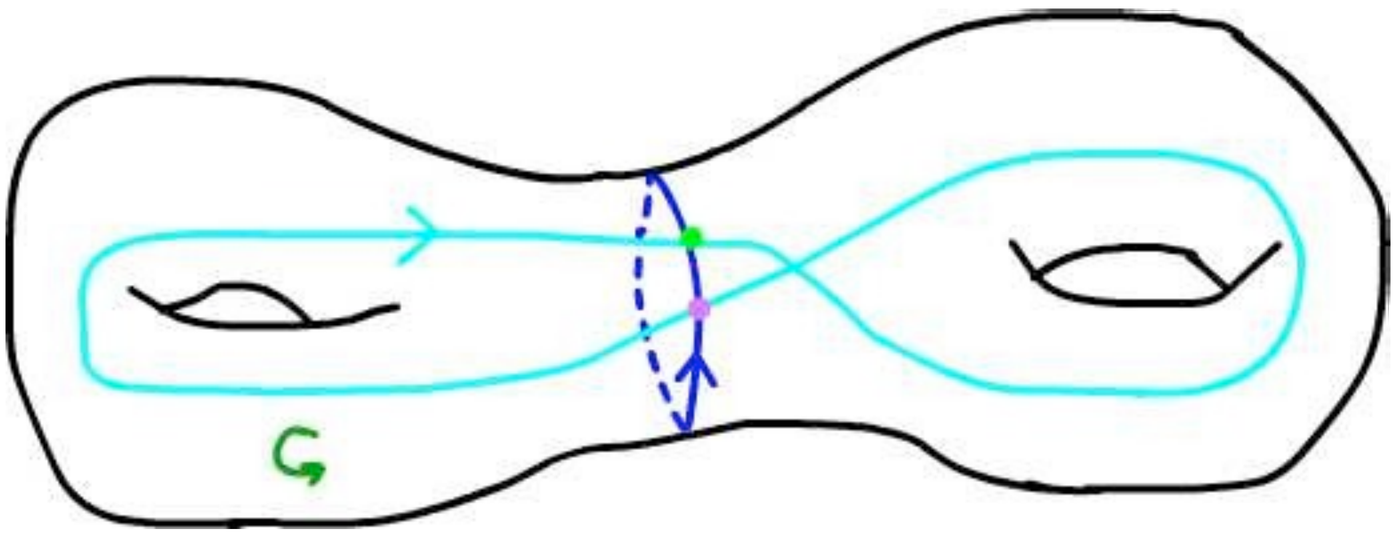

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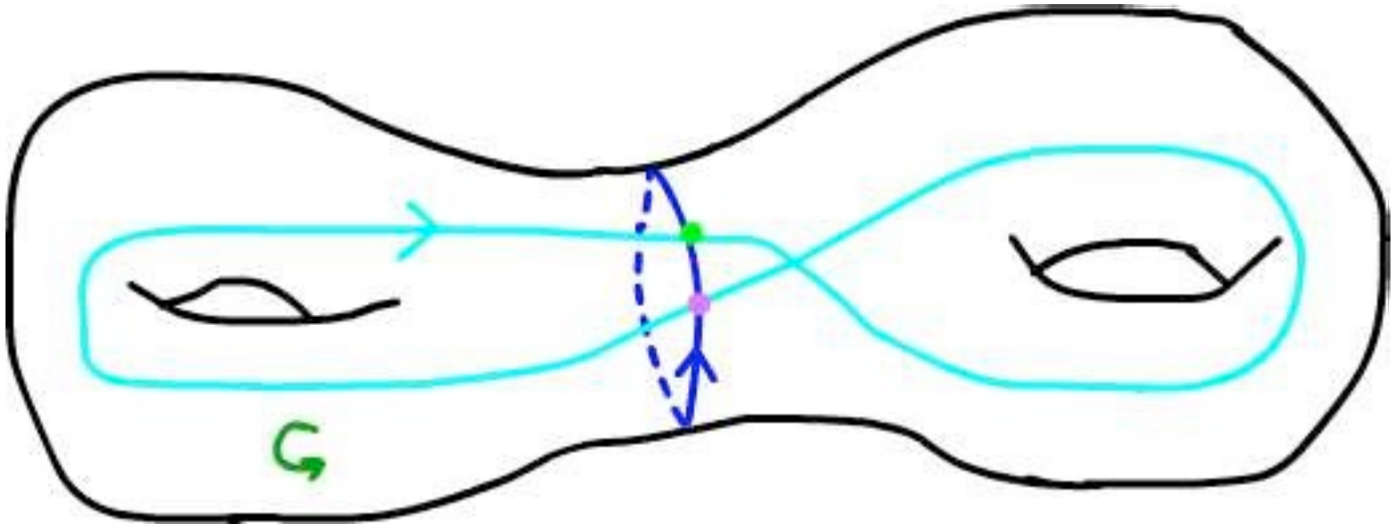

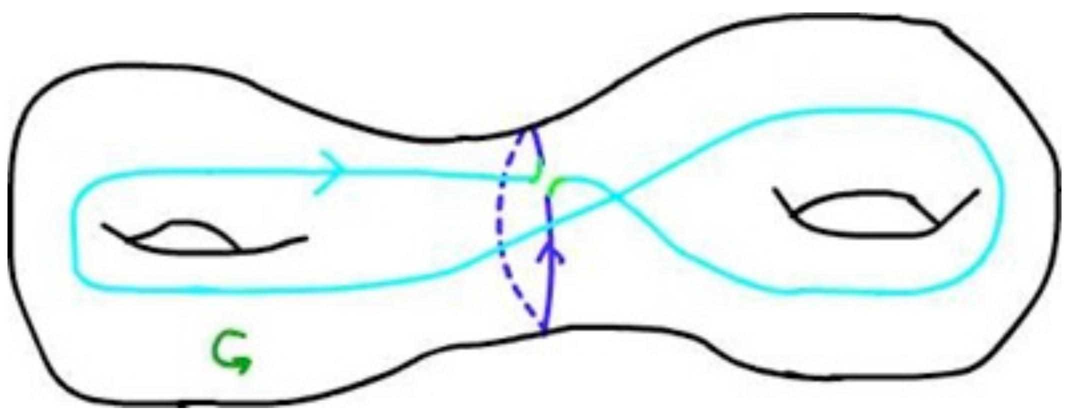

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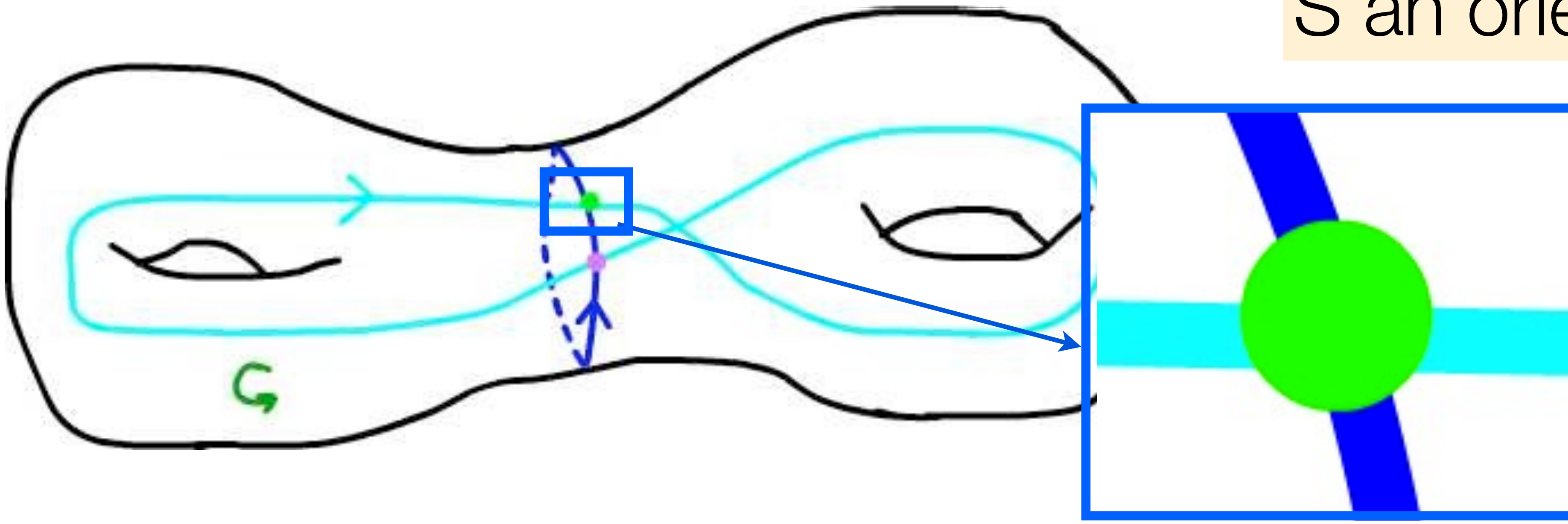

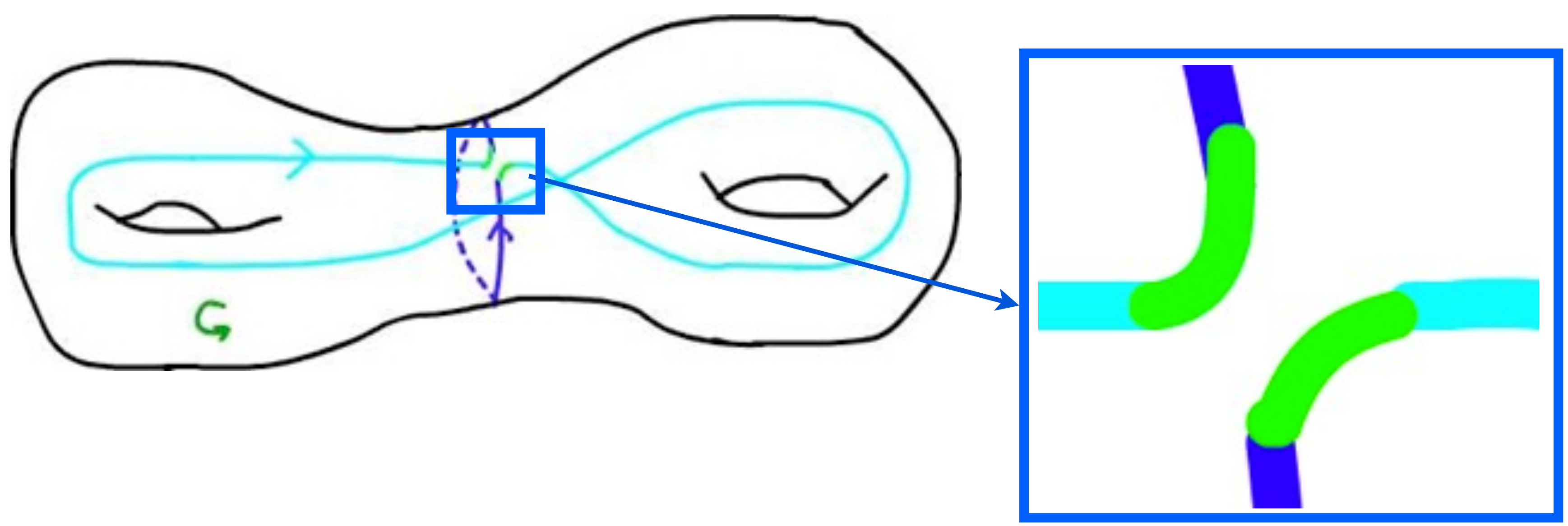

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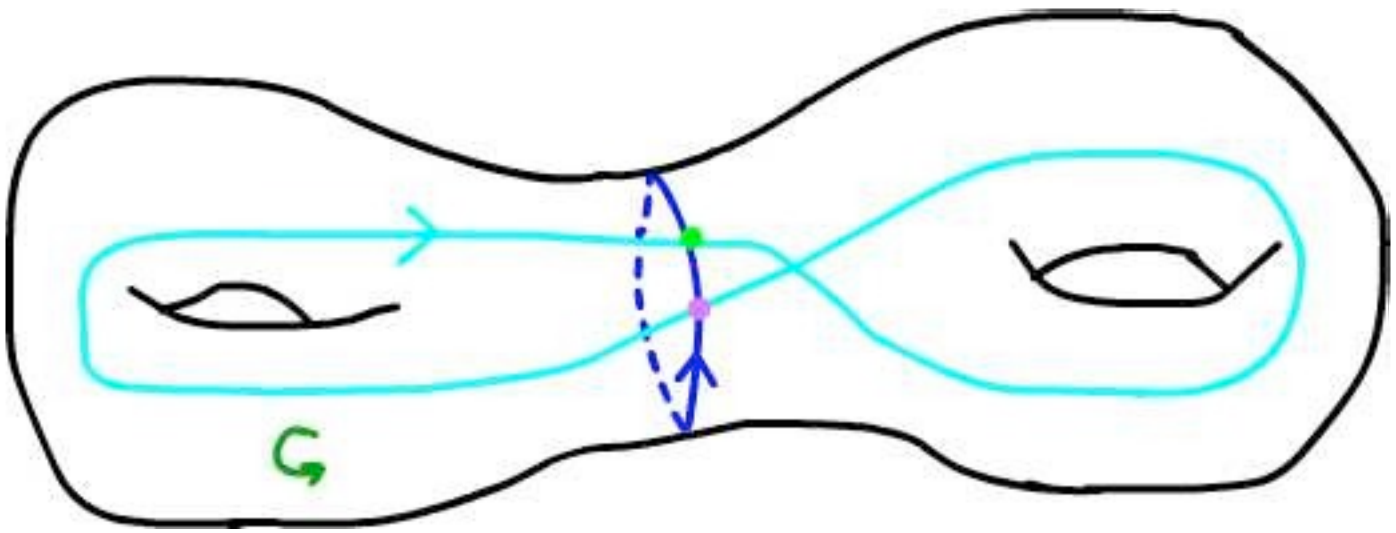

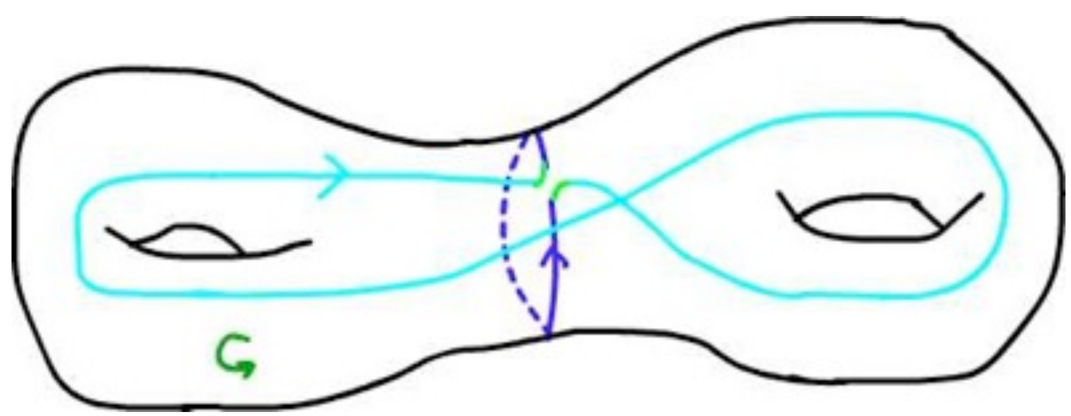

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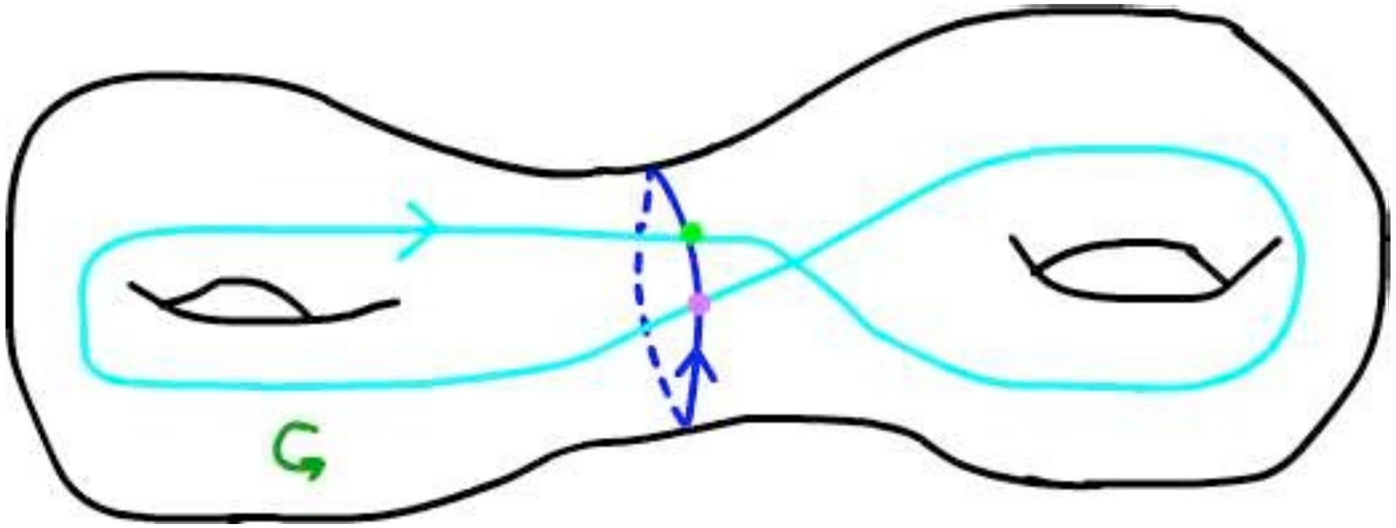
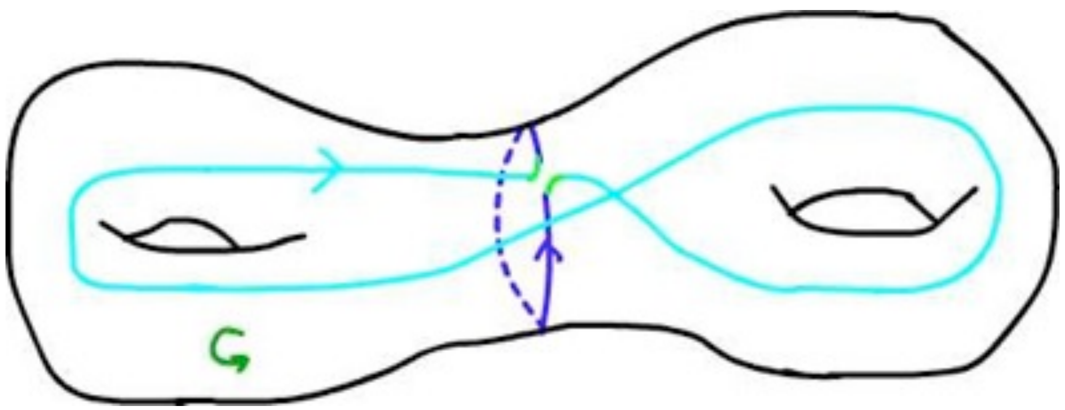


+ 

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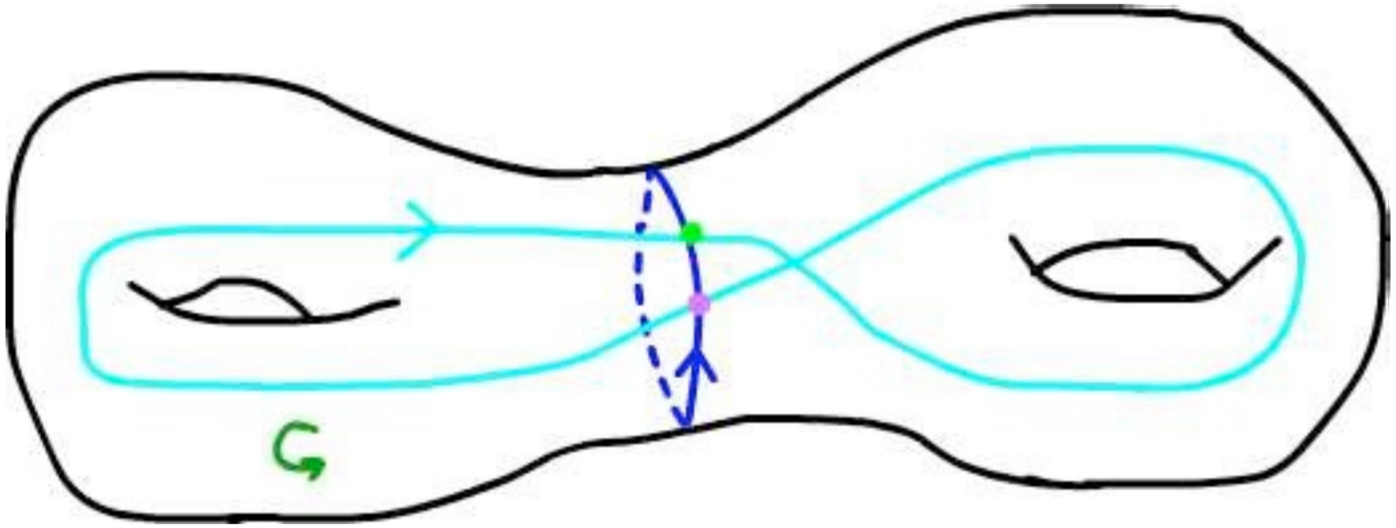

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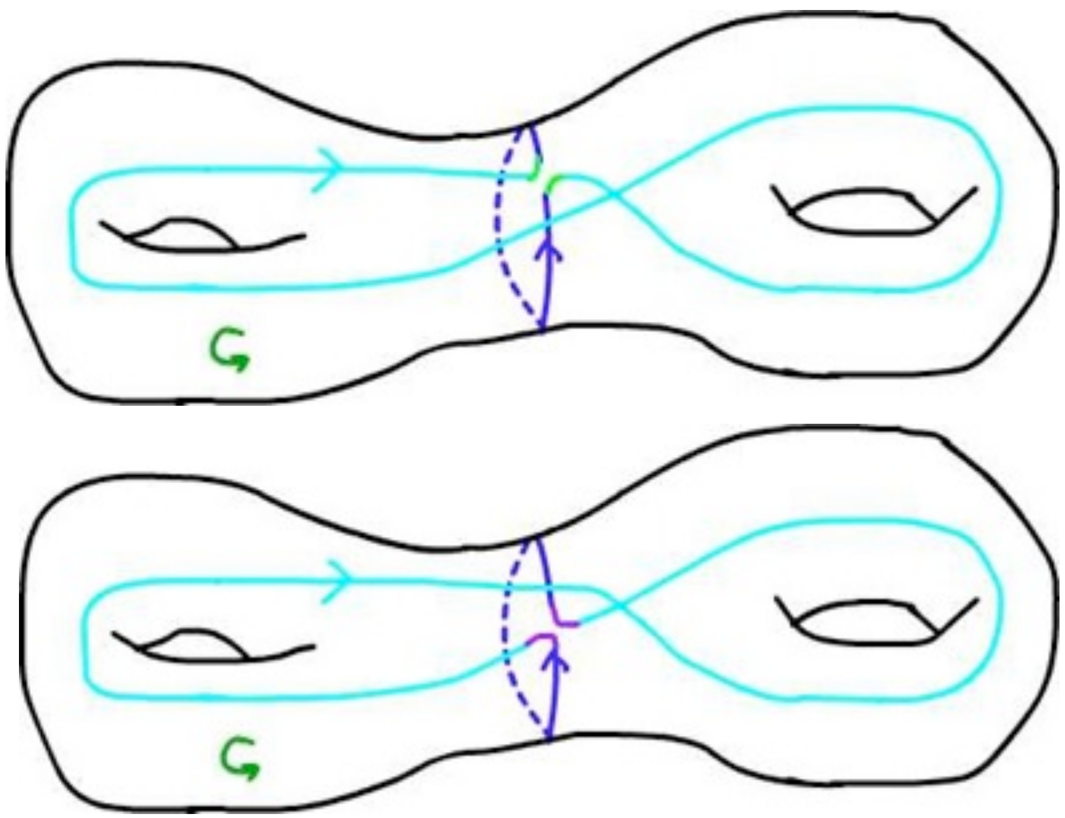

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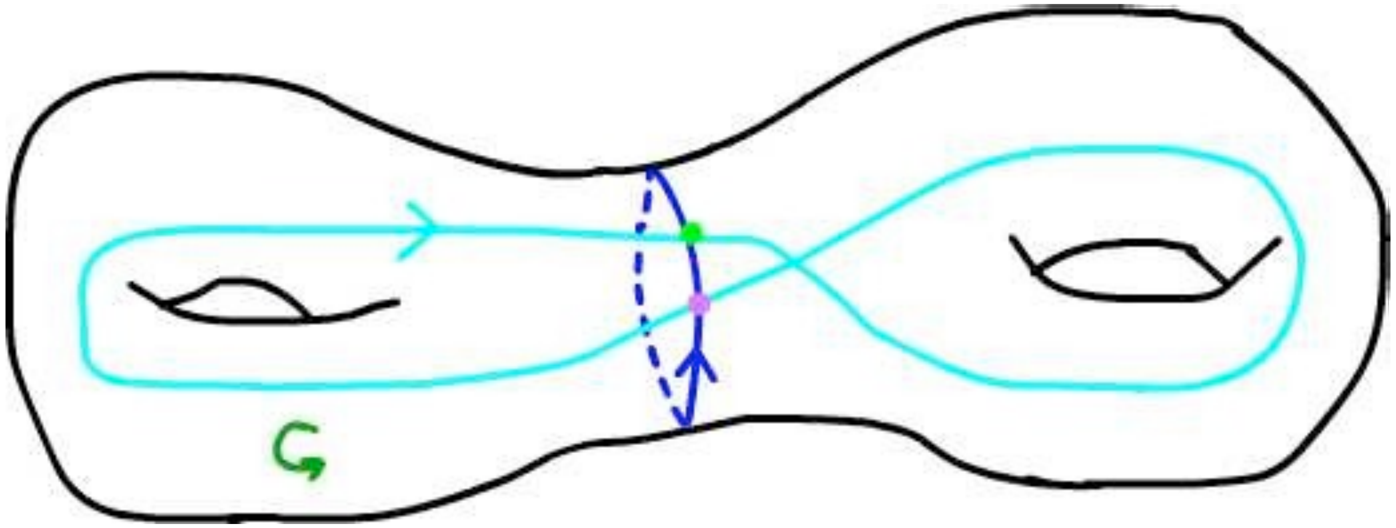

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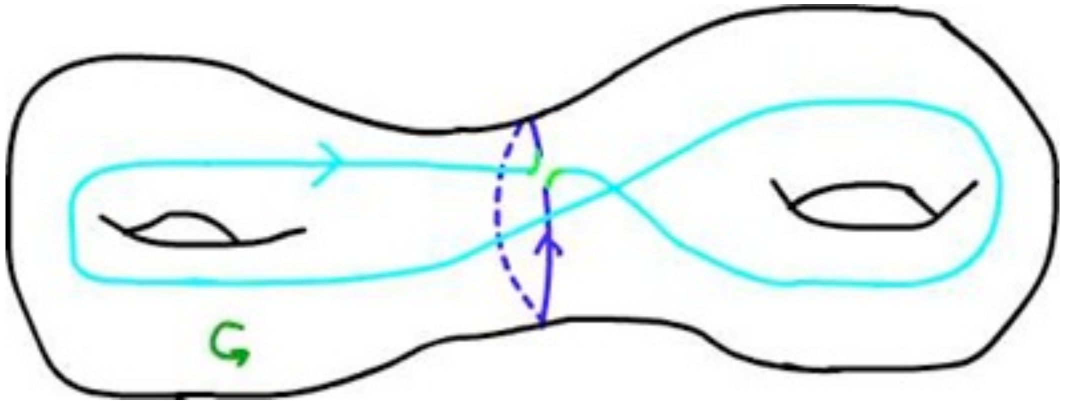

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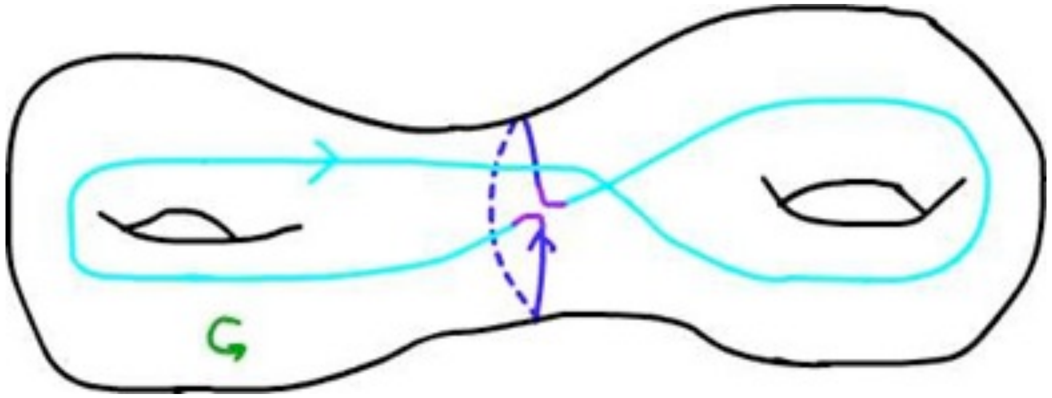

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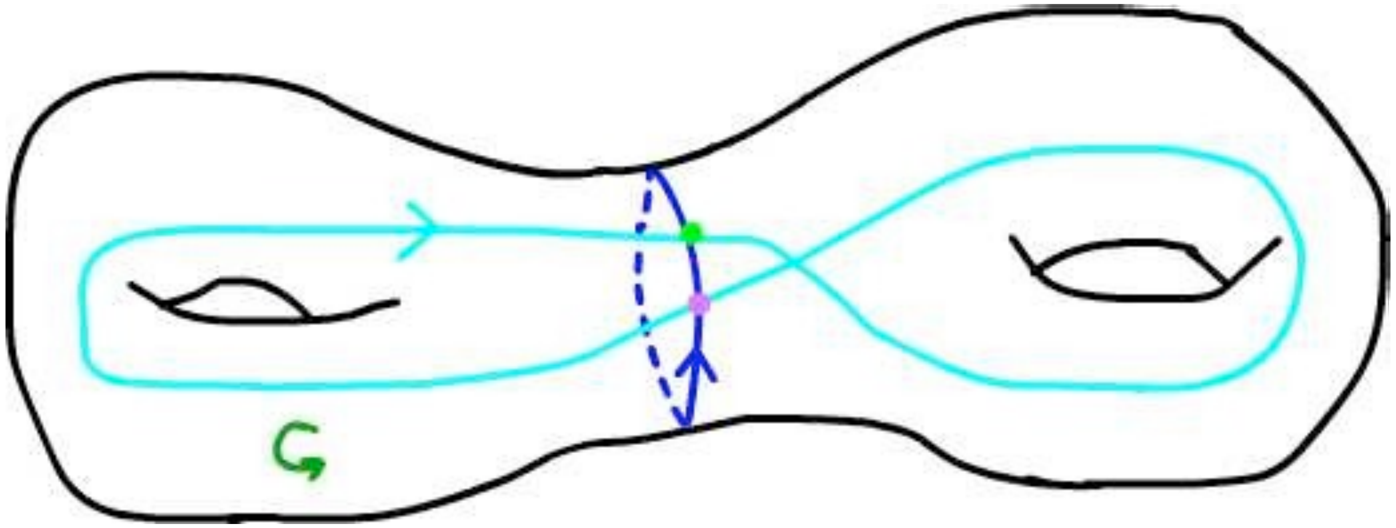

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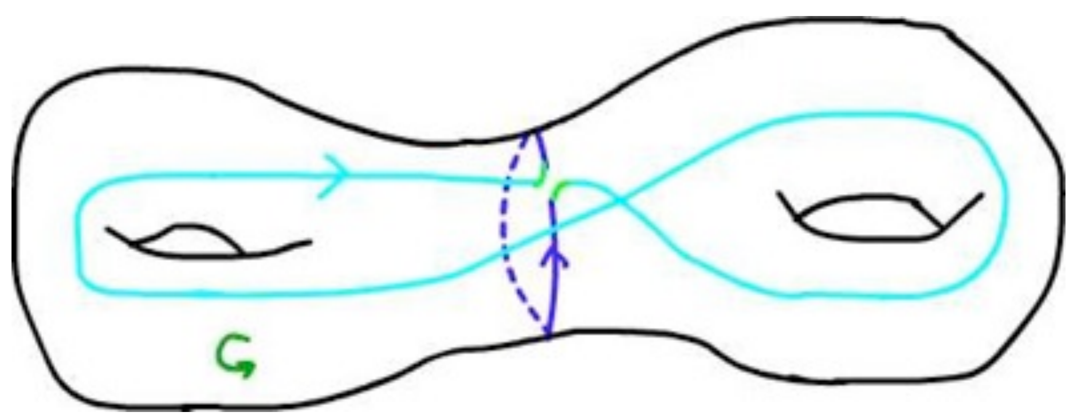

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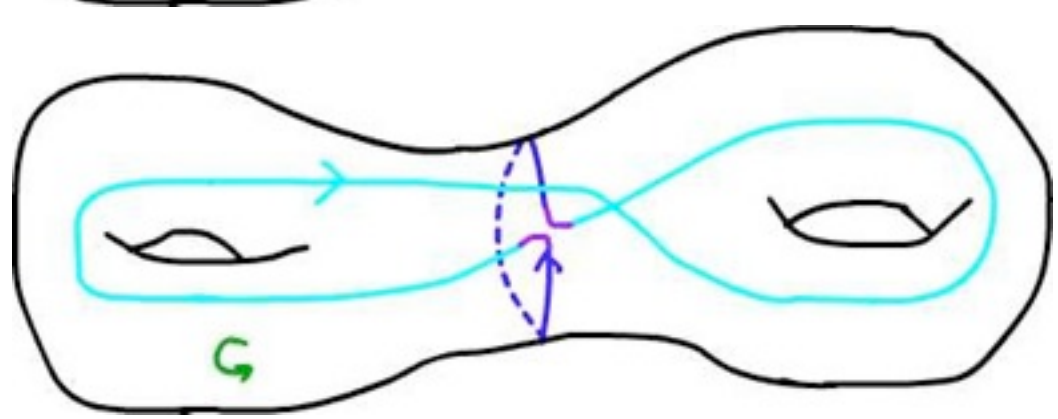

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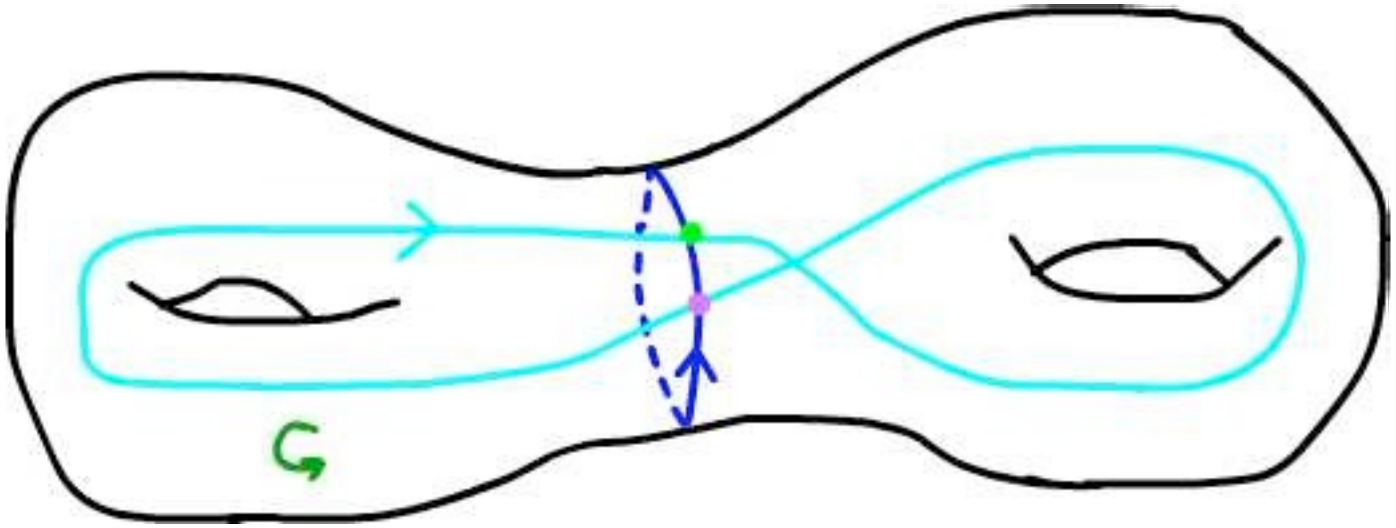

S an orientable surface

π_0 denotes the set of free homotopy classes of closed oriented curves on S.
NOTE: $\pi_0 = \pi_0$(free loop space of the surface)

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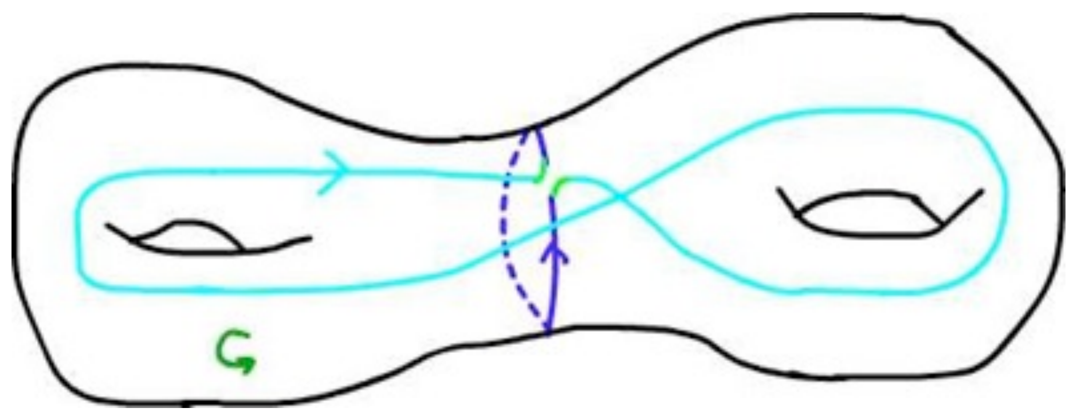

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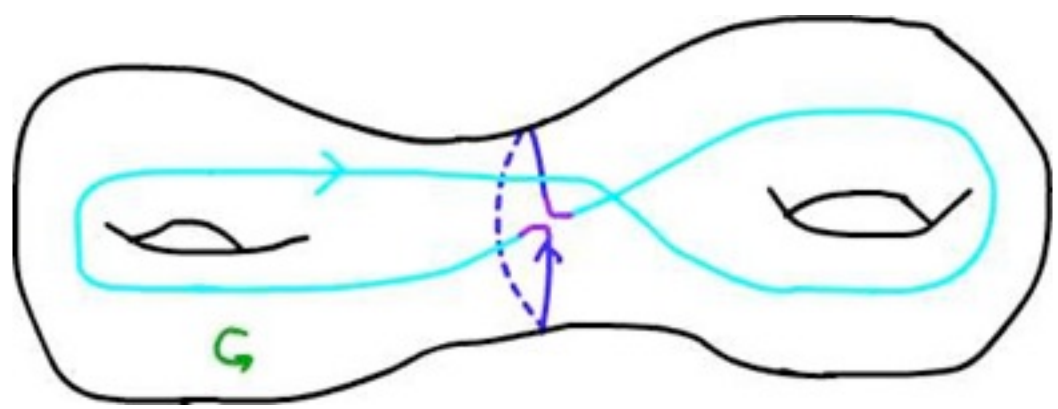

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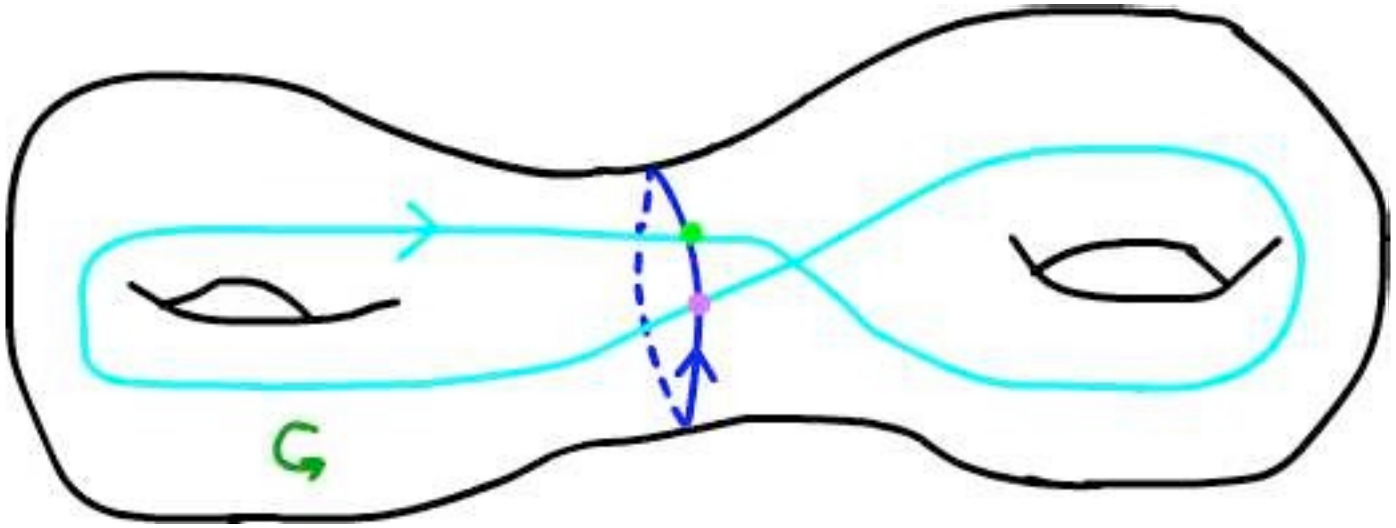

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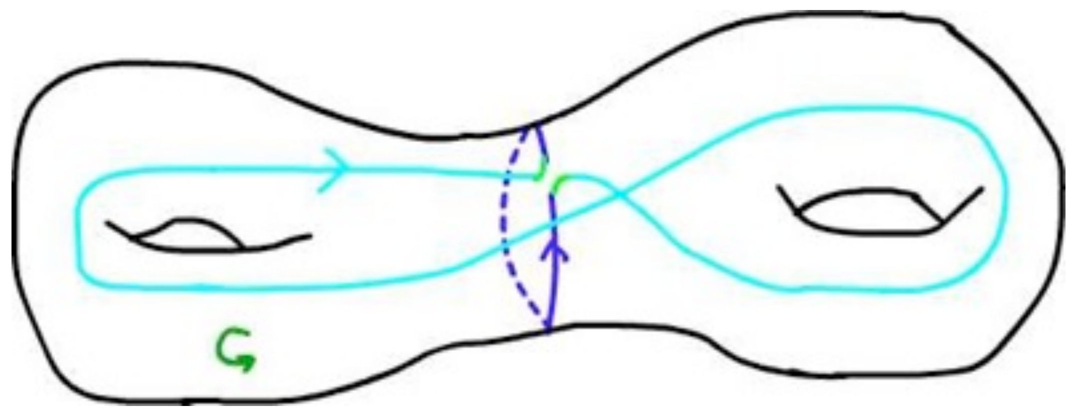
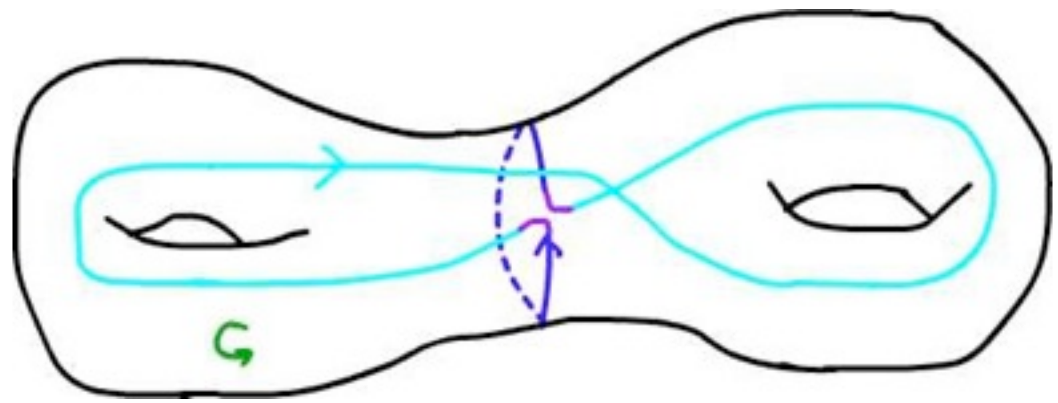
[,] = + 

\- 

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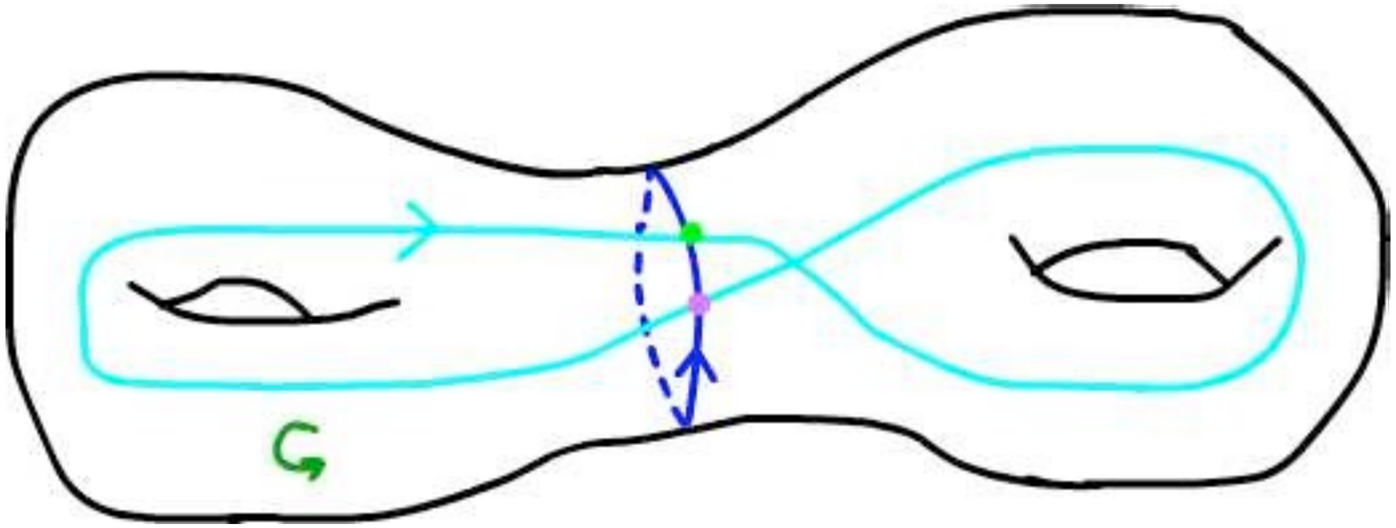

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NOTE: $\pi_0 = \pi_0$(free loop space of the surface)

Theorem: (Goldman, 1986)
The bracket is well defined and satisfies the Jacobi identity.

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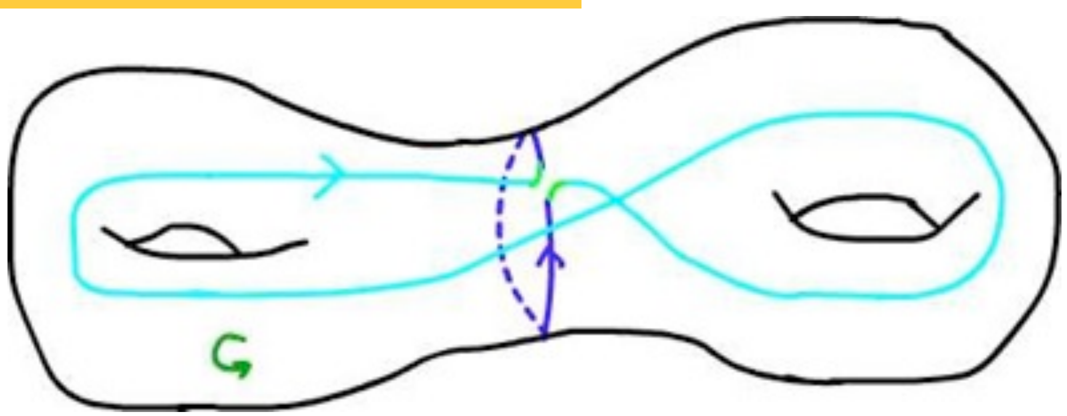

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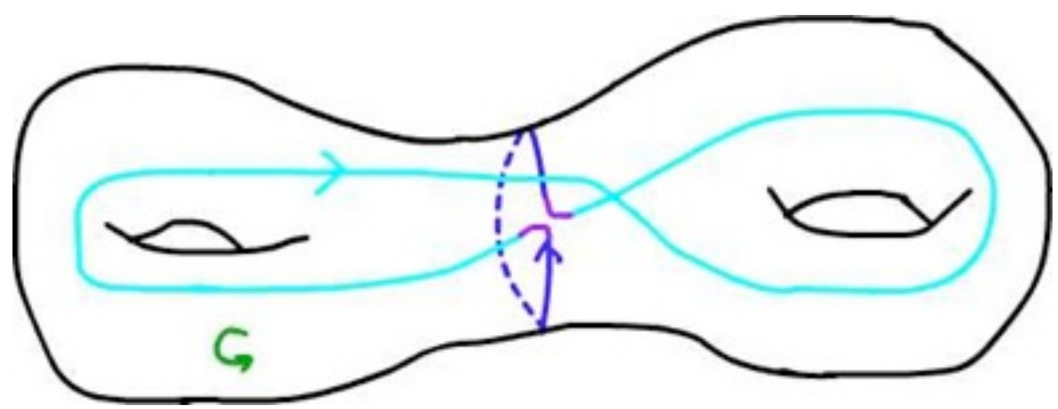

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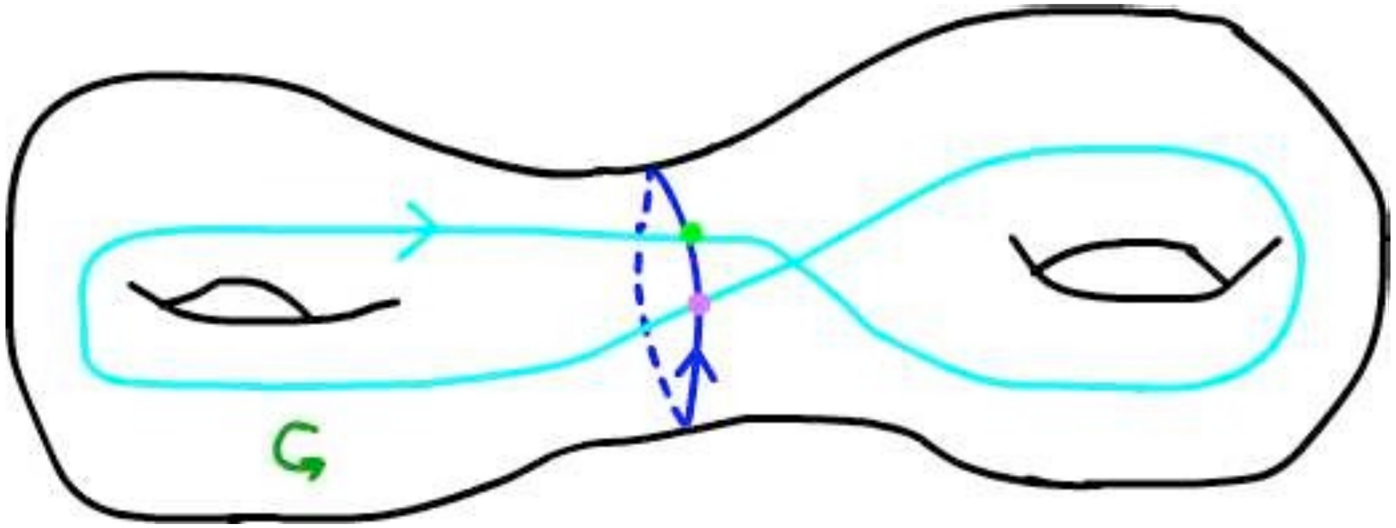

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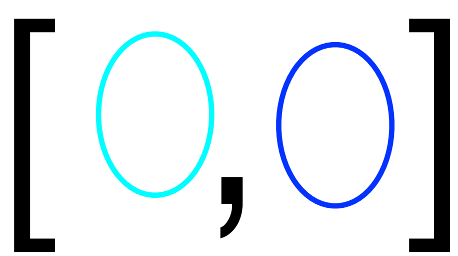

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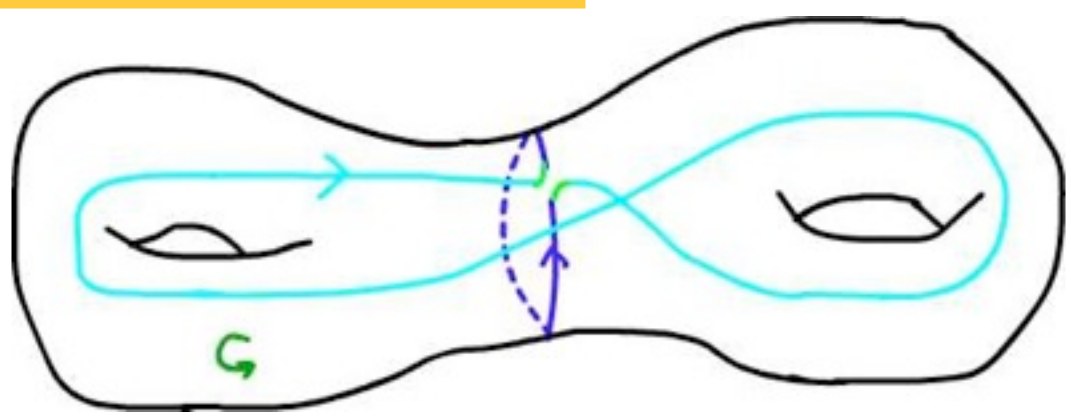

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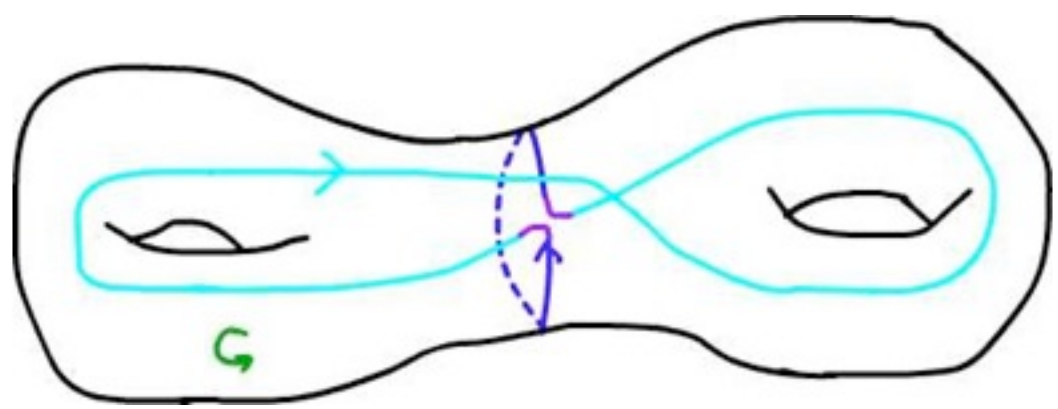

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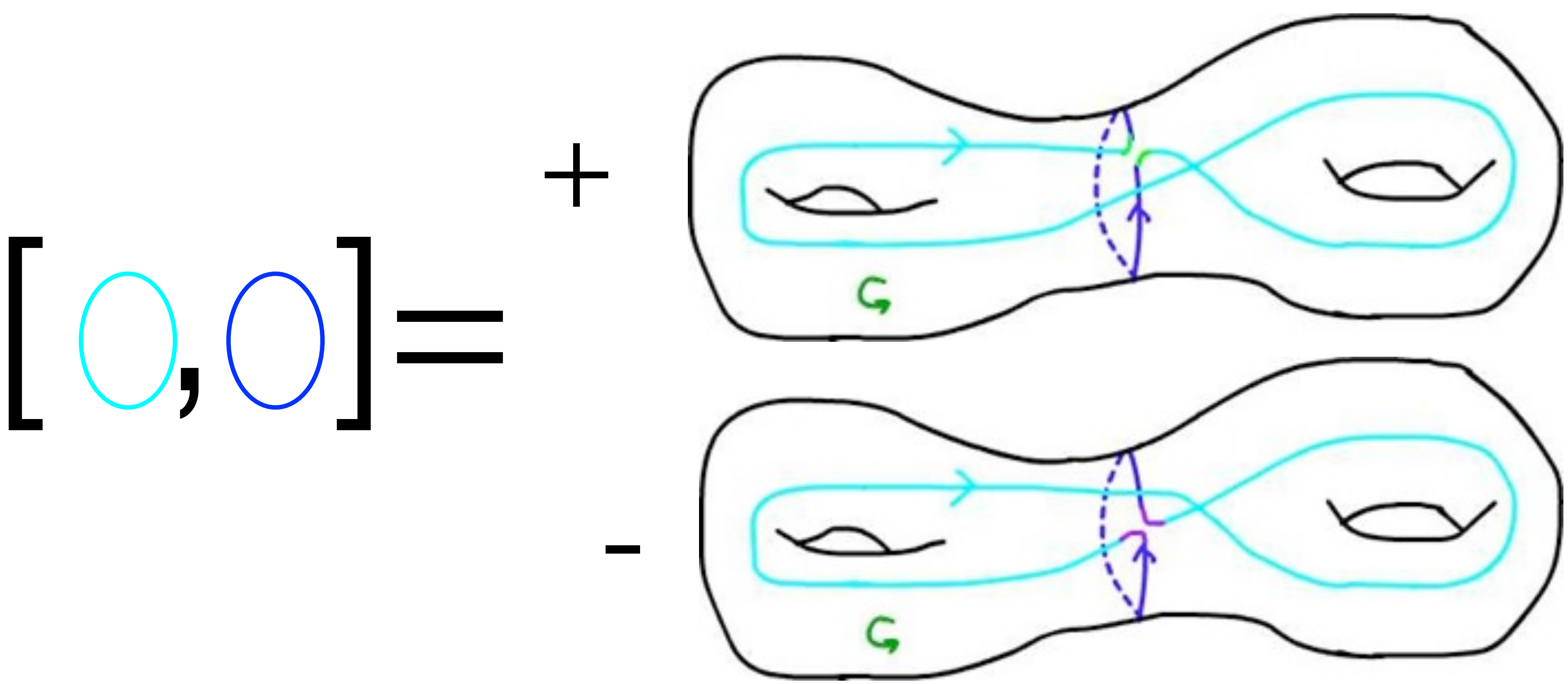

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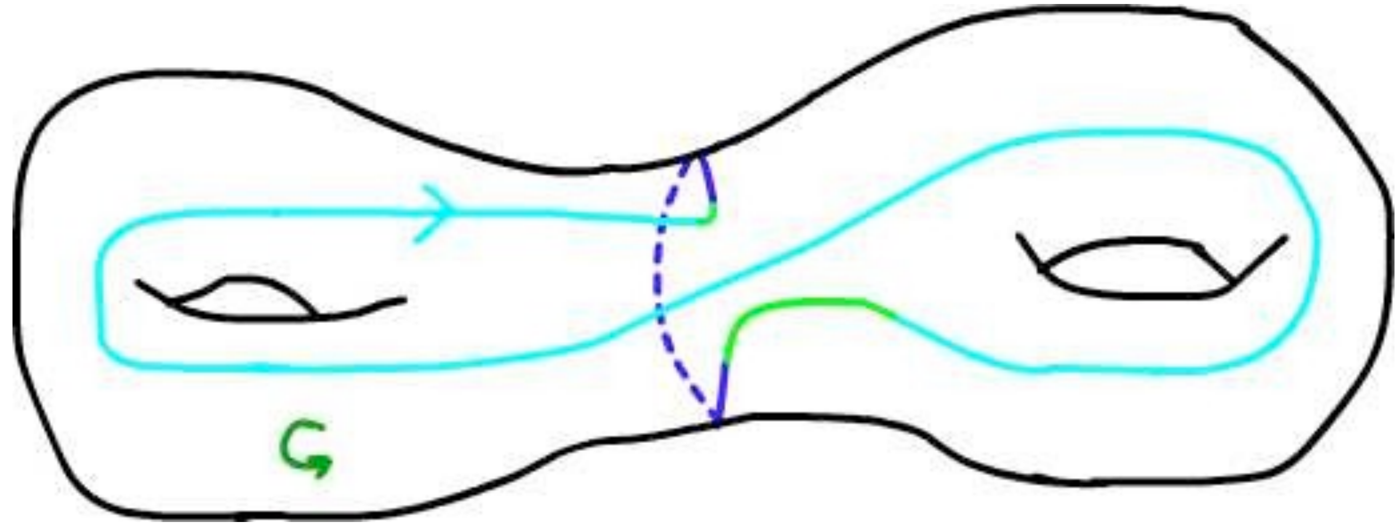

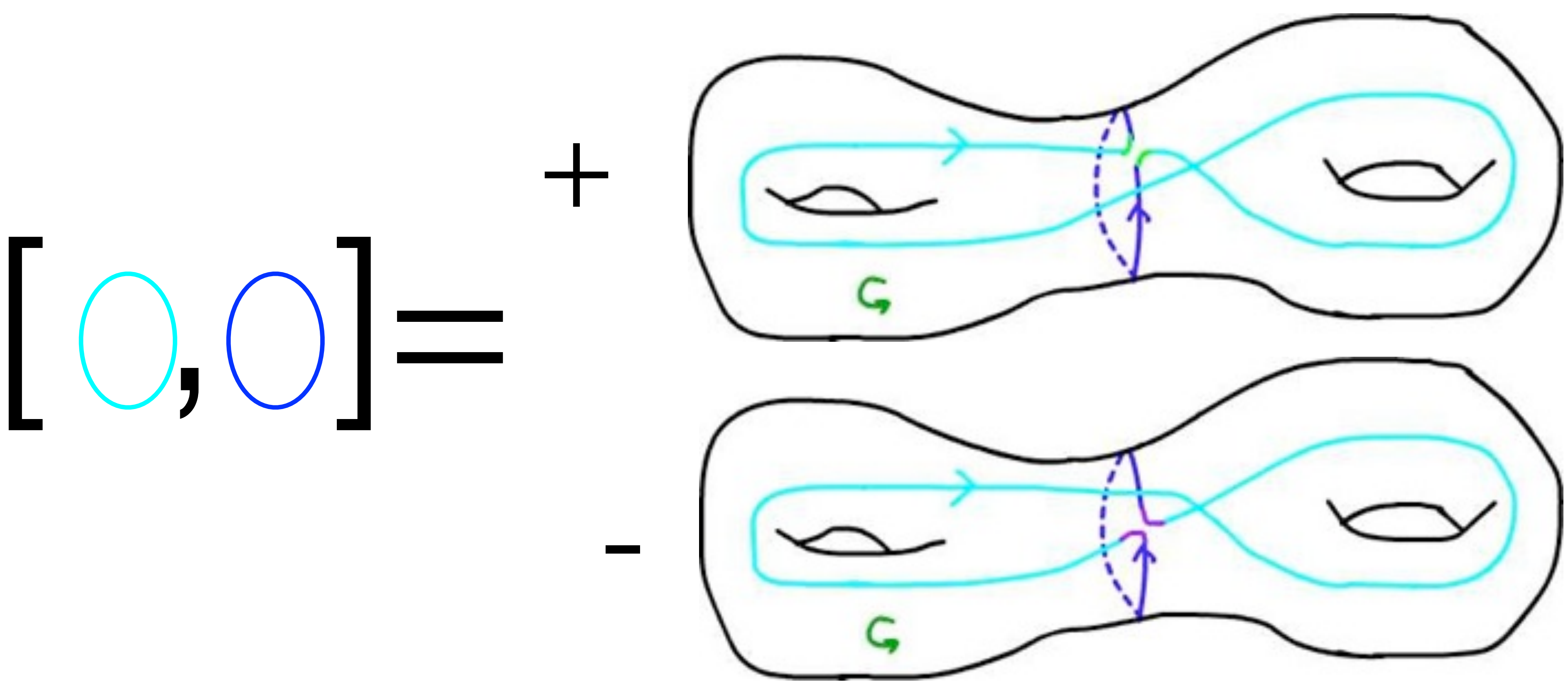

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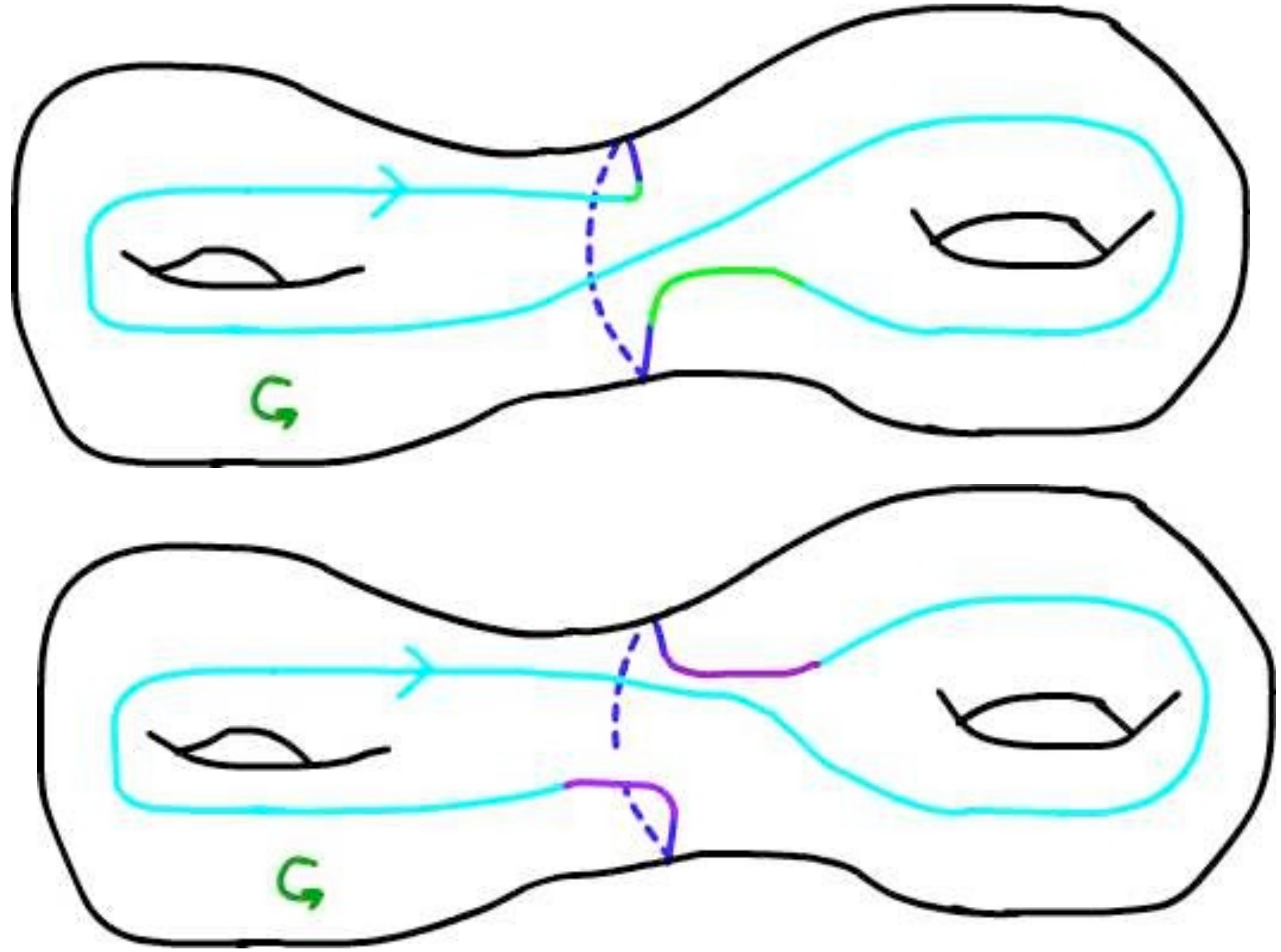
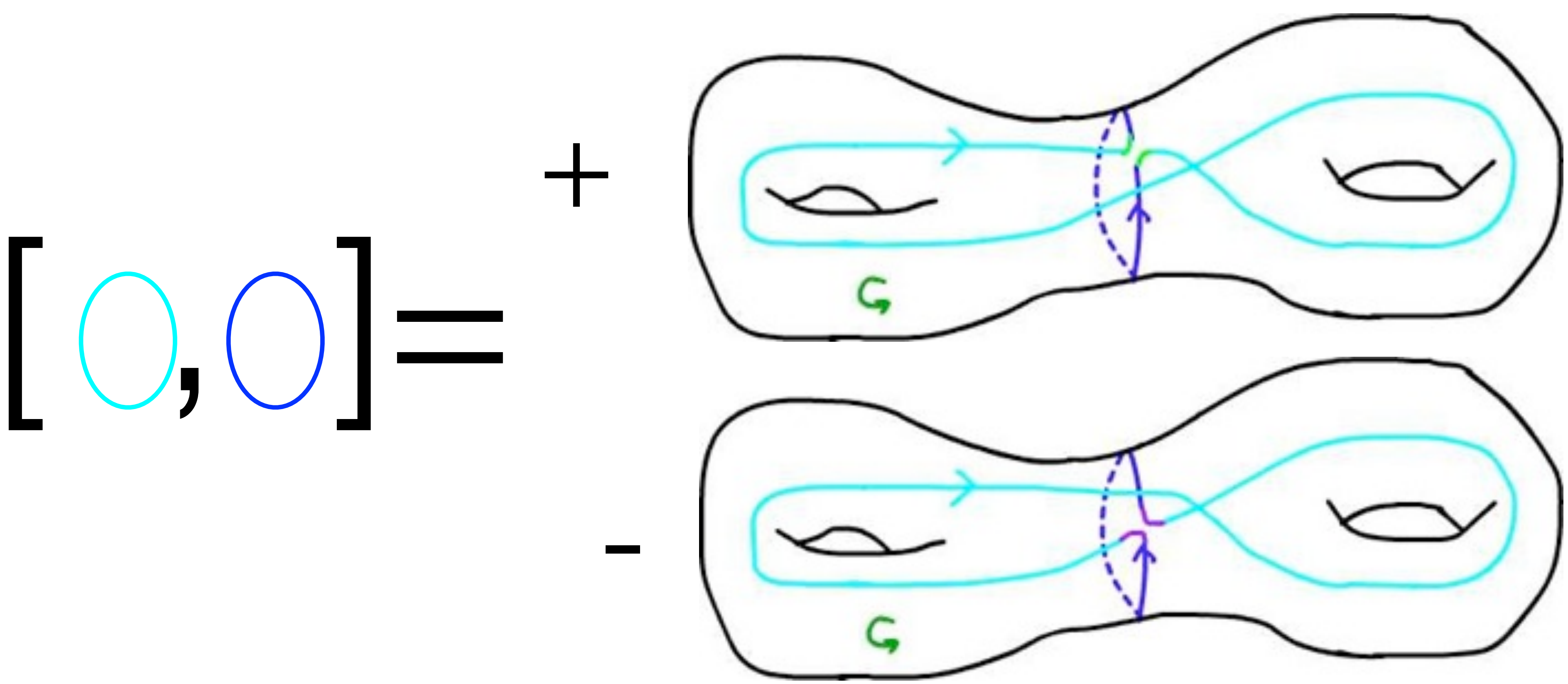
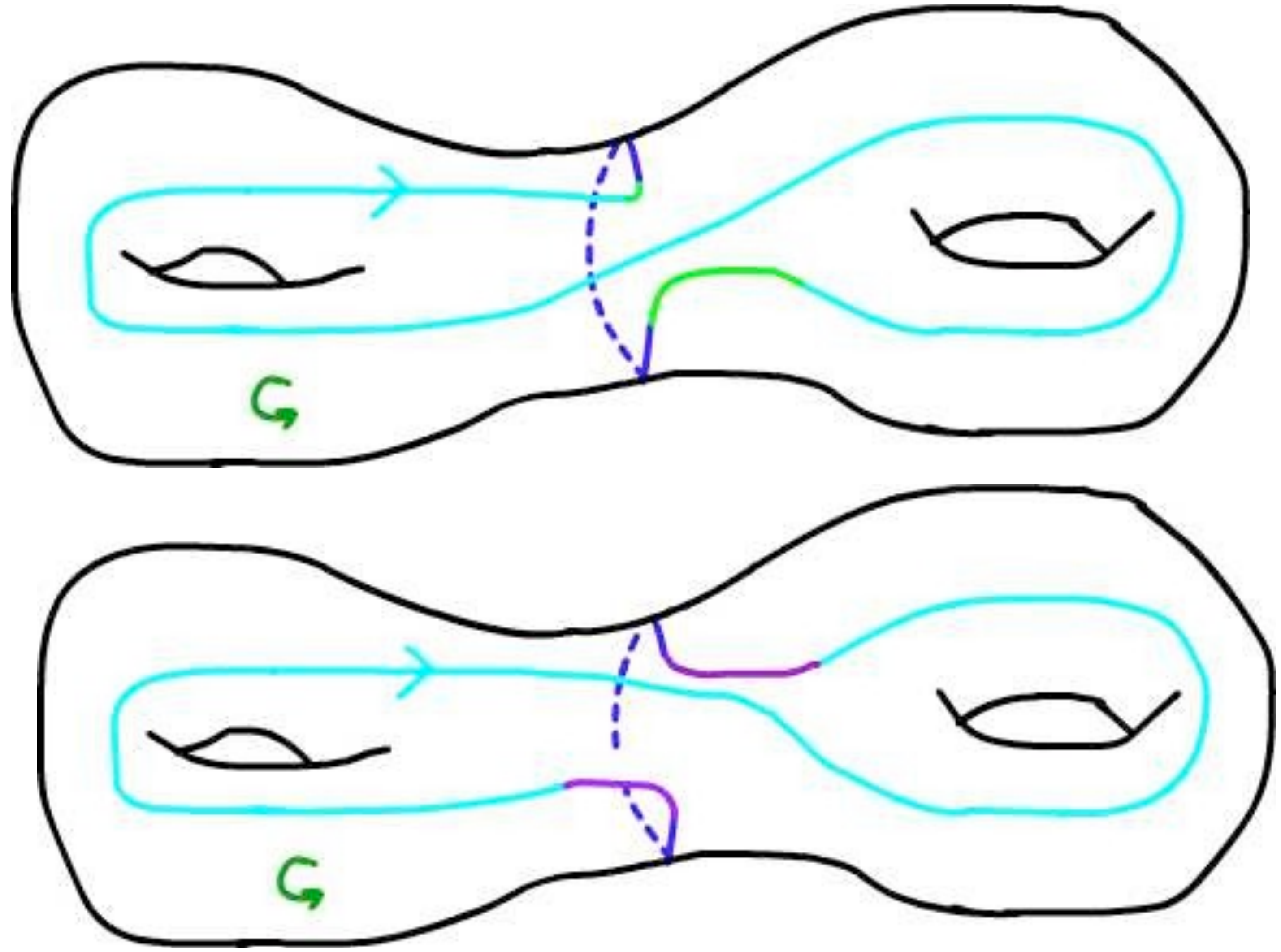
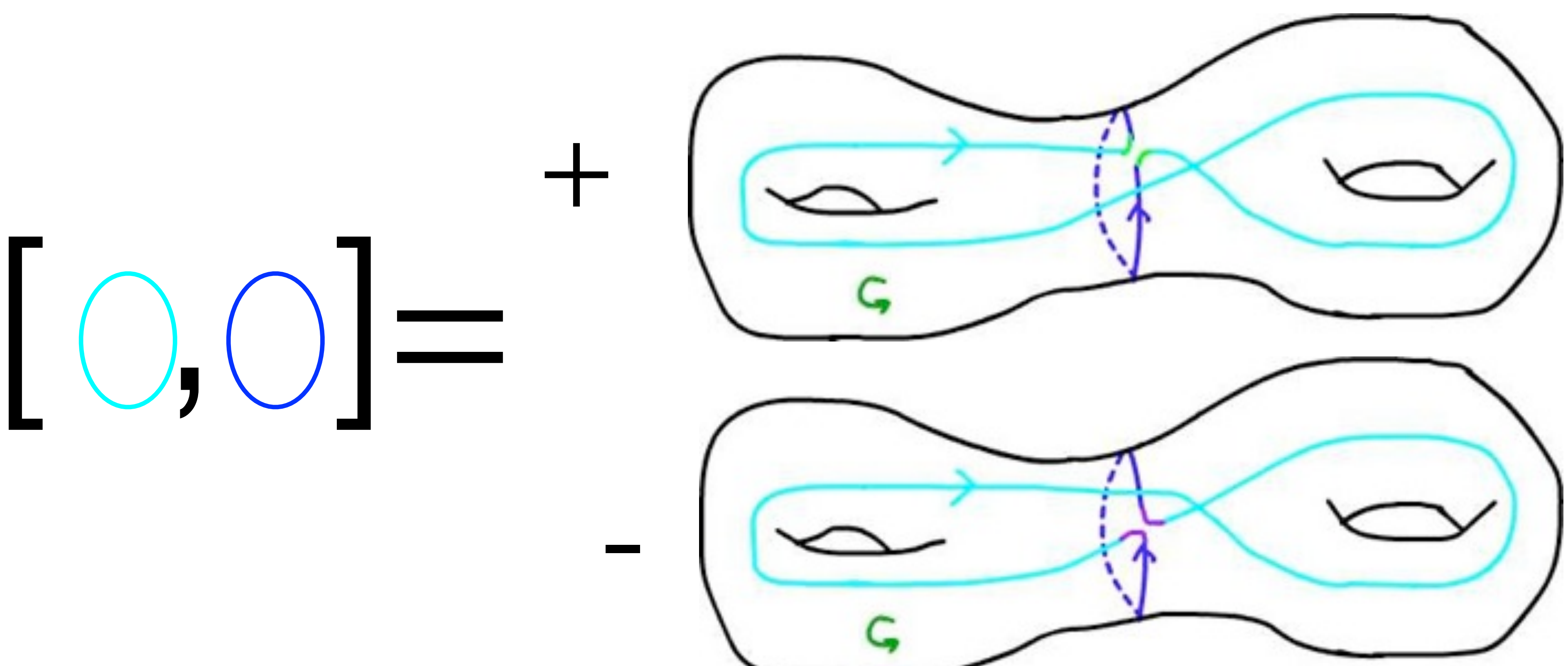




The Goldman bracket

Claim: that these two terms are different.





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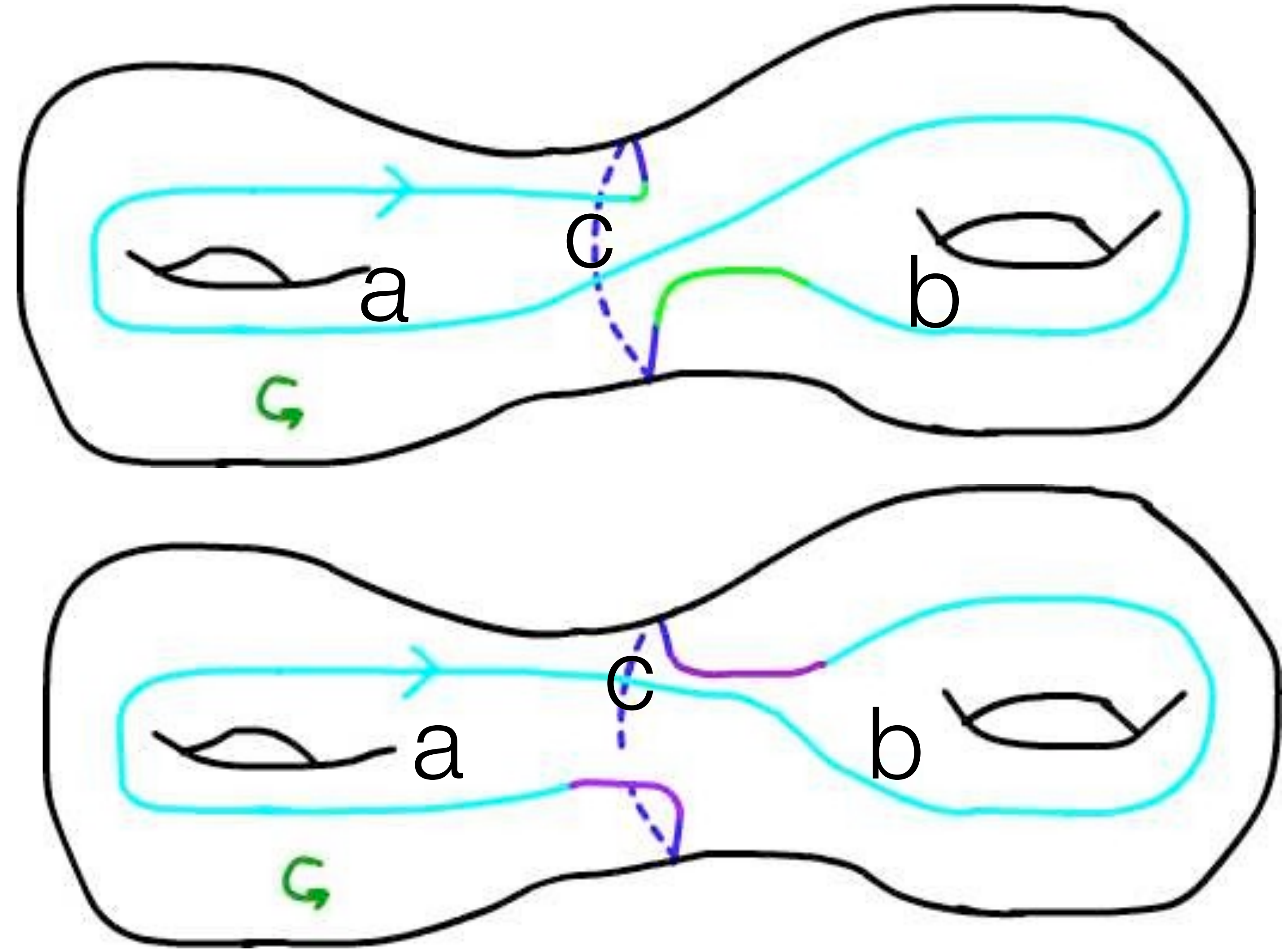

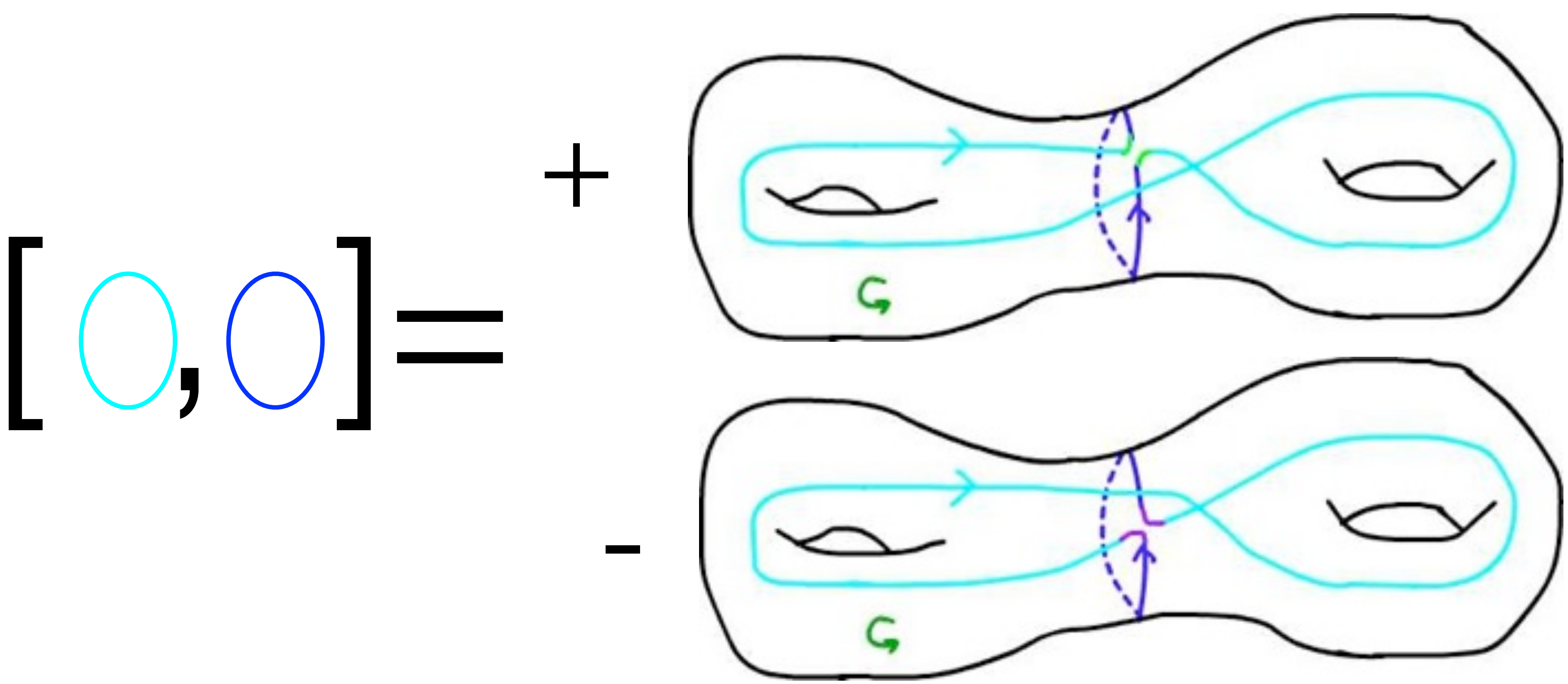

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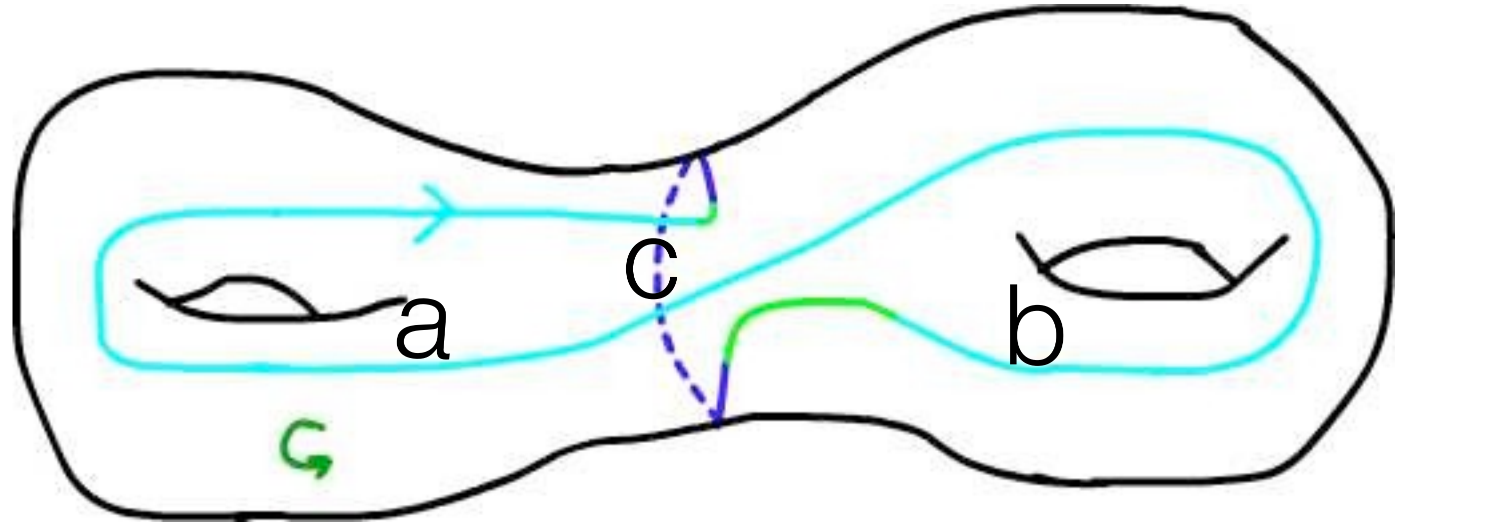

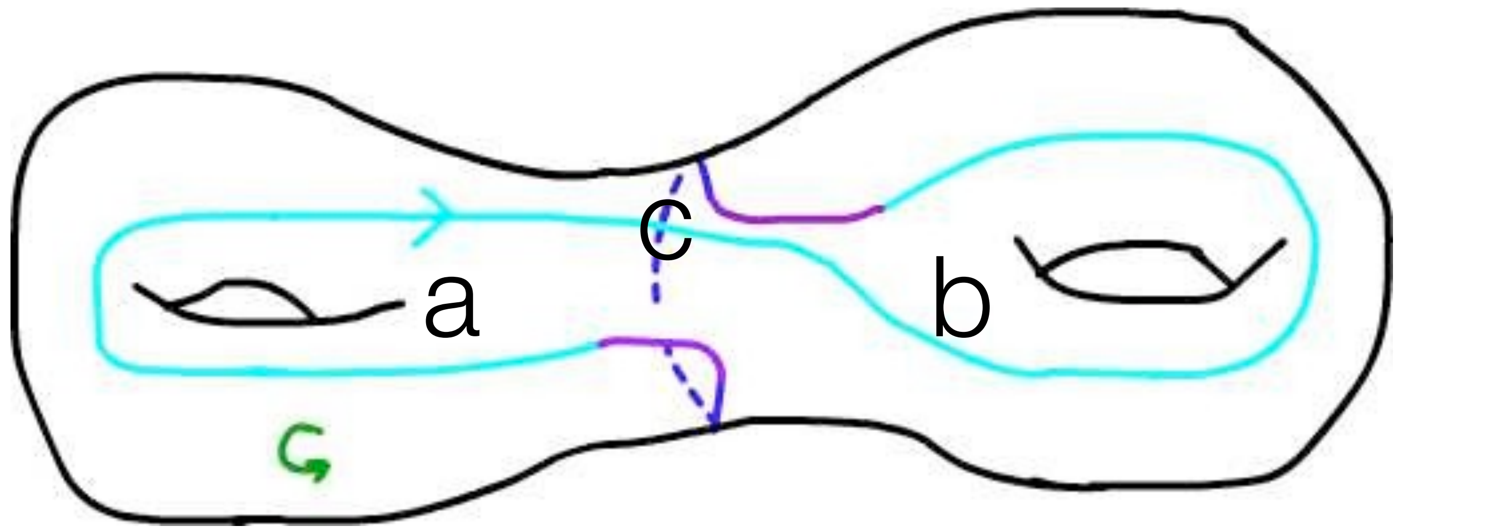

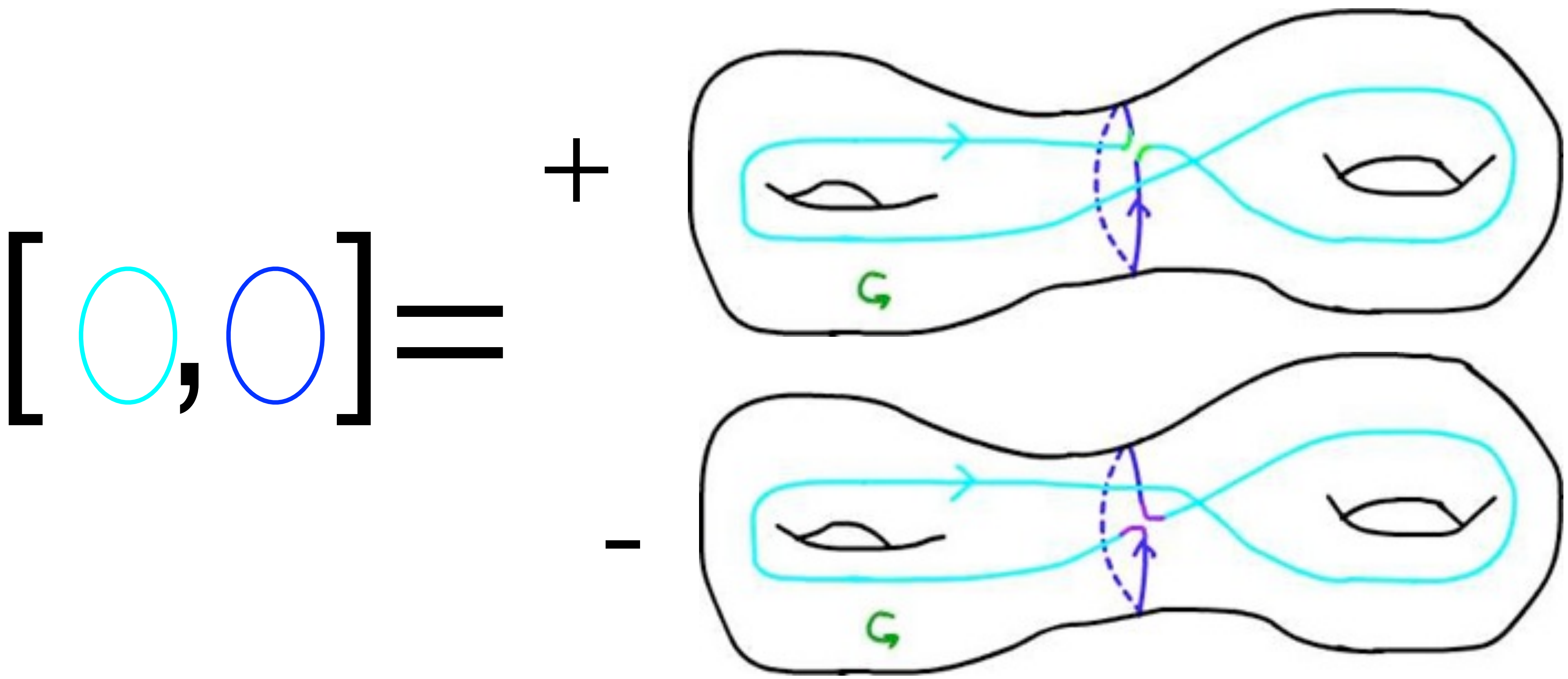

Observe that the two terms are different.

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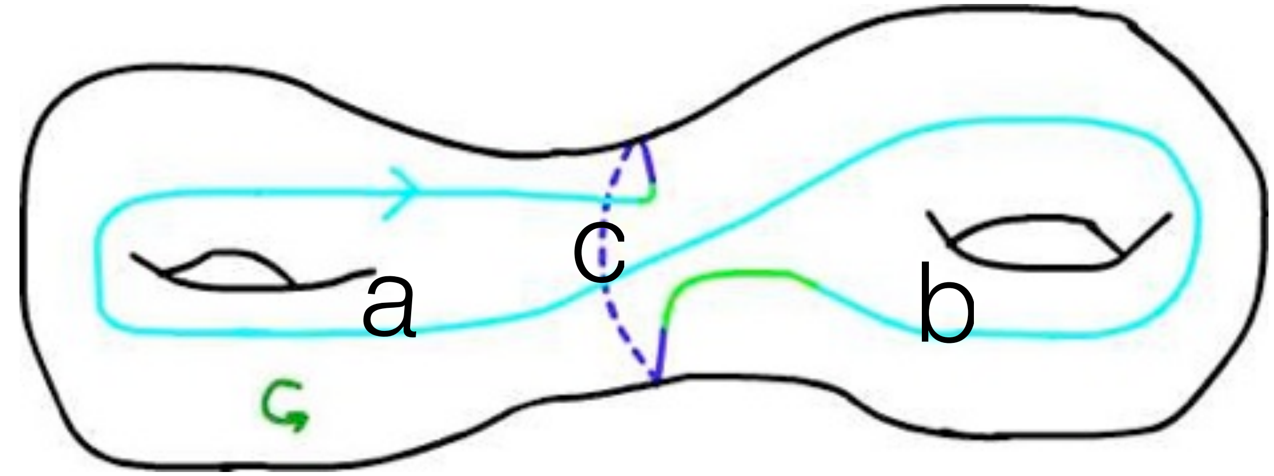

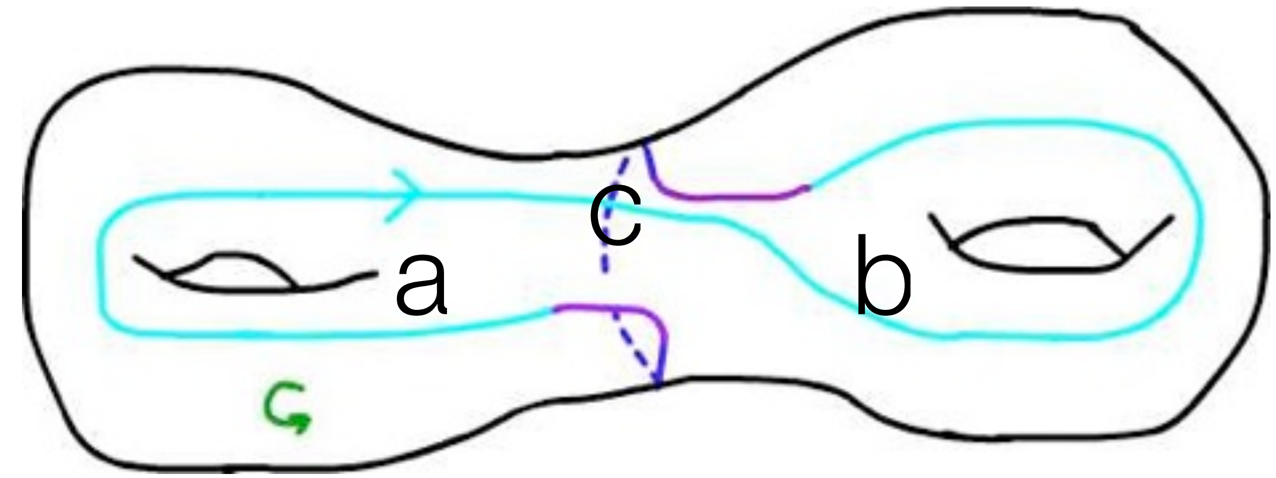

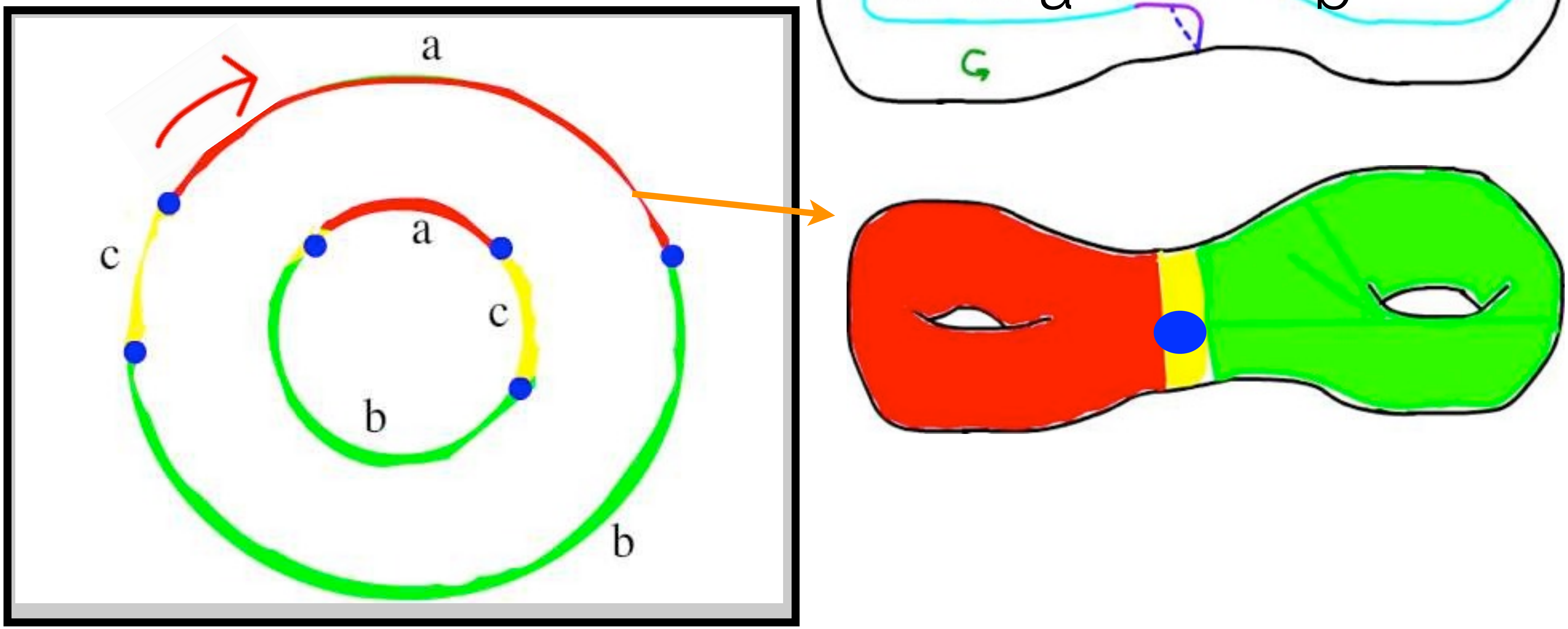

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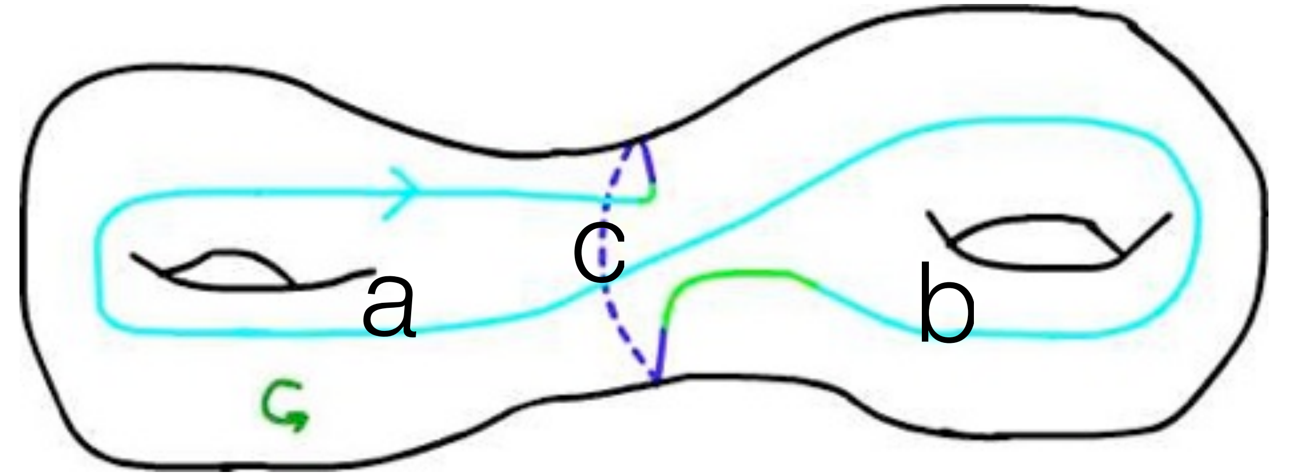

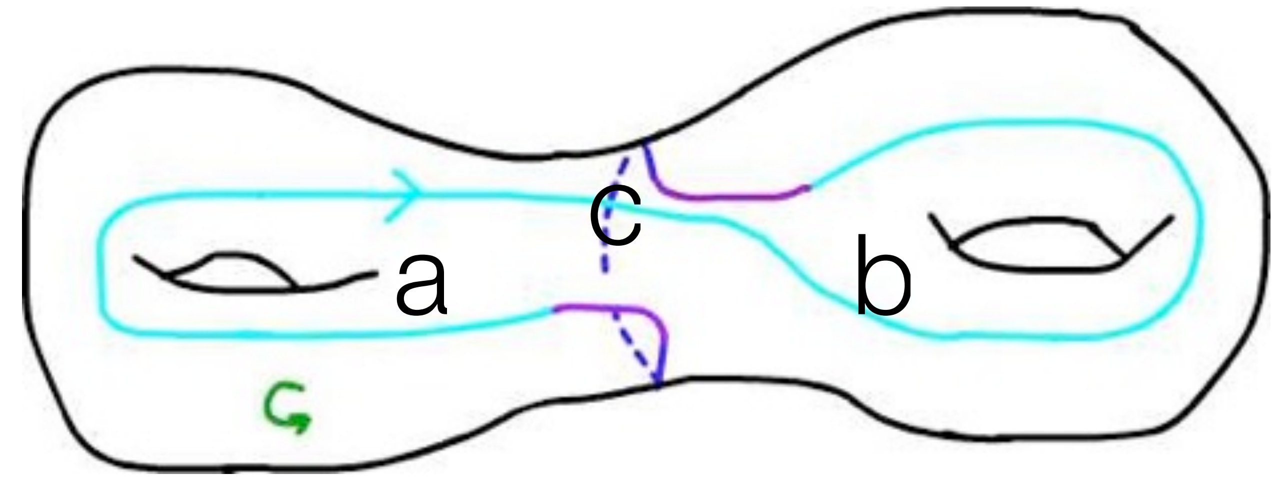

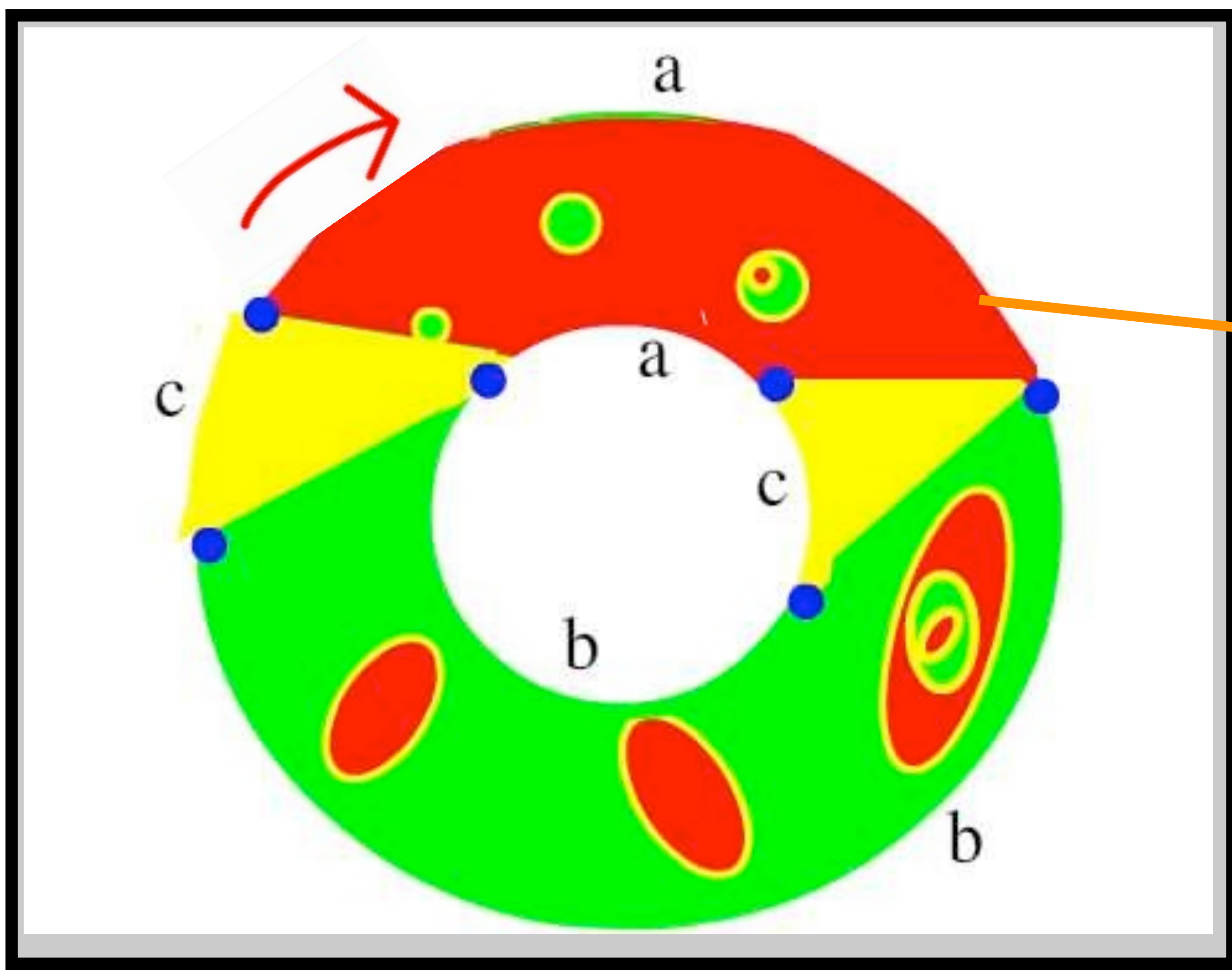

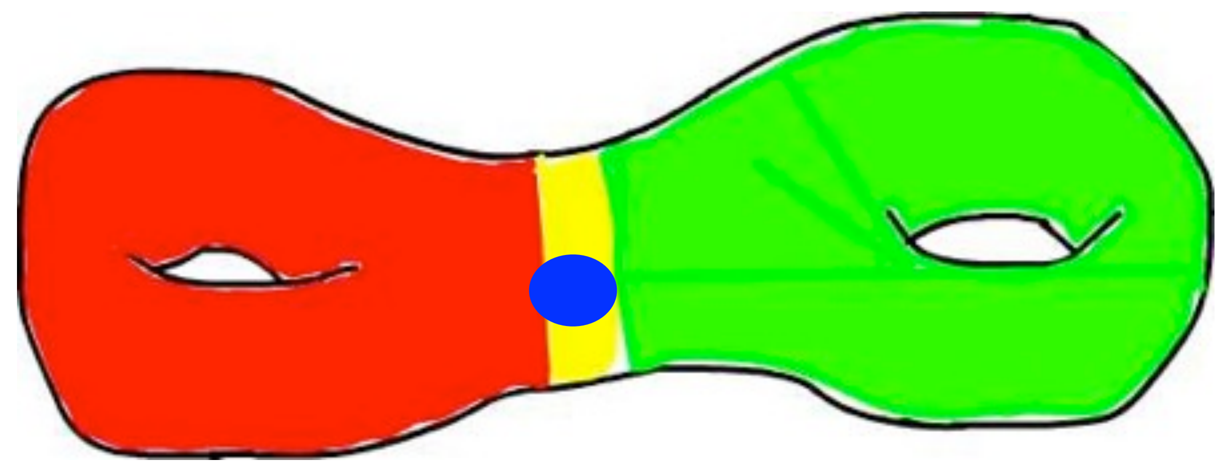

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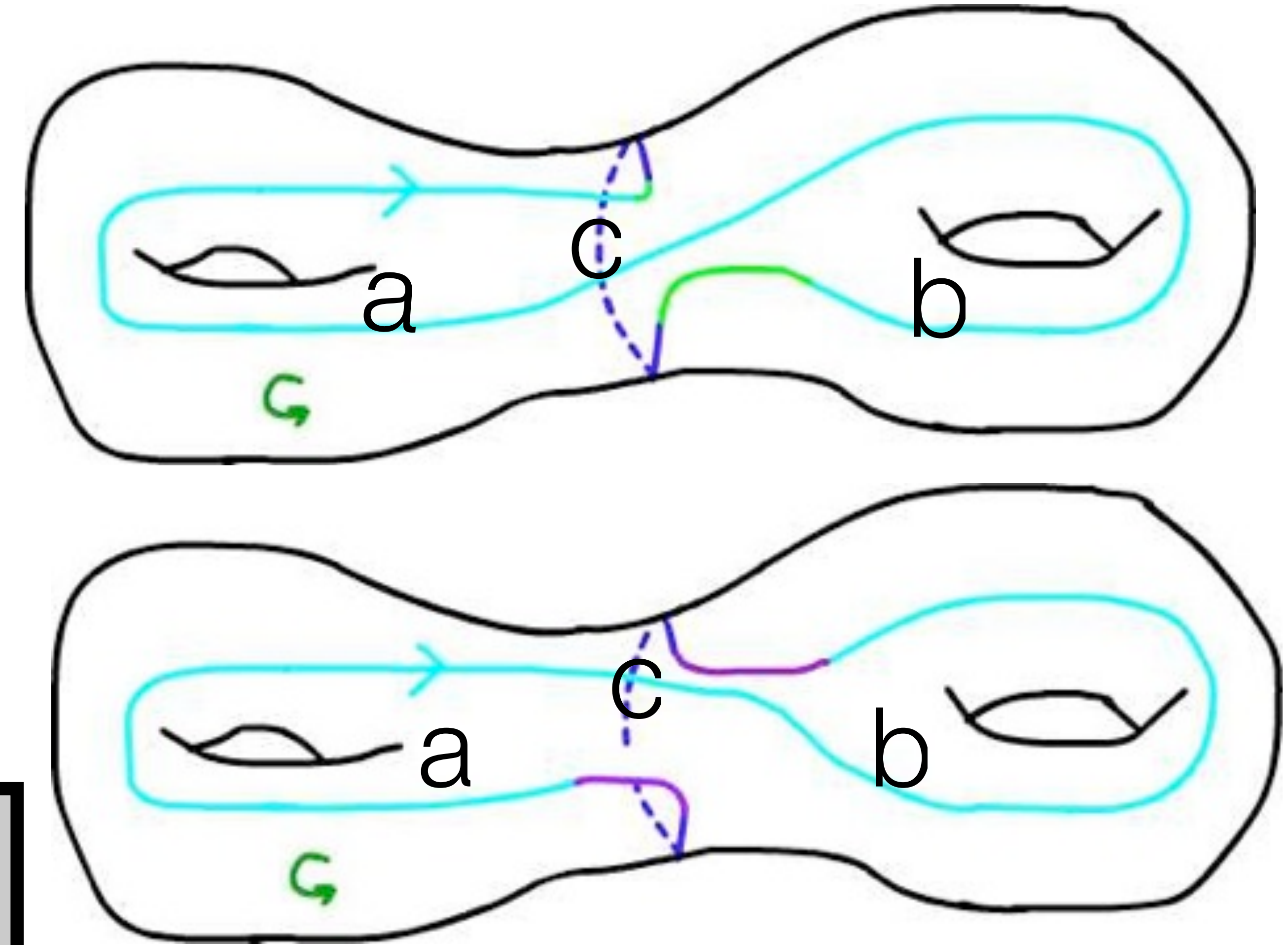

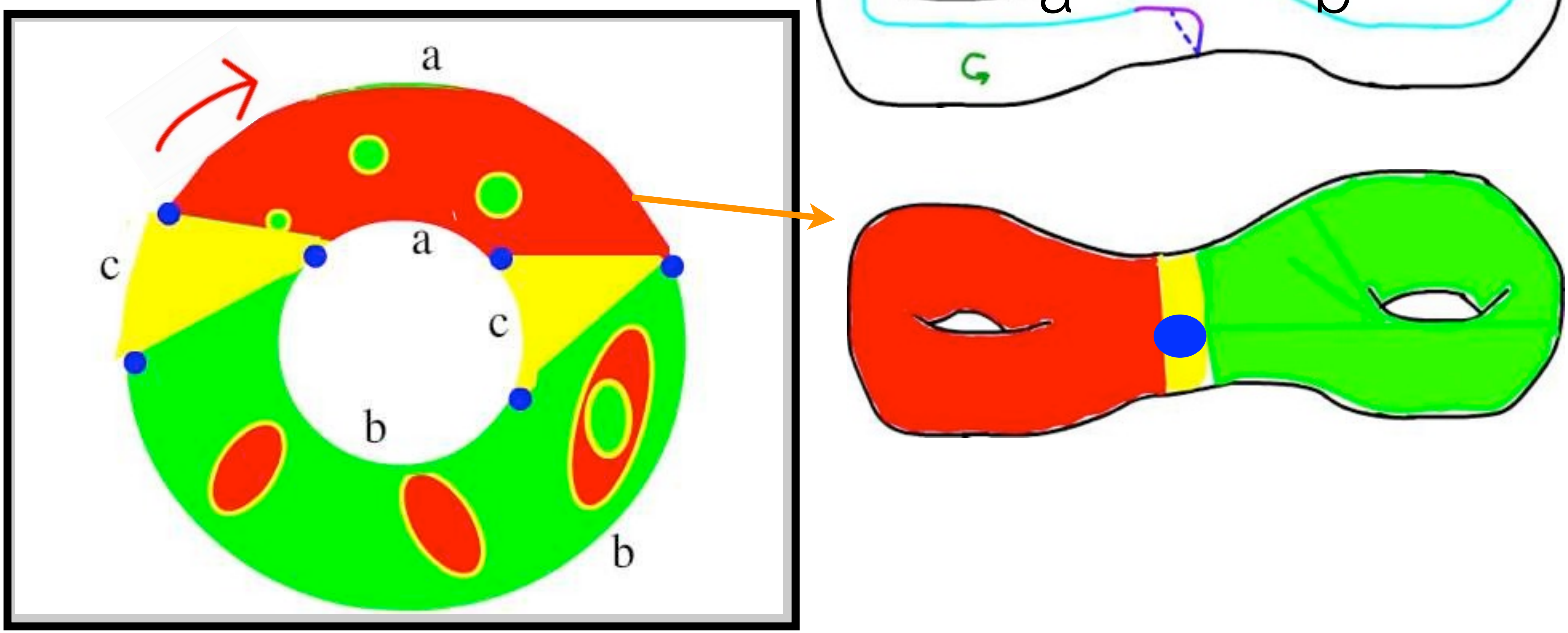

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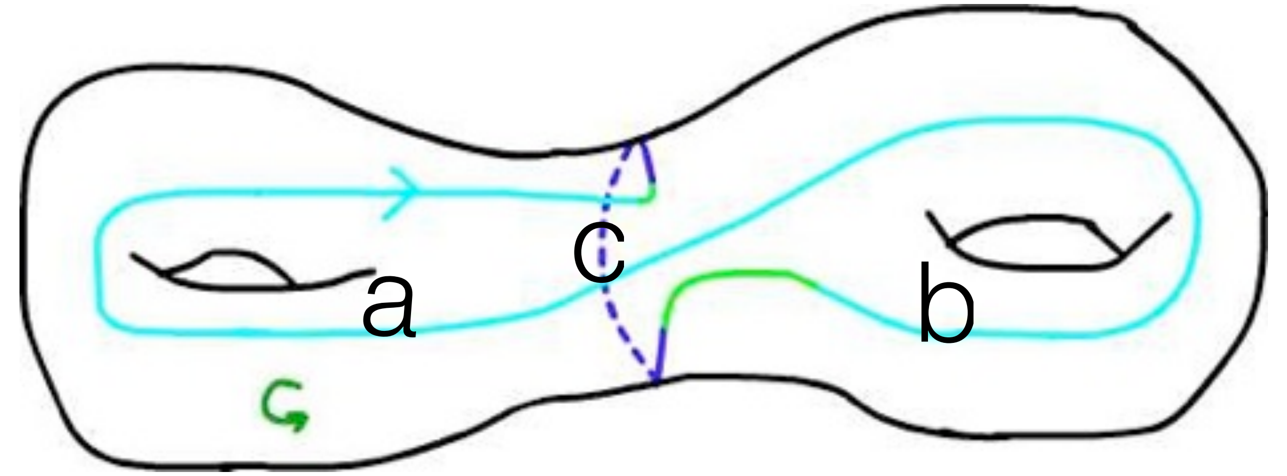

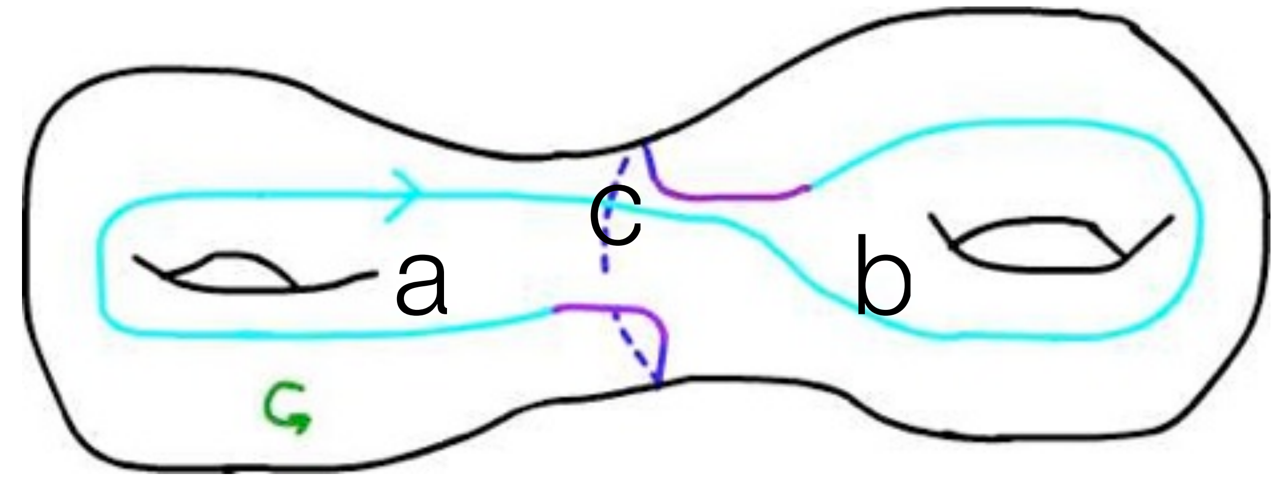

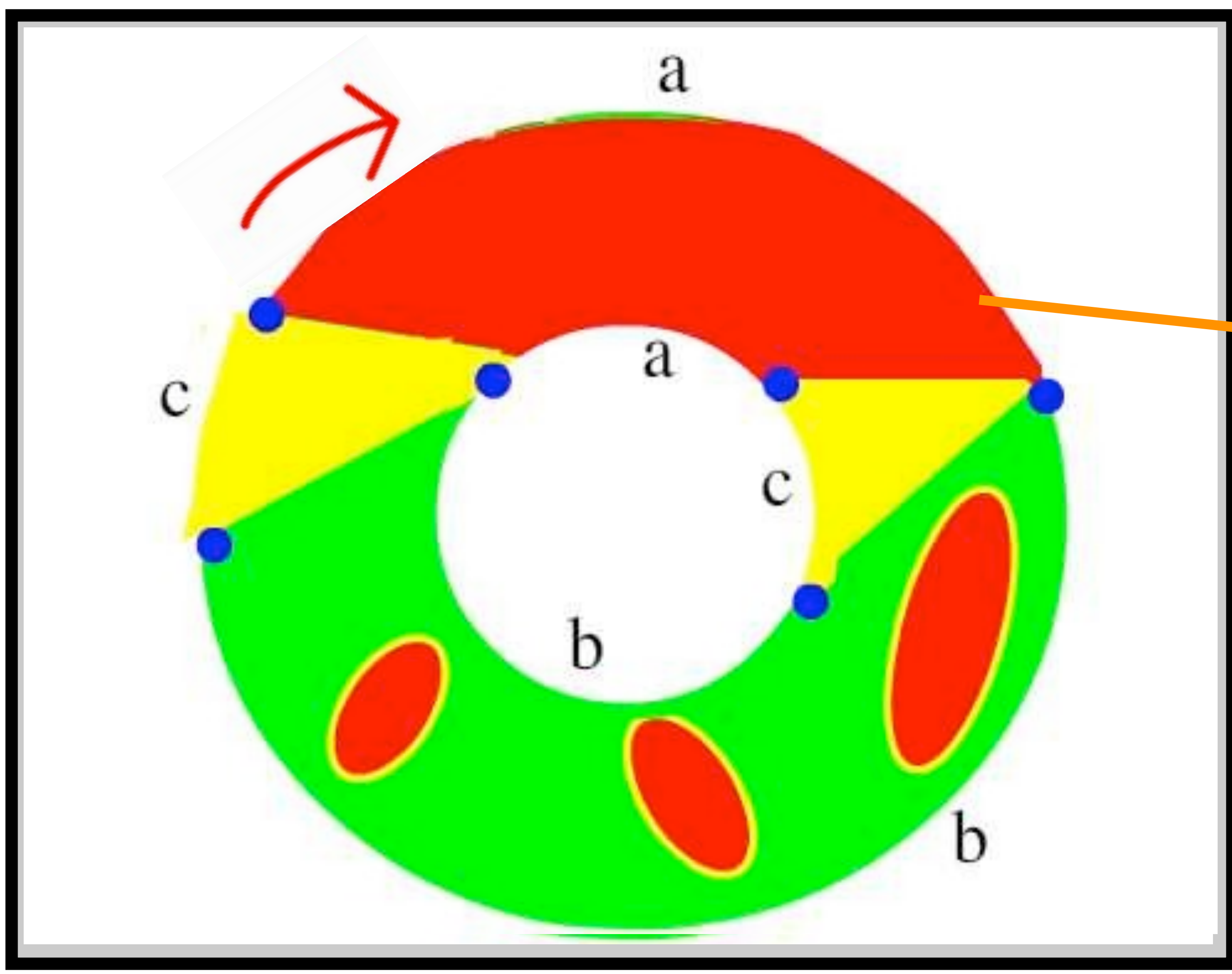

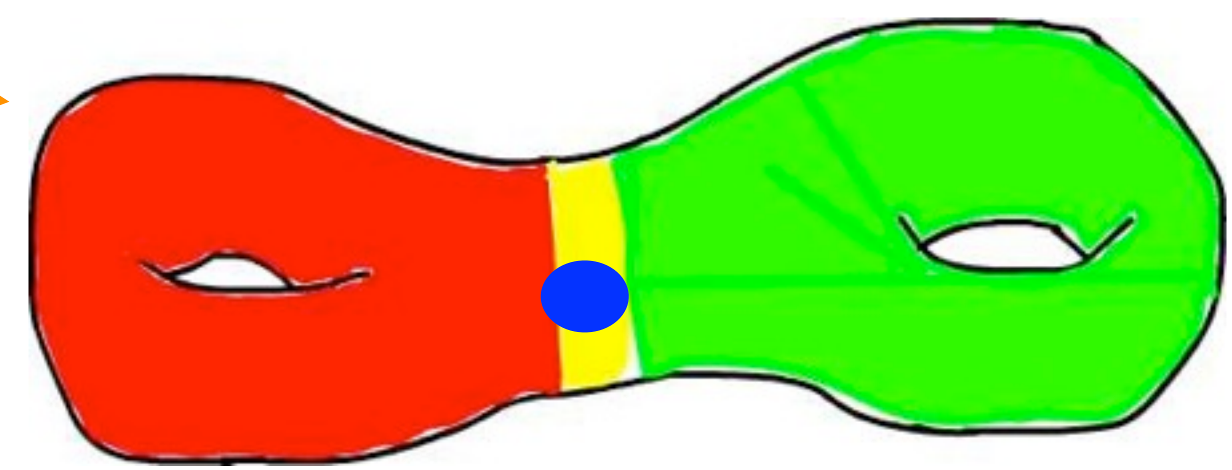

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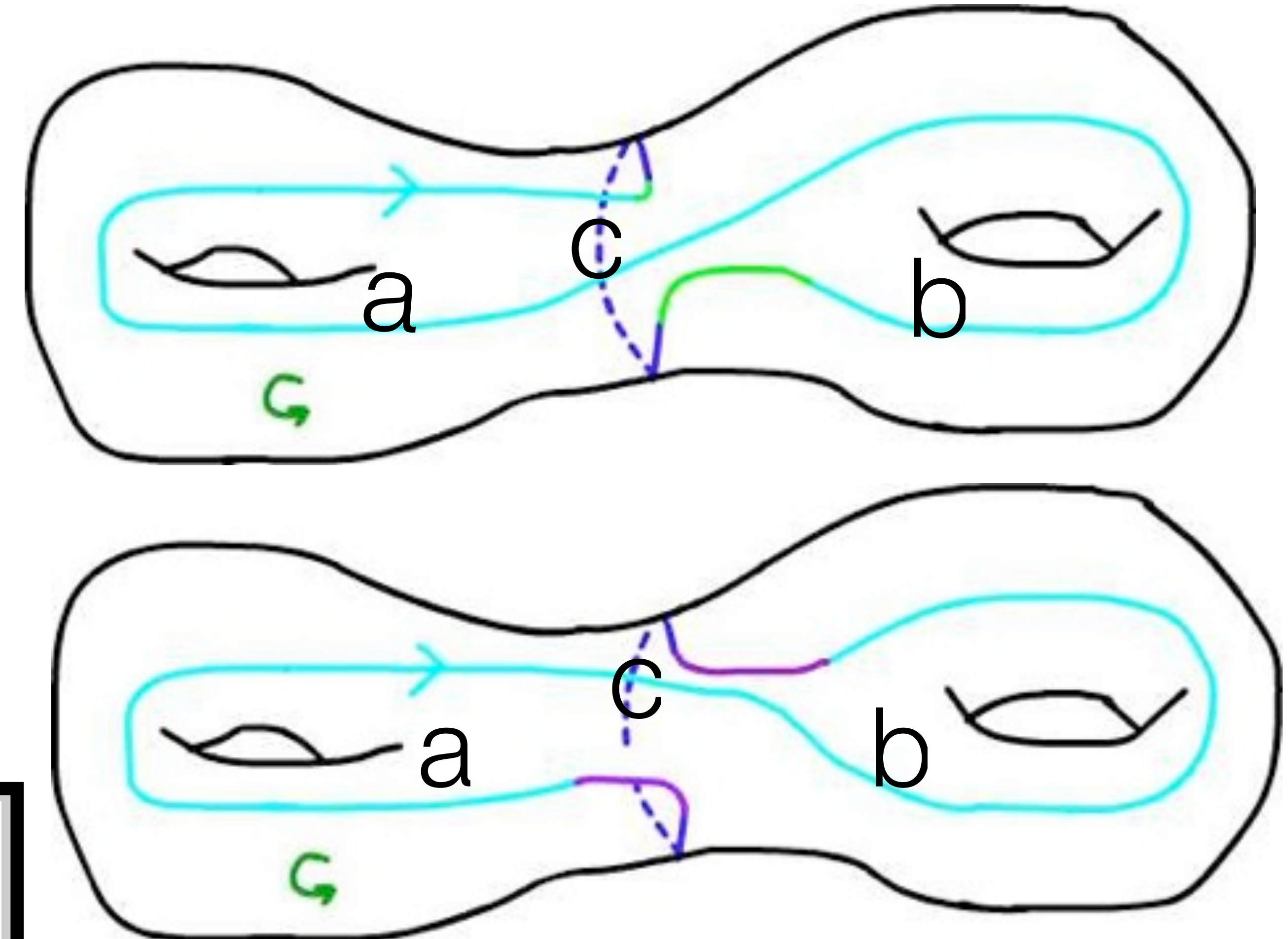

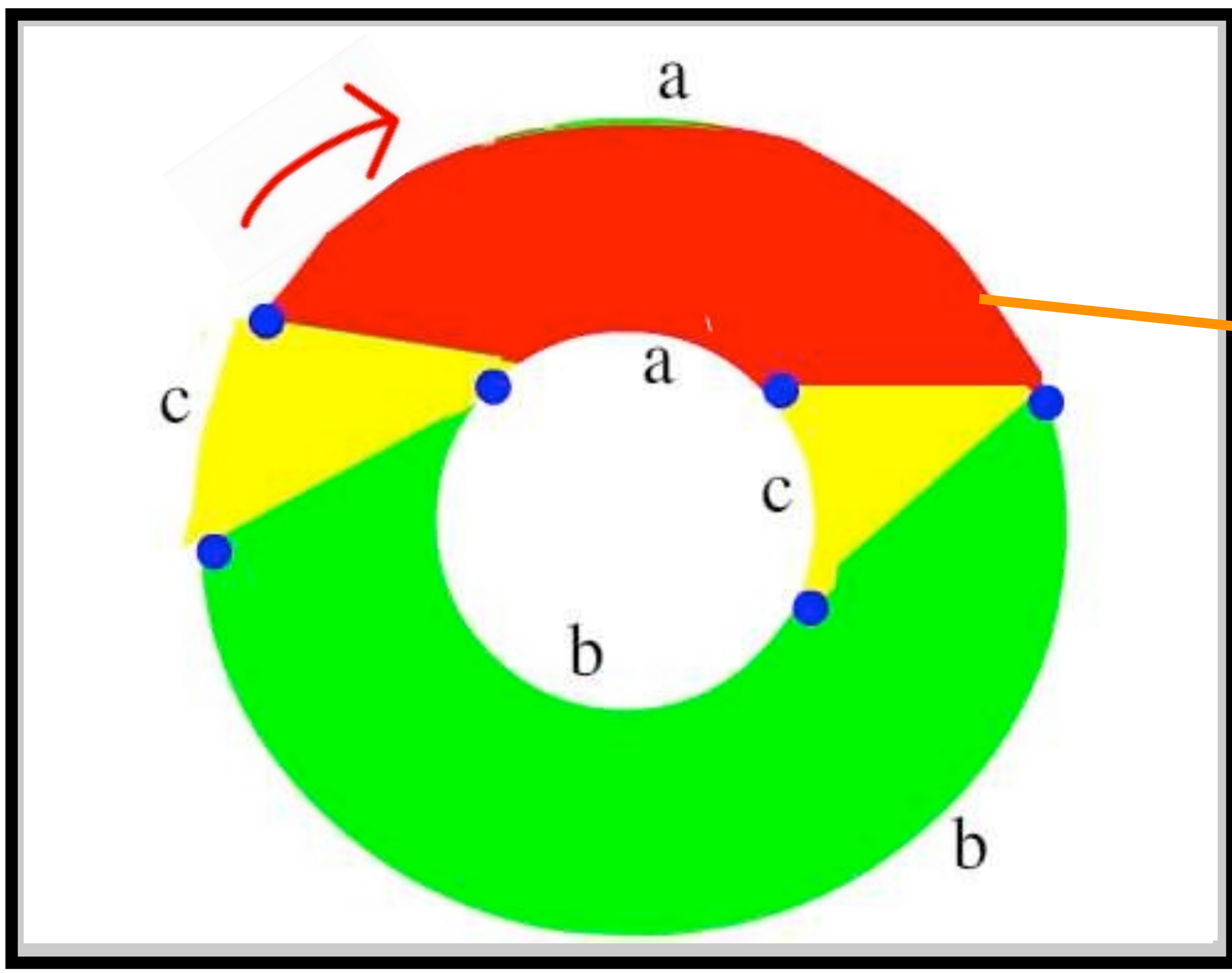

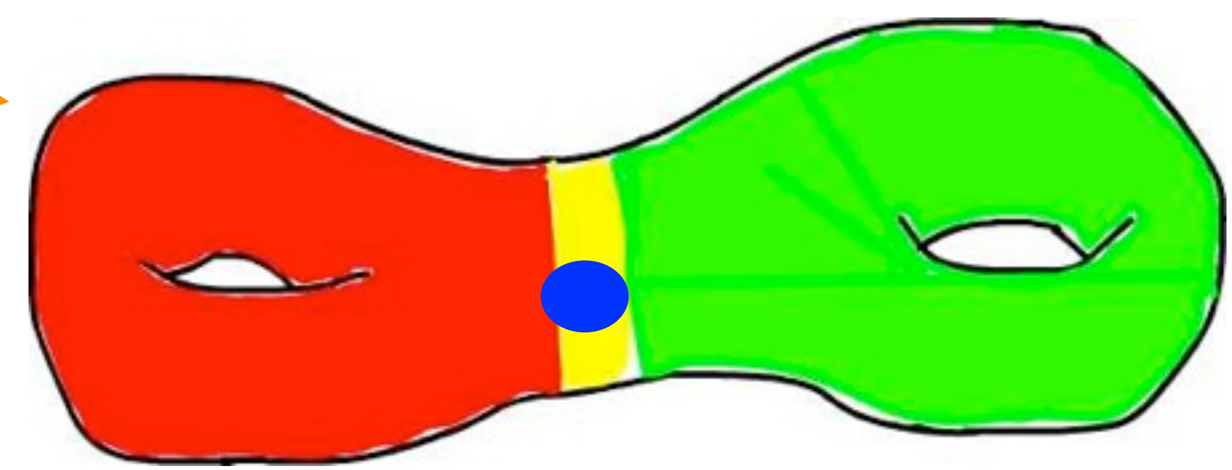

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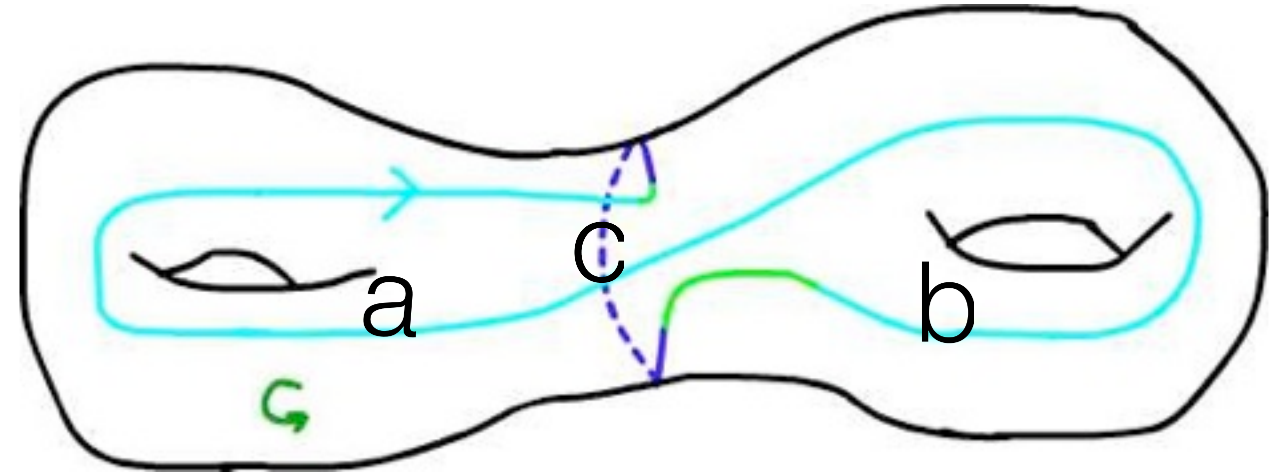

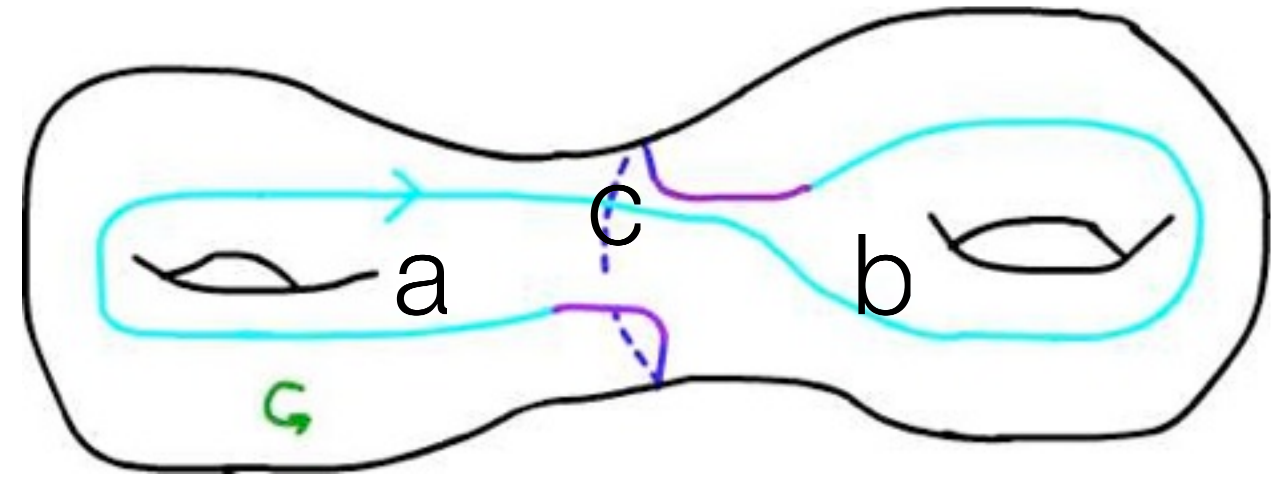

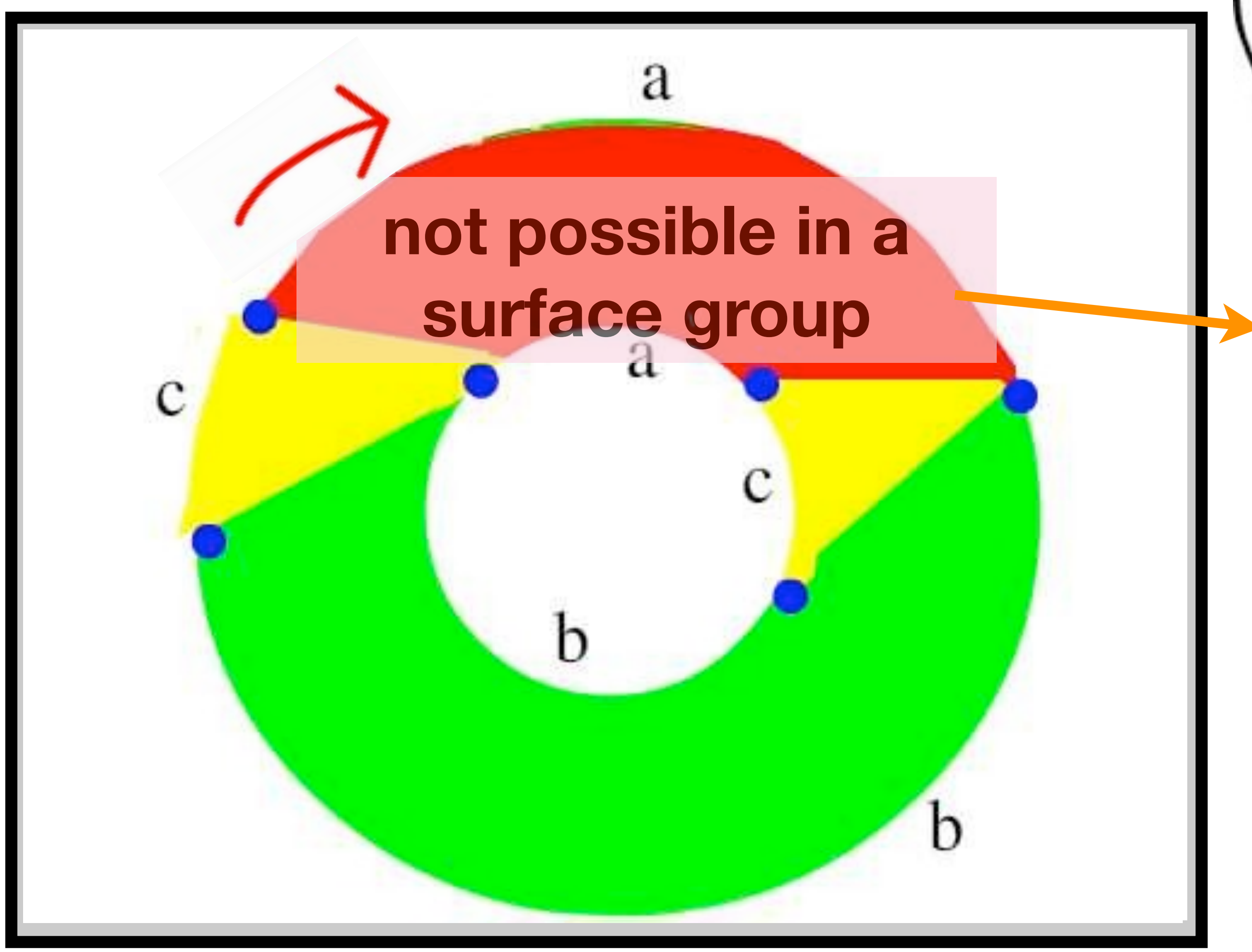

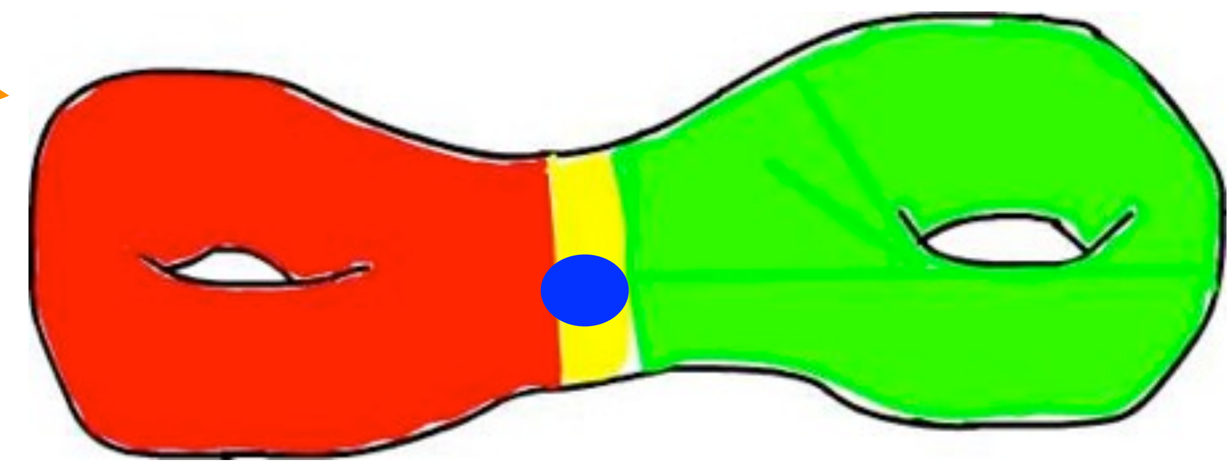

Proof of Jacobi identity

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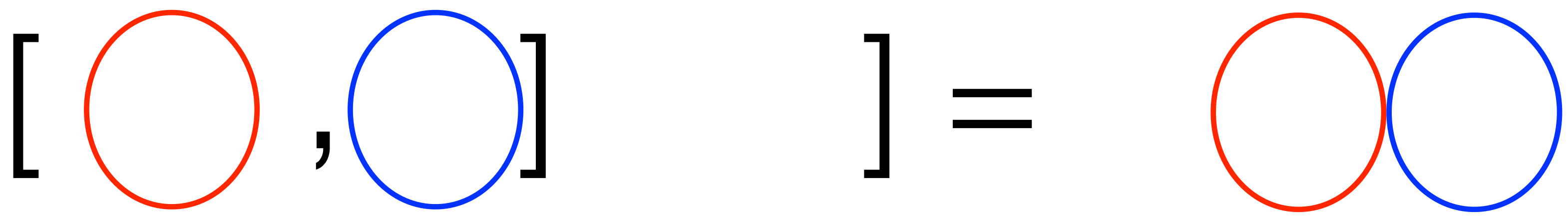

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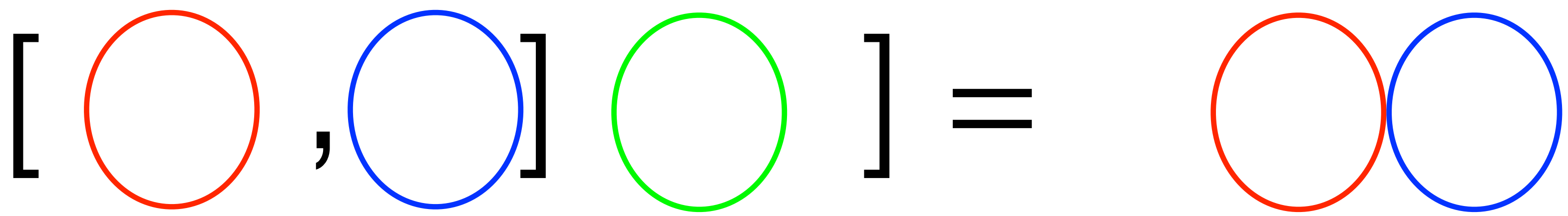

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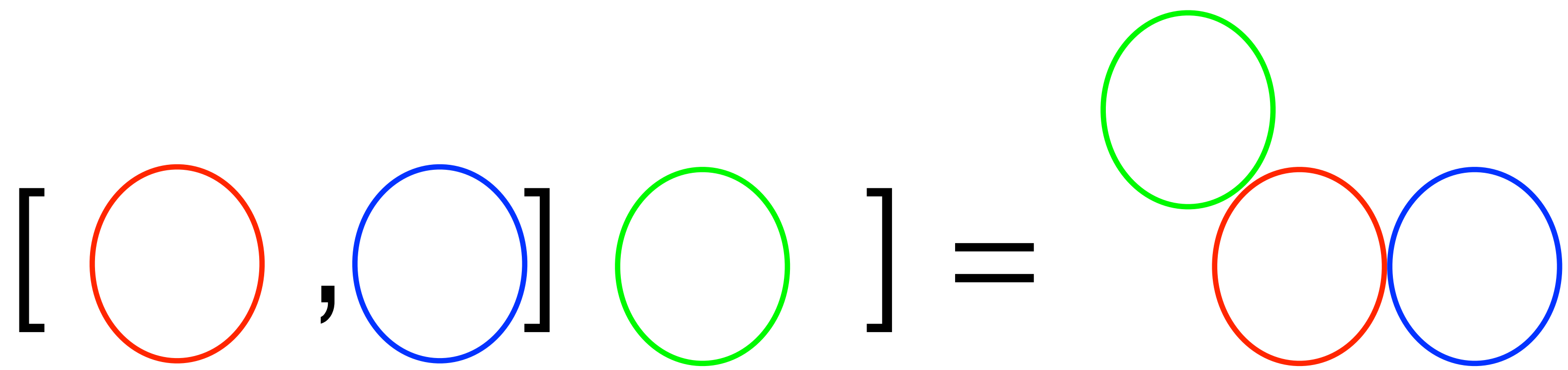

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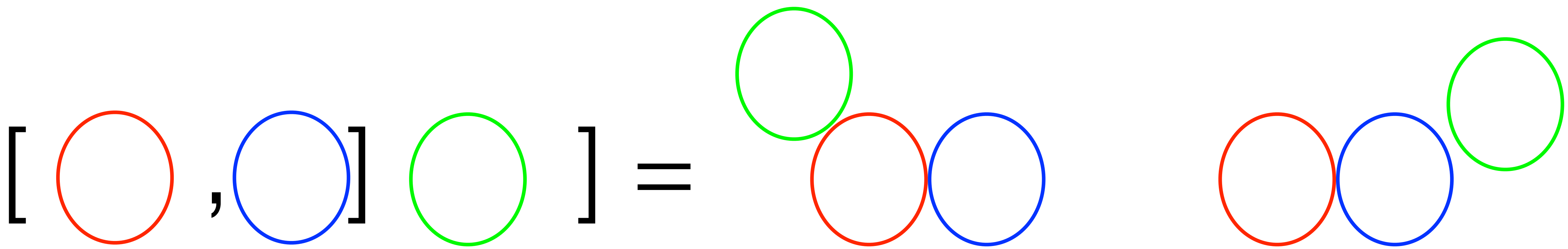

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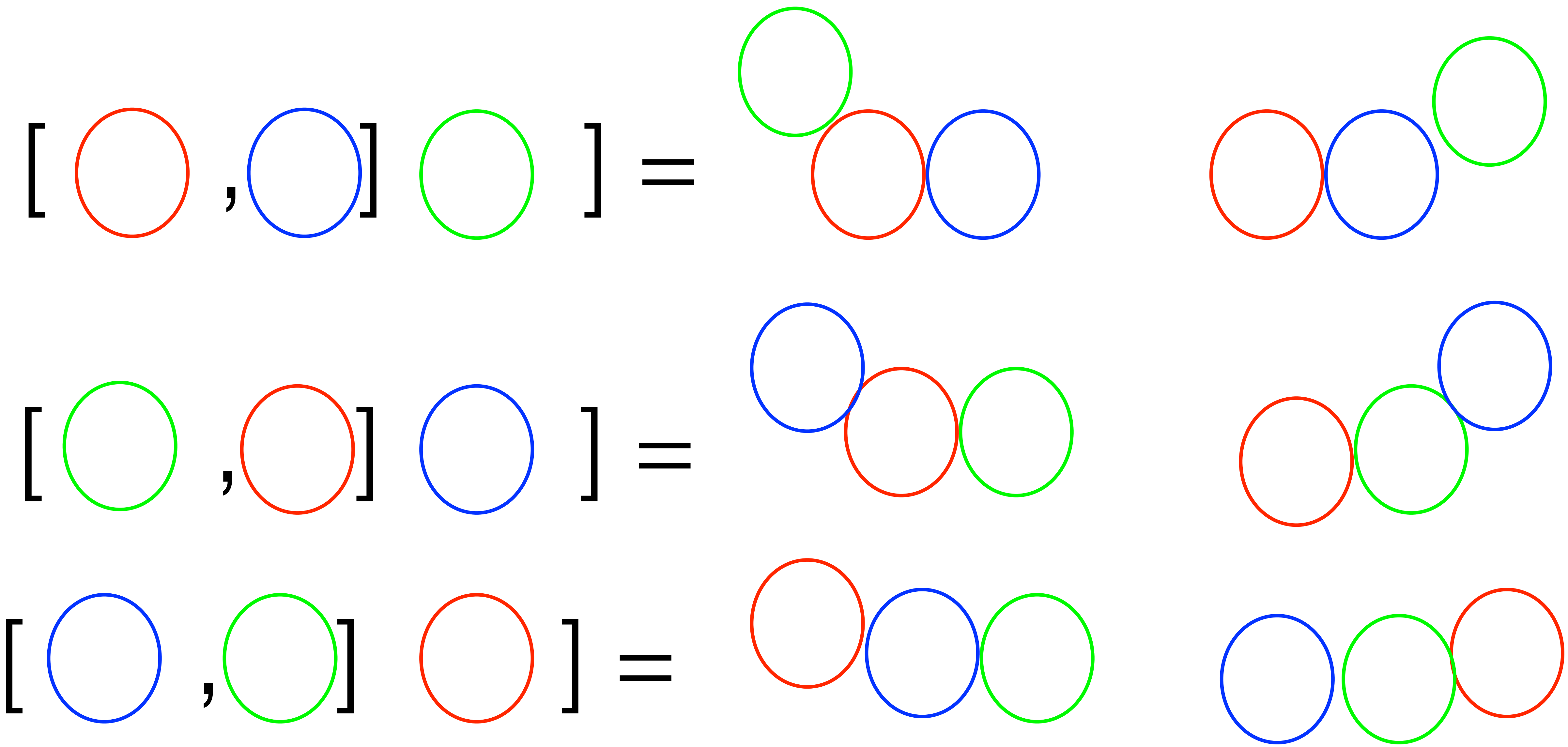

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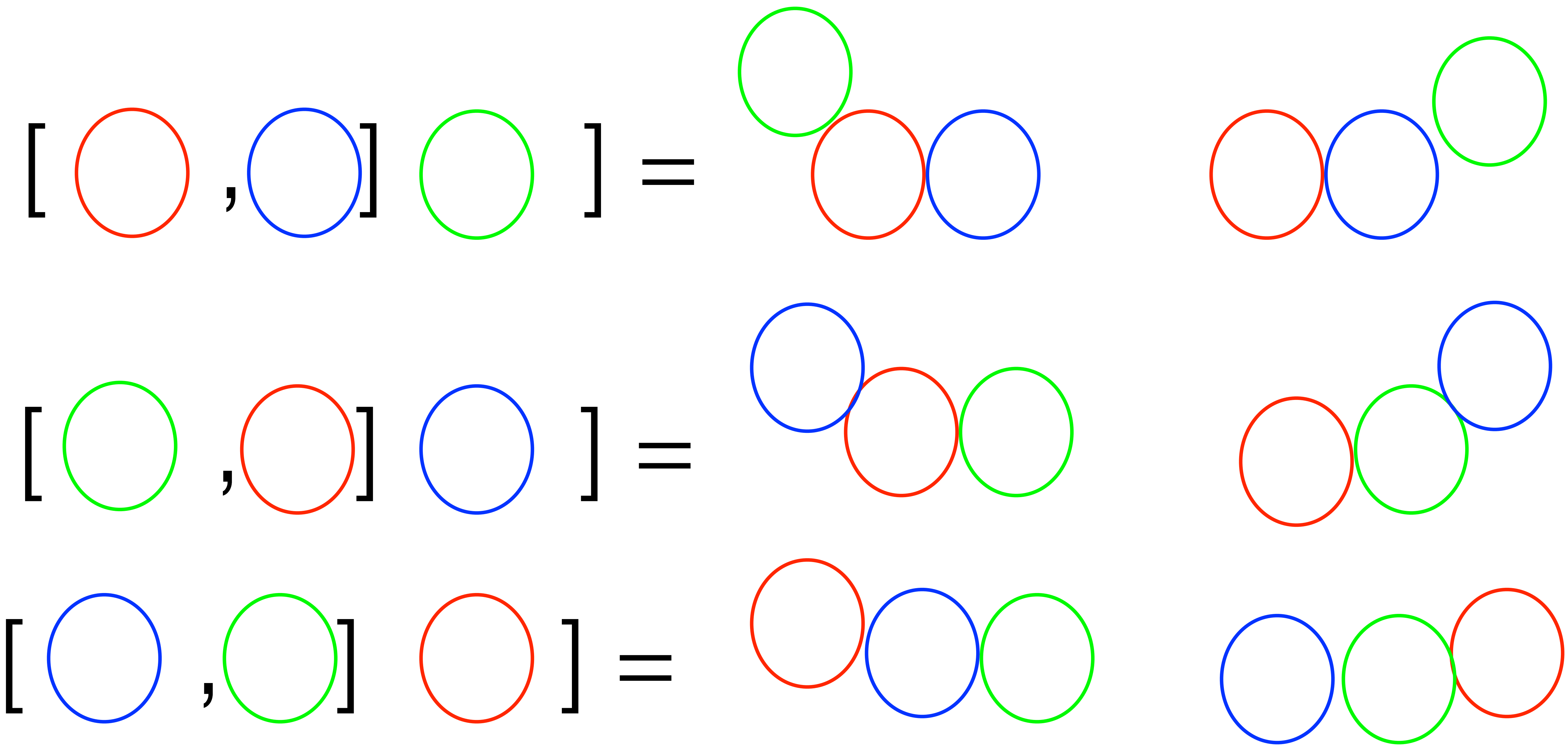
Theorem: (Goldman, 1986)
The bracket is well defined and satisfies the Jacobi identity.



- Write w in $Z[\pi_0]$ as a linear combination of elements of π_0. The sum of the absolute value of the coefficients is the *Manhattan norm of w (or l_1 norm)*, denoted ***M(w)***.

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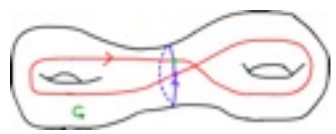
Goal: Study relation between M([x,y]) and i(X,Y).

<table>
<tr><td>S = orientable surface
(or orbifold)</td><td>M^3 = compact, orientable, irreducible, with contractible universal cover.</td></tr>
<tr><td>Goldman Bracket: 
Lie Bracket on (linear combination of) closed, oriented free homotopy classes of curves.</td><td>String bracket: Lie bracket on (linear combination of) families of oriented closed curves. </td></tr>
<tr><td colspan="2">Combinatorial presentation</td></tr>
<tr><td colspan="2">The bracket encodes the intersection structure in terms of the Manhattan norm.</td></tr>
<tr><td>Different surfaces have different Goldman Lie algebras</td><td>String bracket gives the H-S graph of the graph of groups in the celebrated torus decomposition</td></tr>
</table>

We are not unaware of the connections with geometrization.

$$M\,[X,Y] \leq i(X,Y)$$

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$M[X,Y] = i(X,Y)$?

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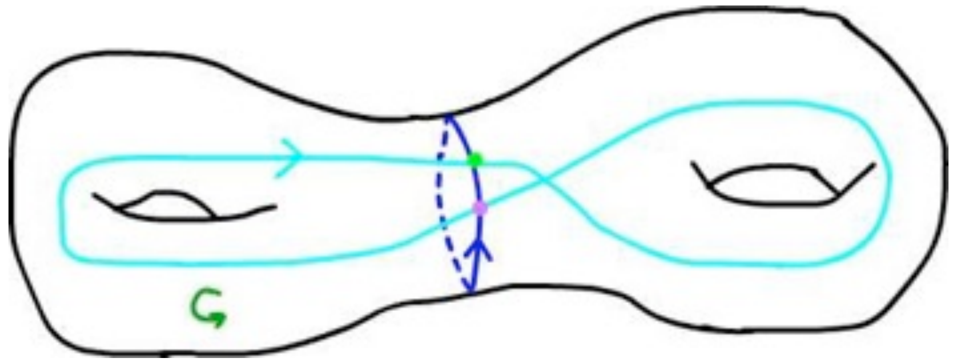
In this
example,
yes.



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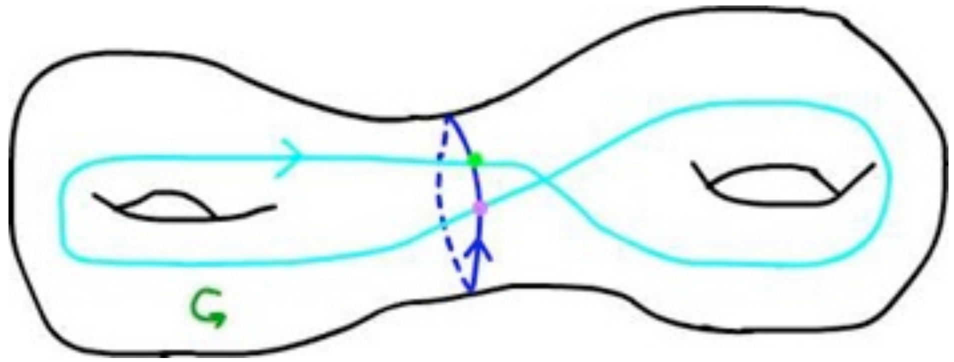

But not always...

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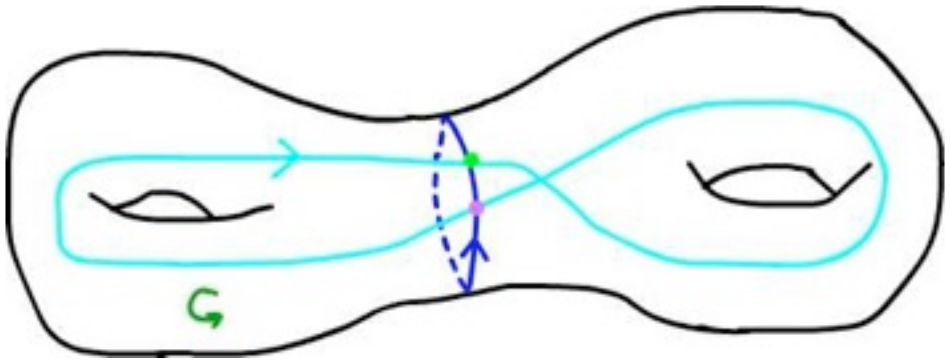

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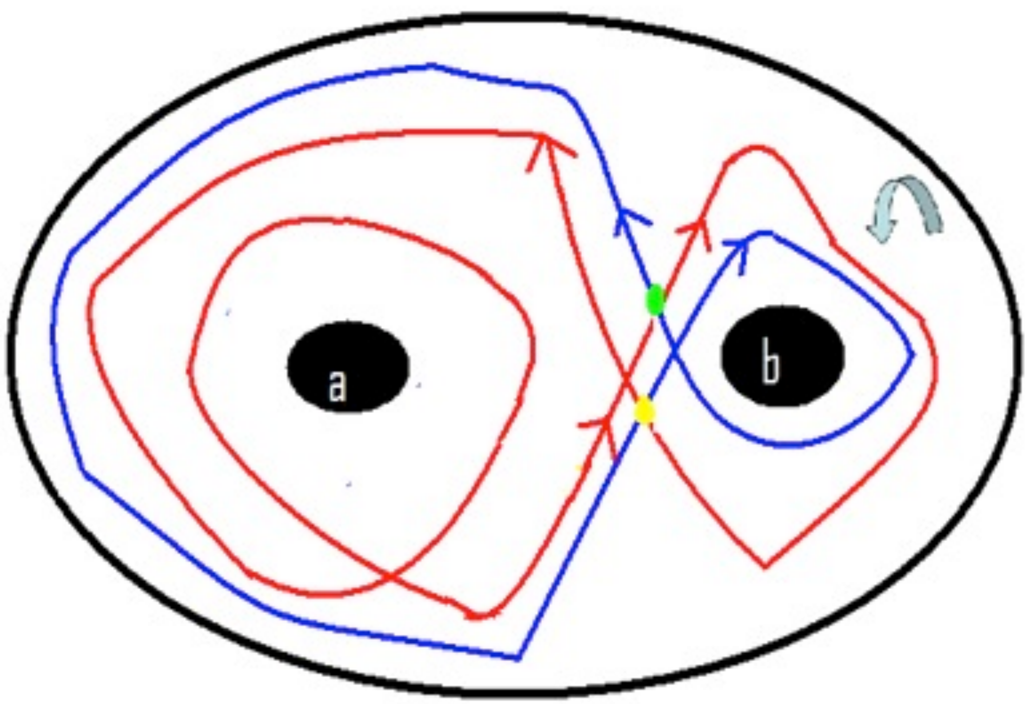

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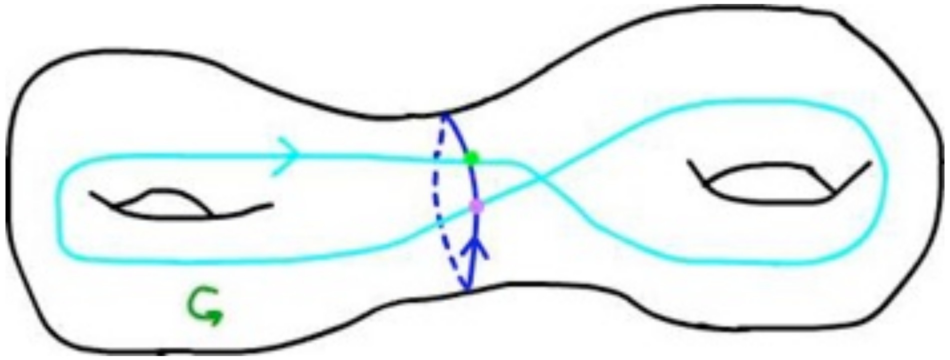

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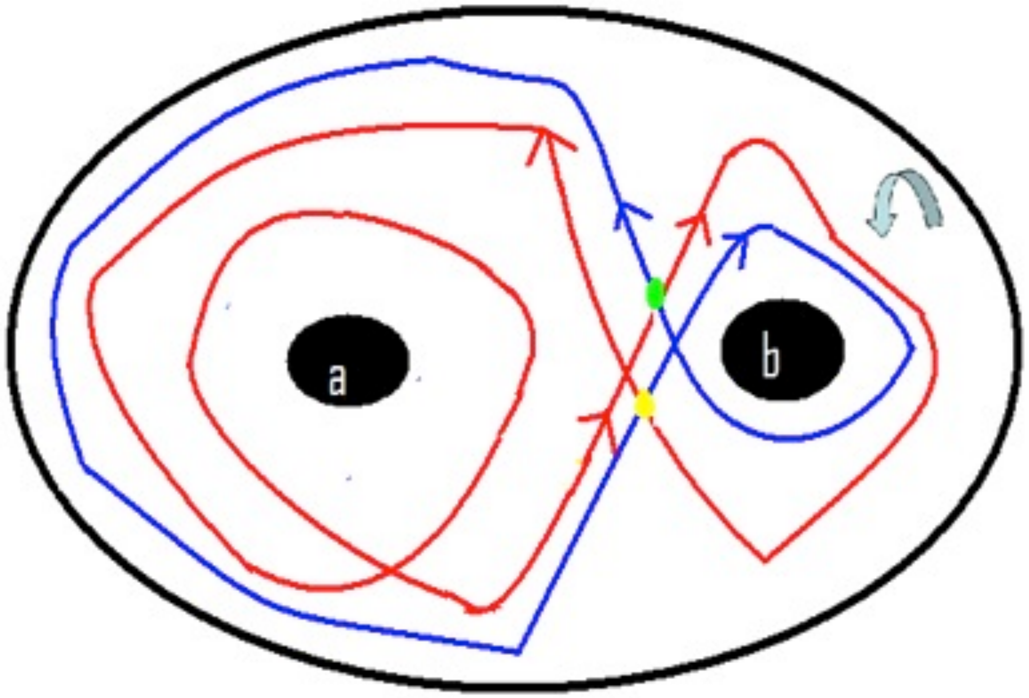

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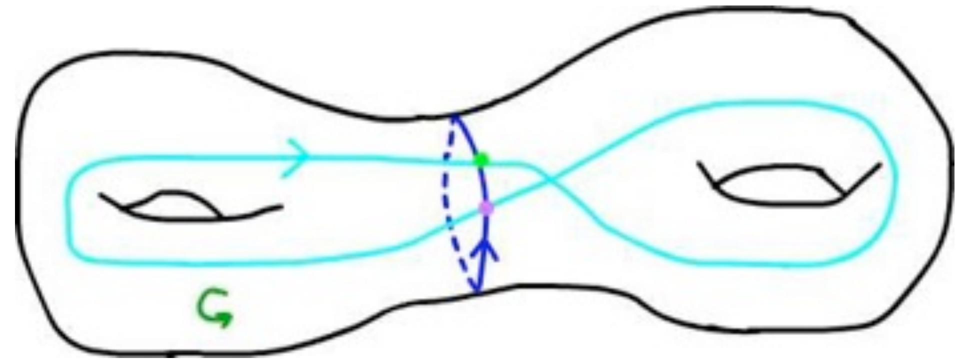

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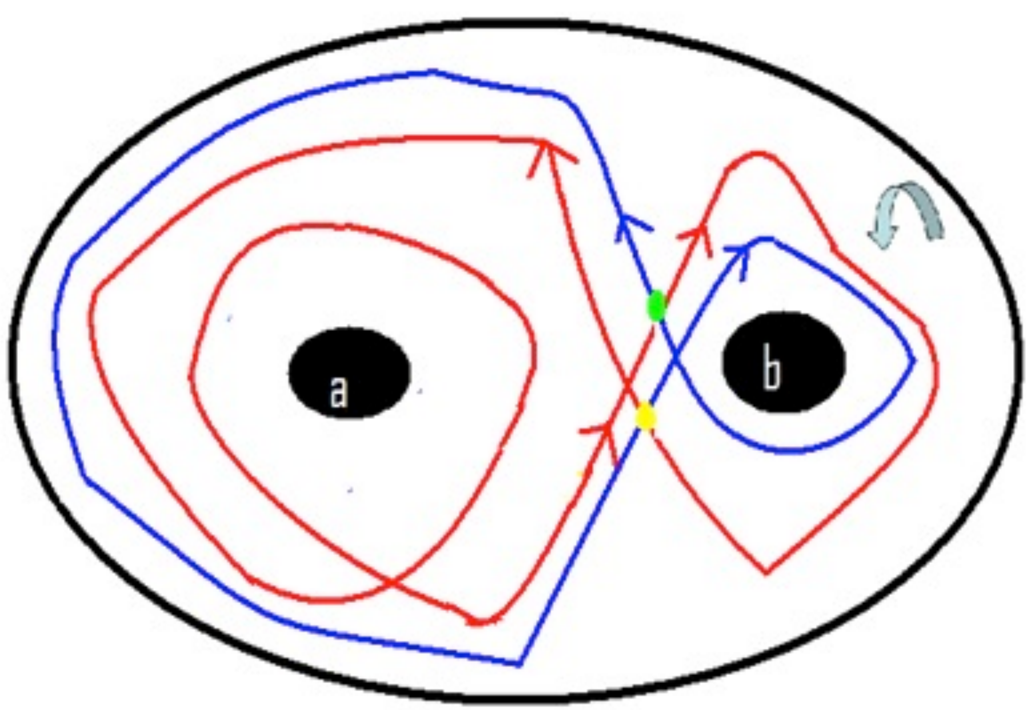

- aab ba = aabba

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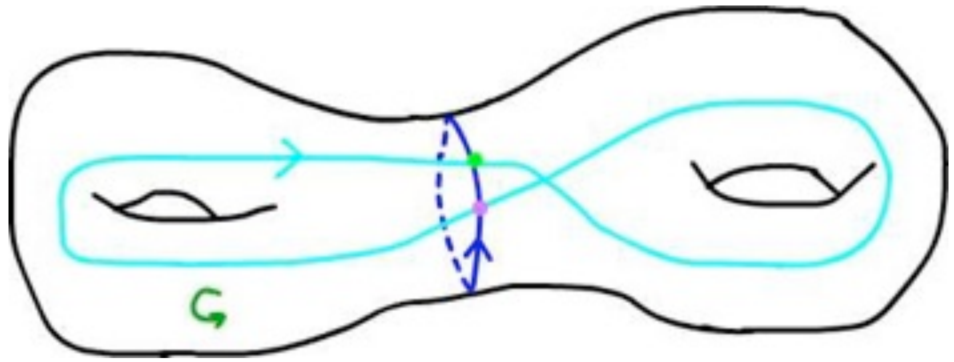

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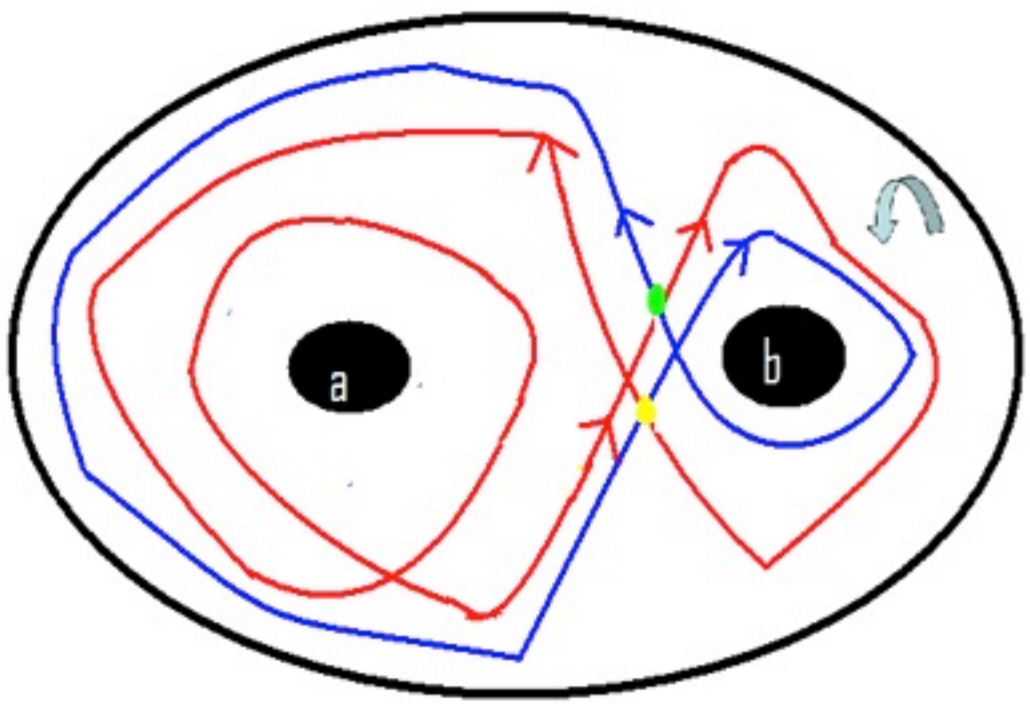

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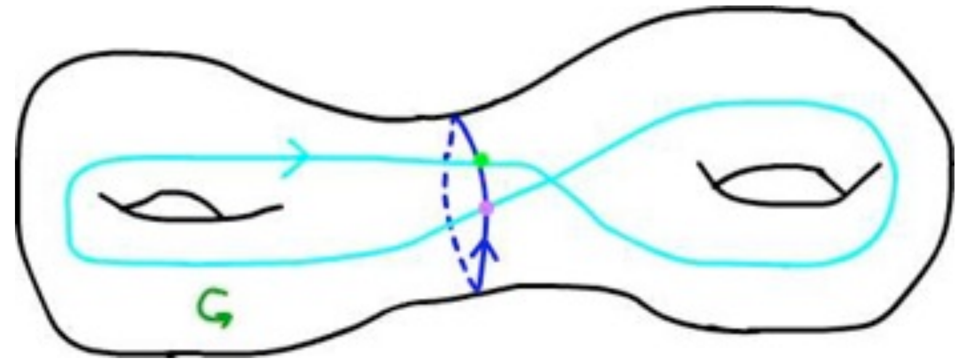

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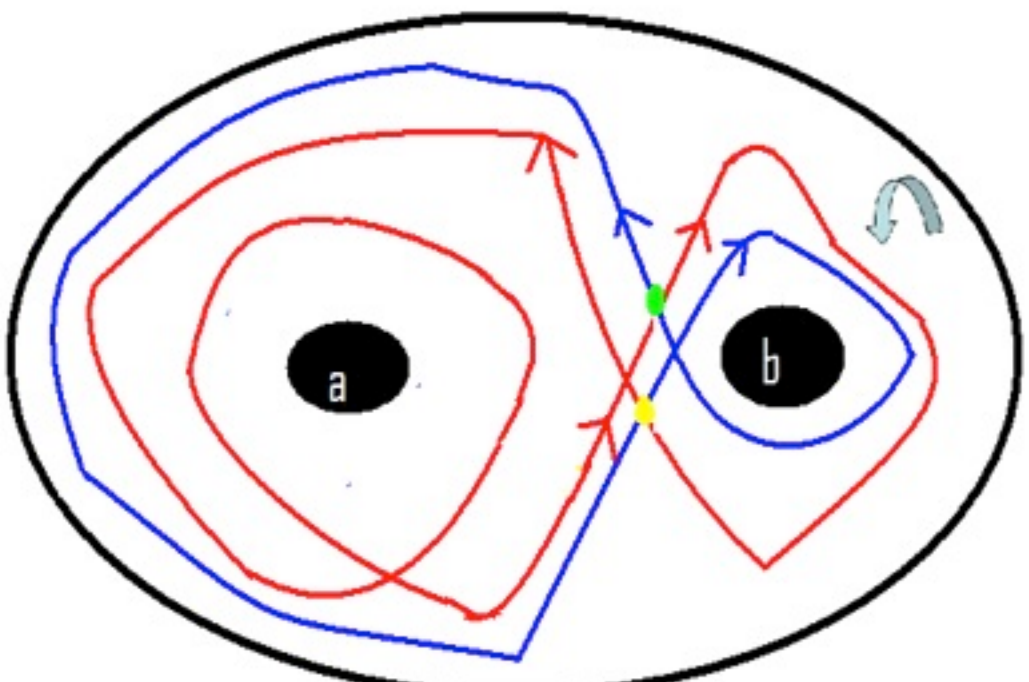

- aab ba = aabba
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[aab,ab]=0

i(aab,ab)=2

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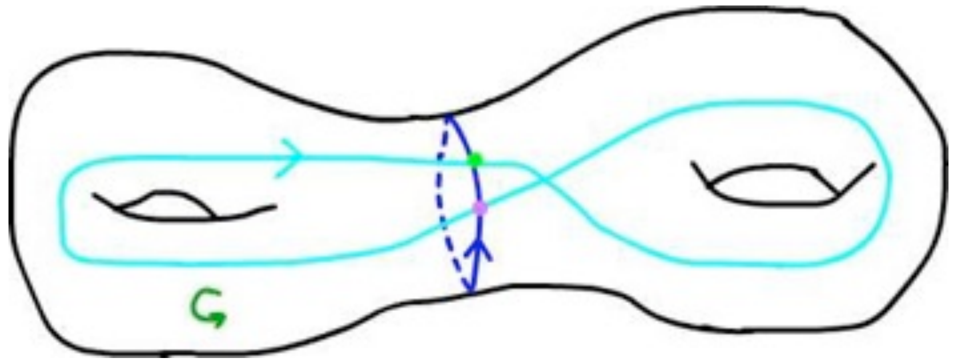

Combinatorial presentation of the Goldman bracket

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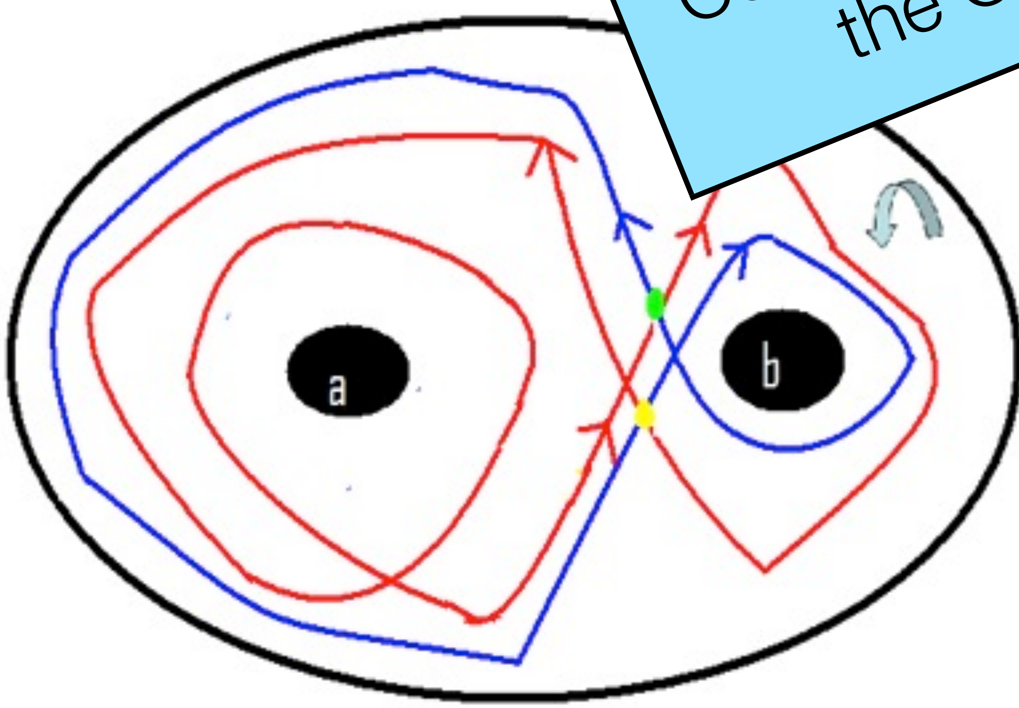

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Consider X and Y are free homotopy classes of closed curves, such that X has a representative with no self-intersection.

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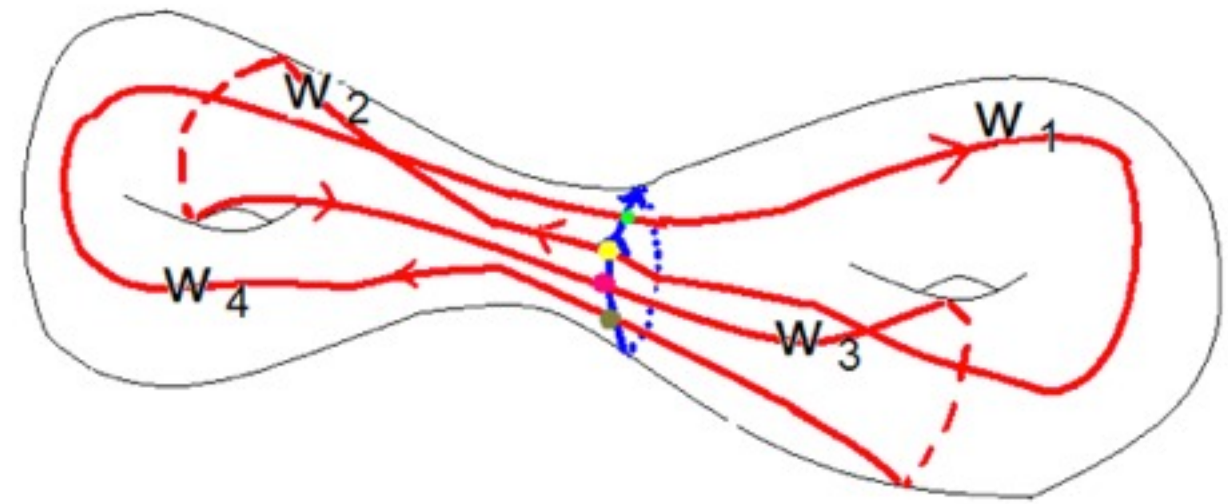

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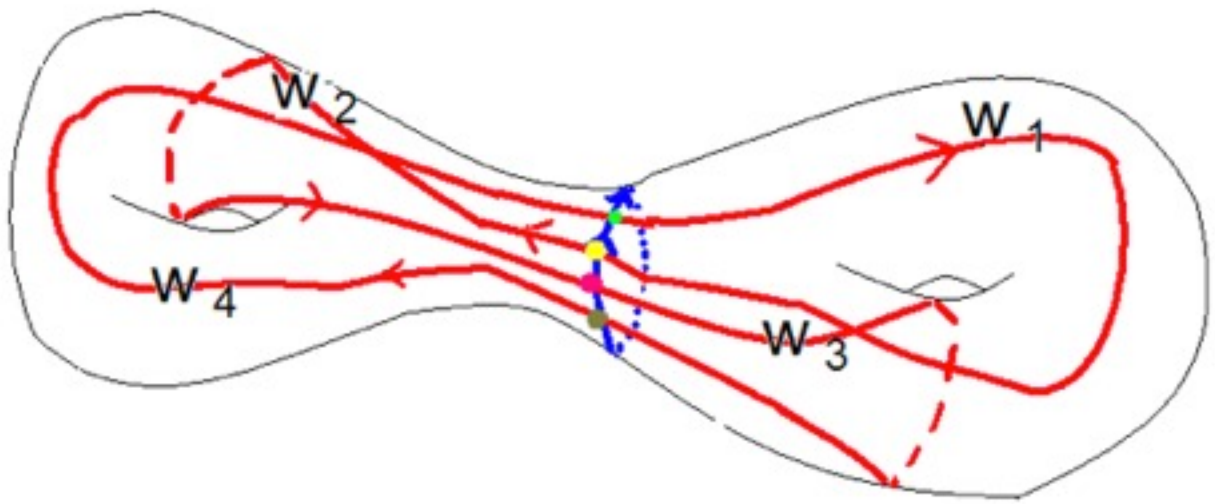

Proof: Free product with amalgamation (or HNN structure)

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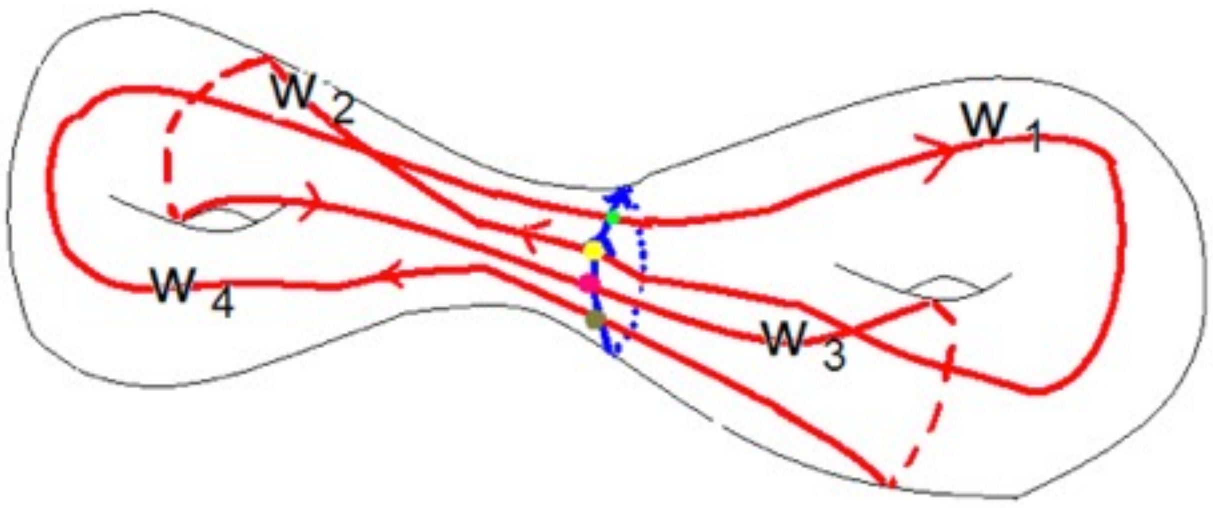

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$\pm[\, w_1w_2w_3w_4,\ X\,]=$

$w_1Xw_2w_3w_4 - w_1w_2Xw_3w_4 + w_1w_2w_3\,X\,w_4 - w_1w_2w_3w_4\,X$

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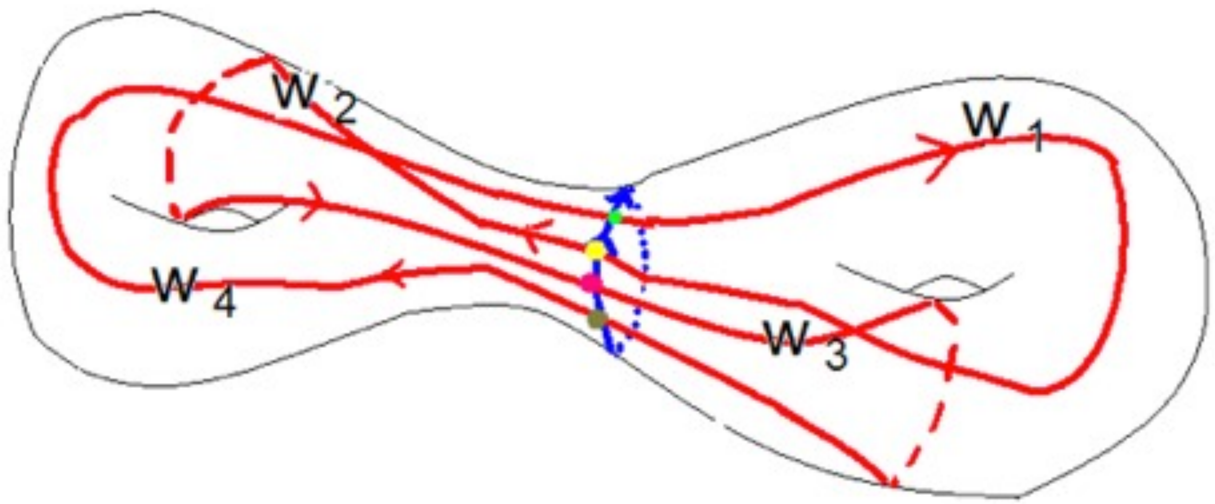

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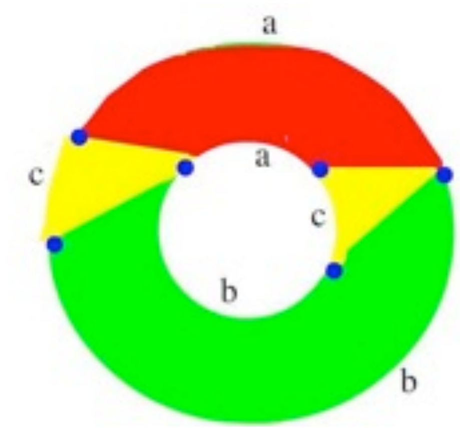

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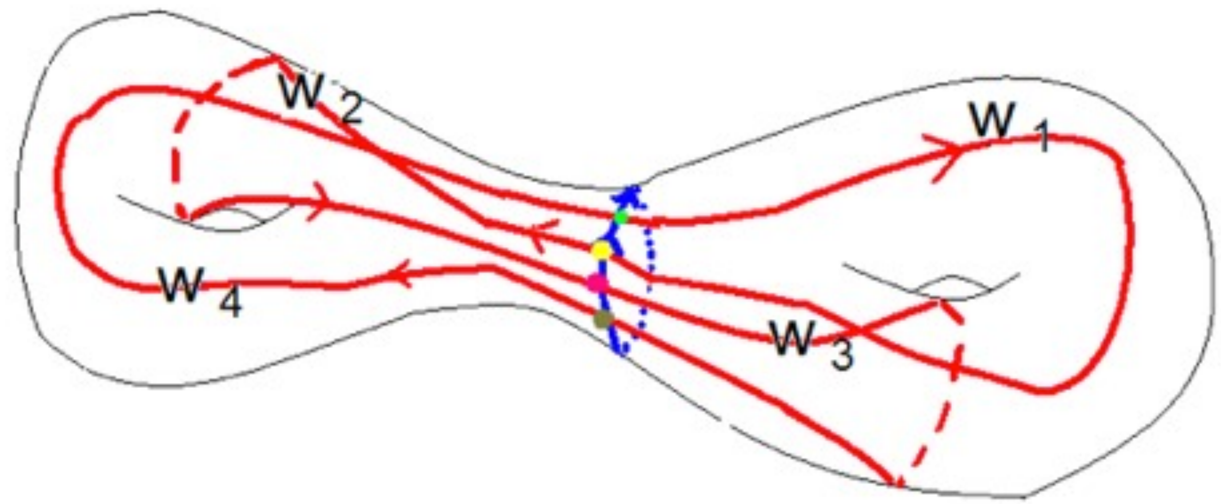

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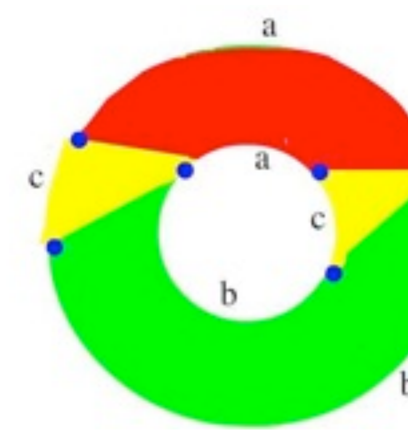

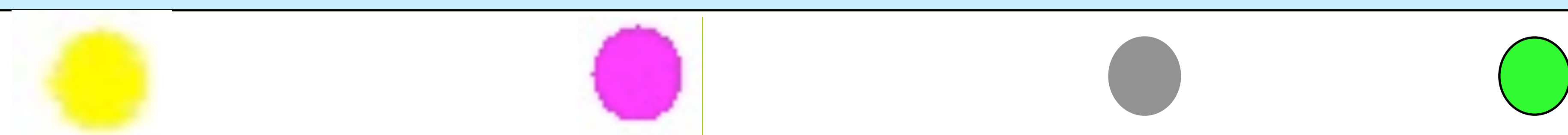

Combinatorial presentation of the Goldman bracket
Counting intersections theorem.

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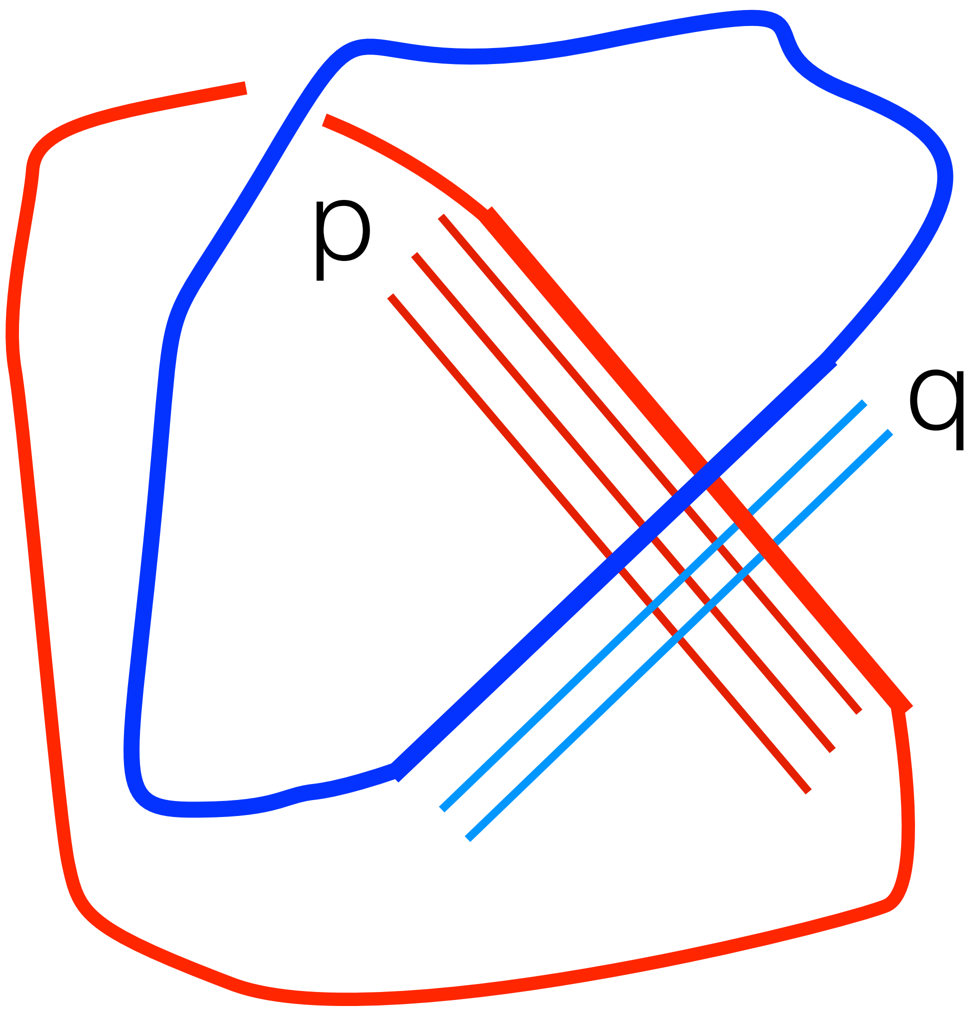

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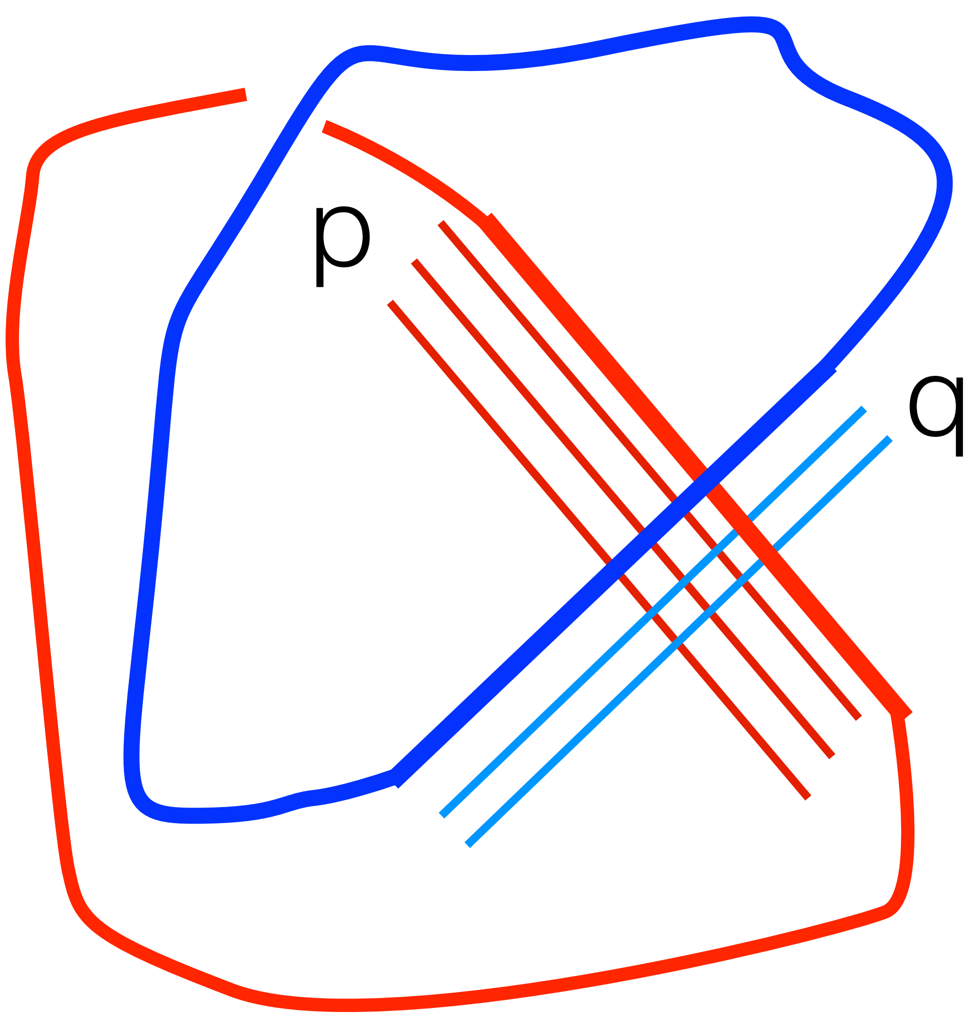

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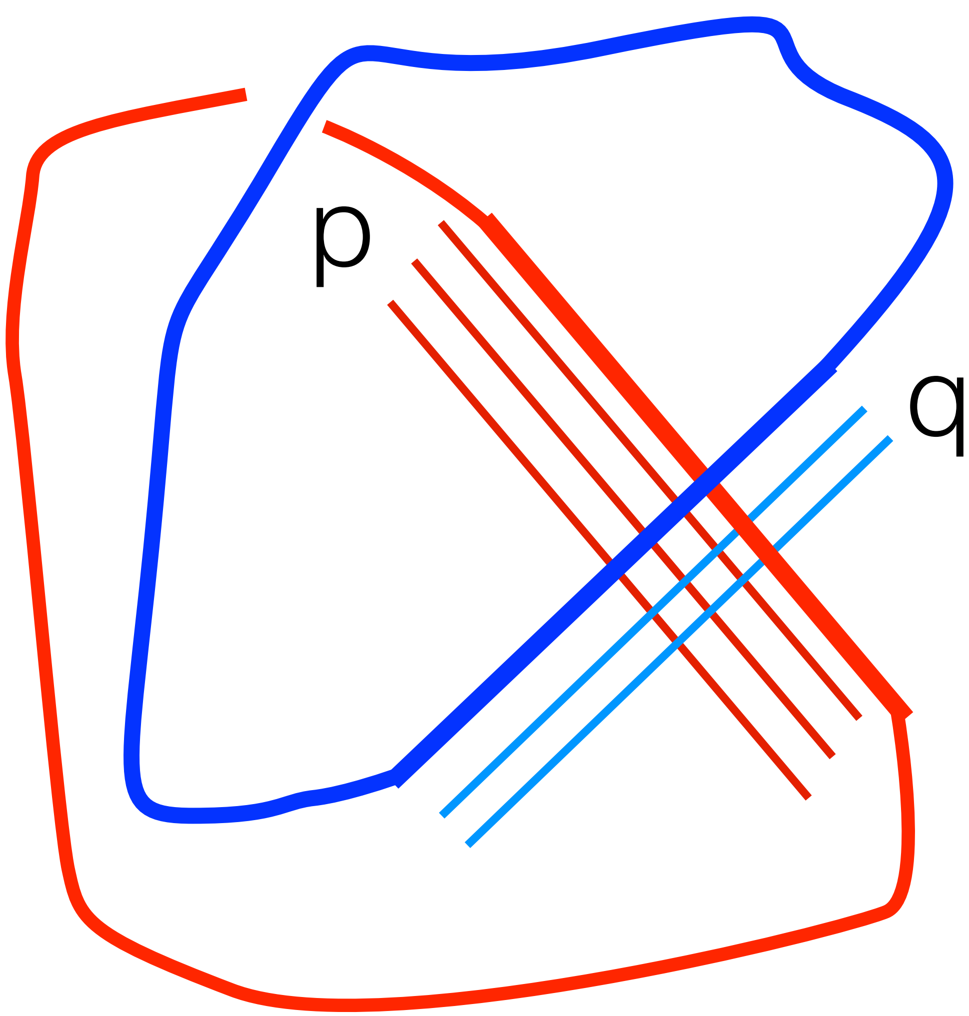

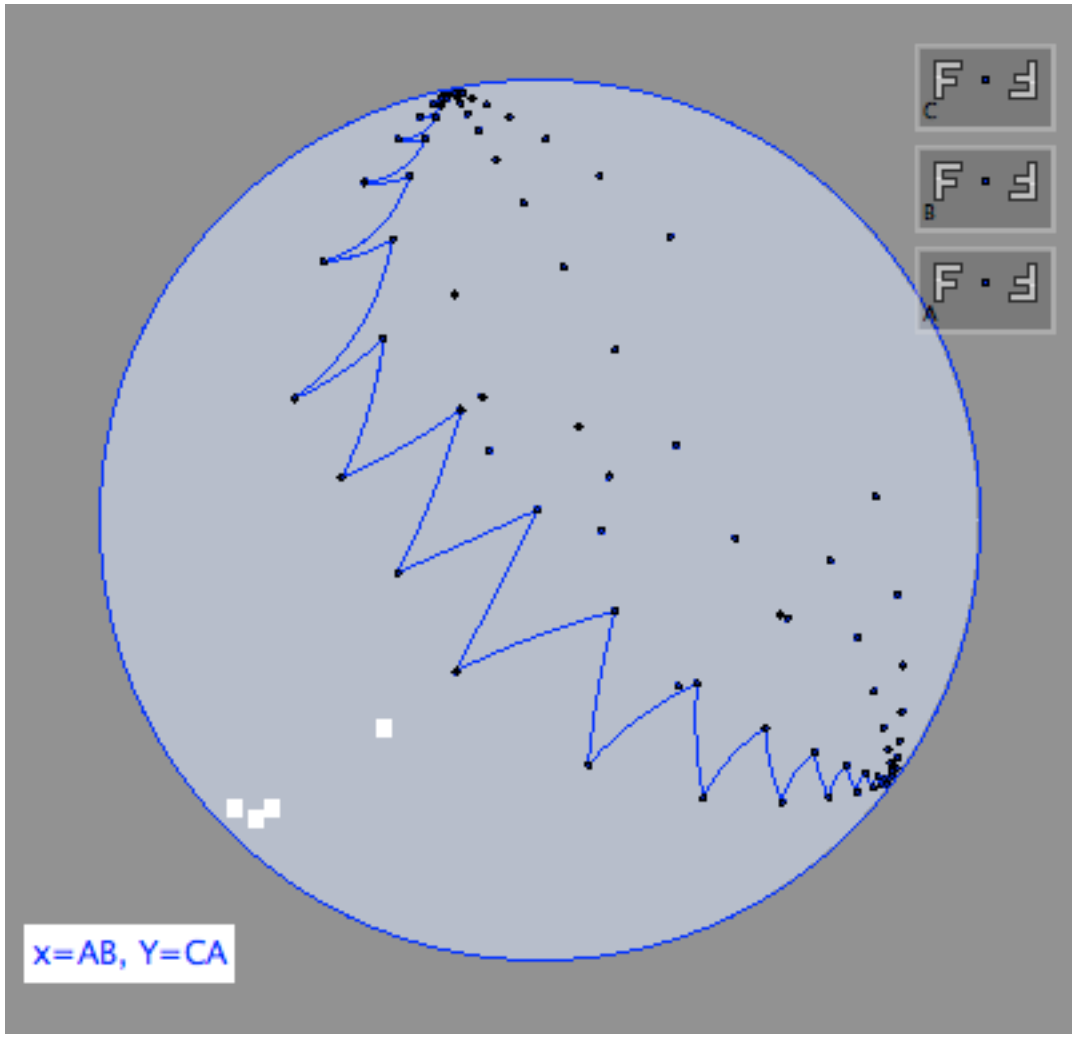

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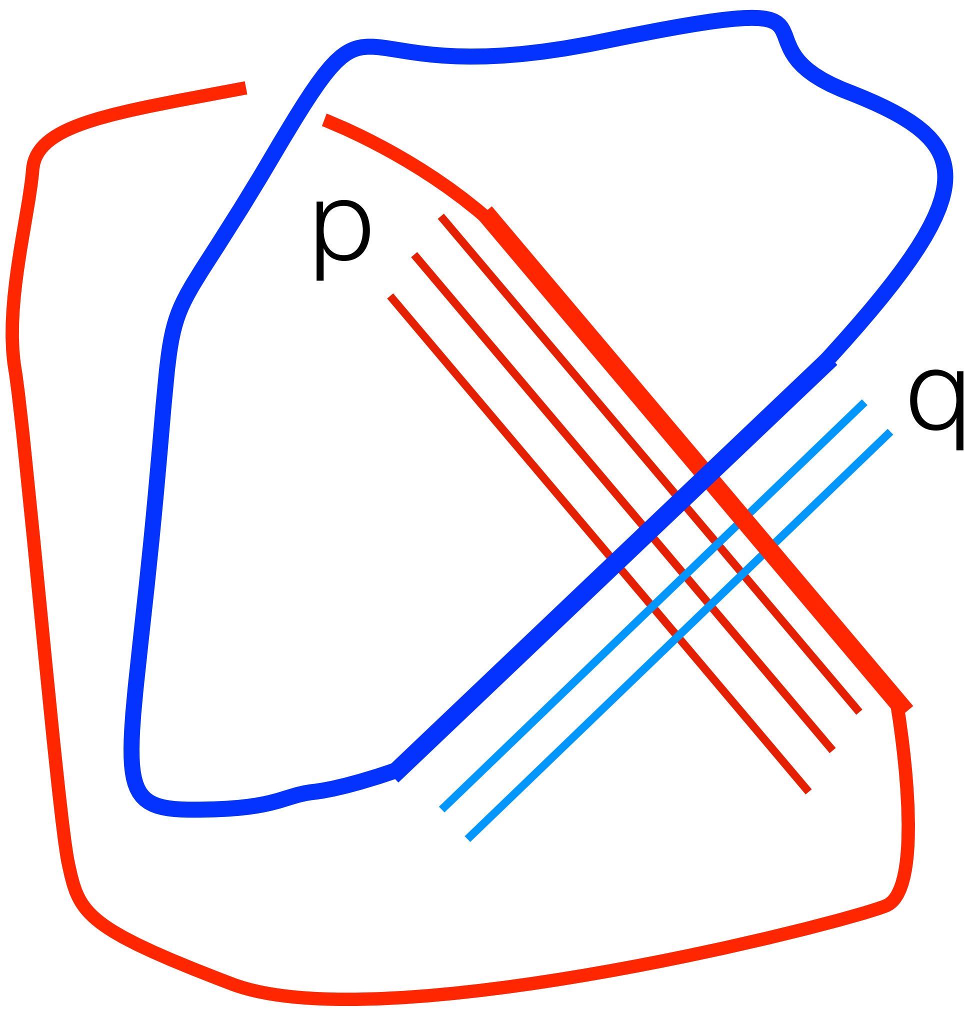
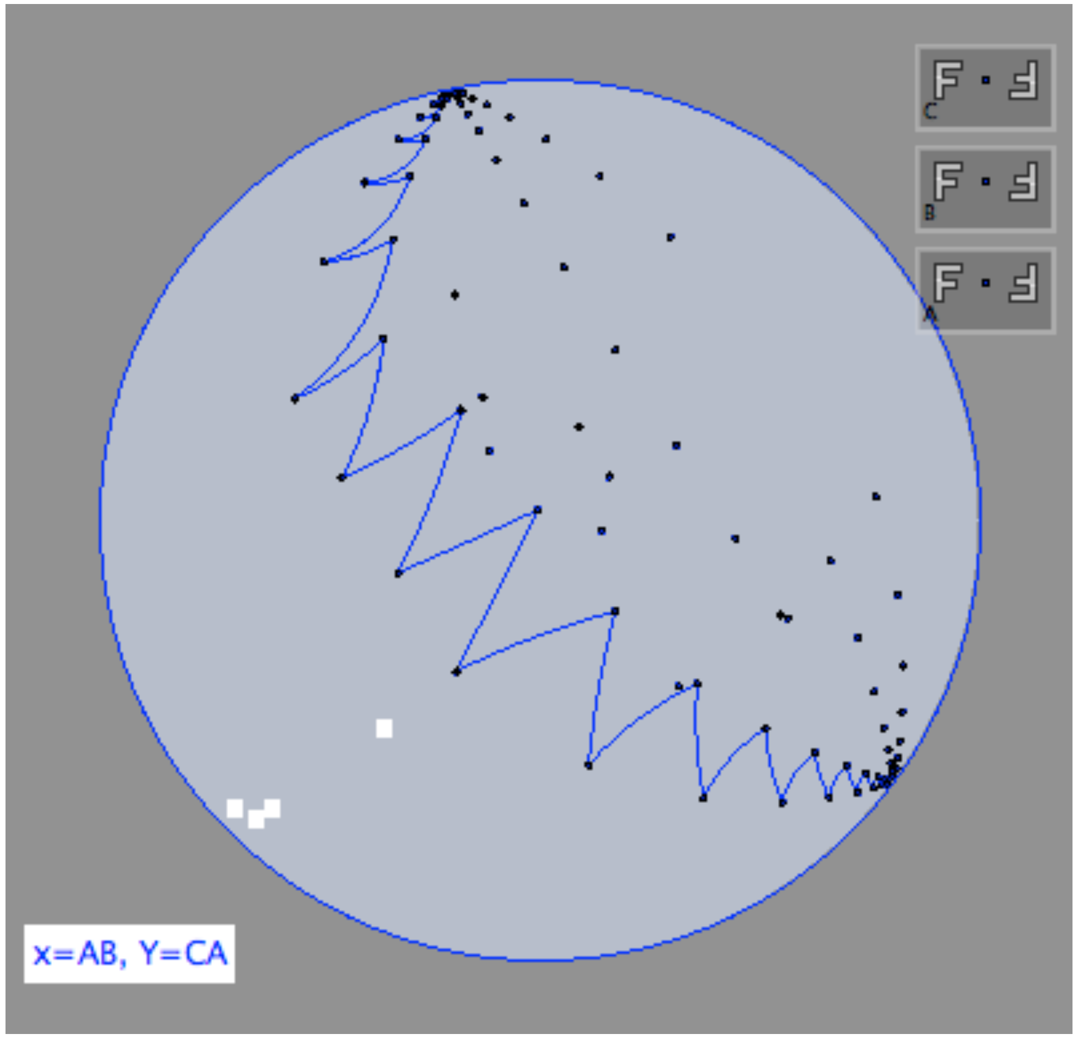
Holds for X=Y, (with p ≠ q).





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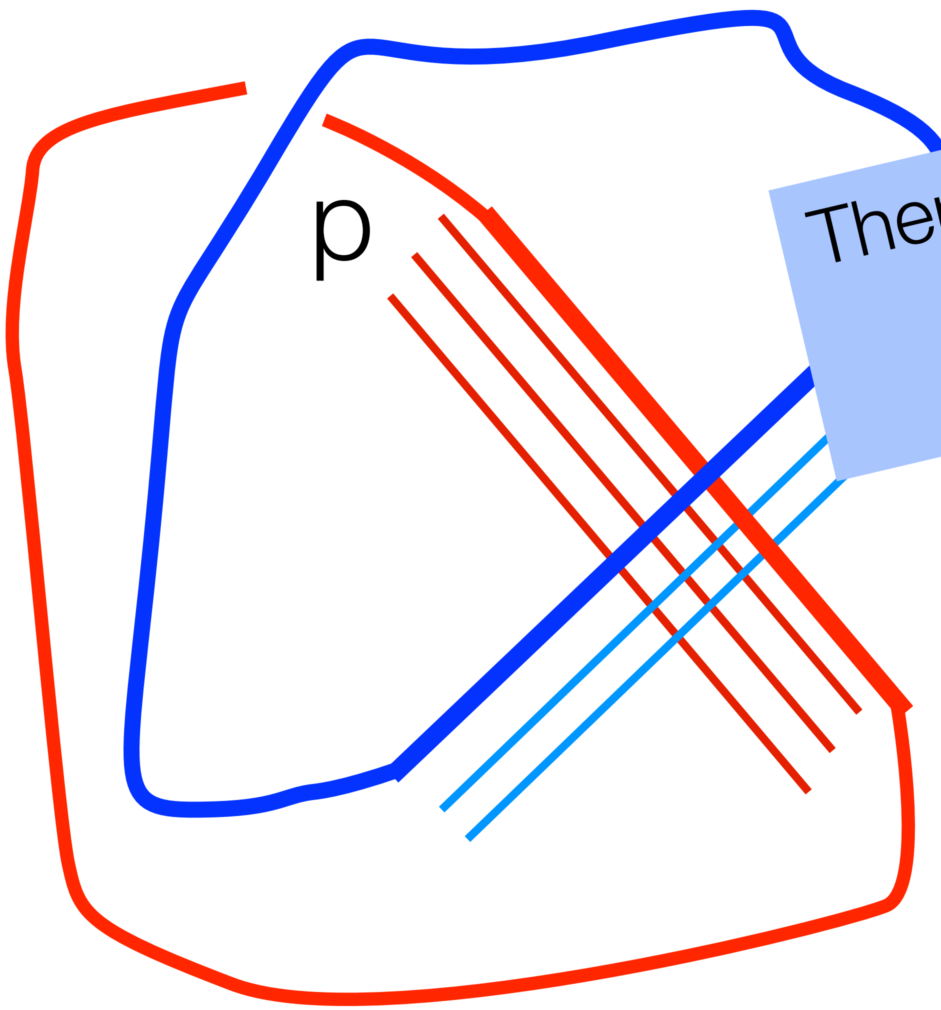
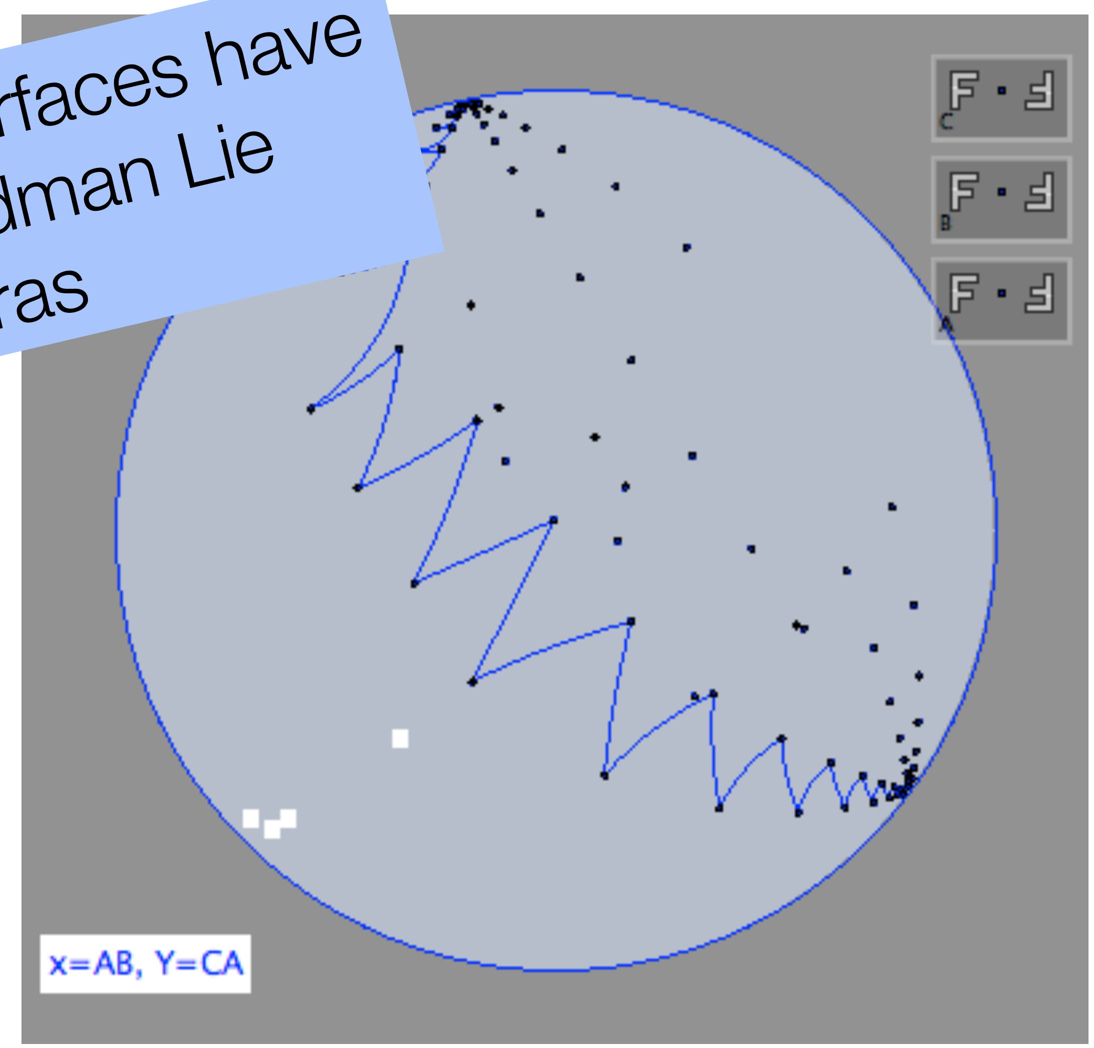


Then different surfaces have different Goldman Lie algebras



M is a 3 manifold

LM=space of maps from the circle to M.

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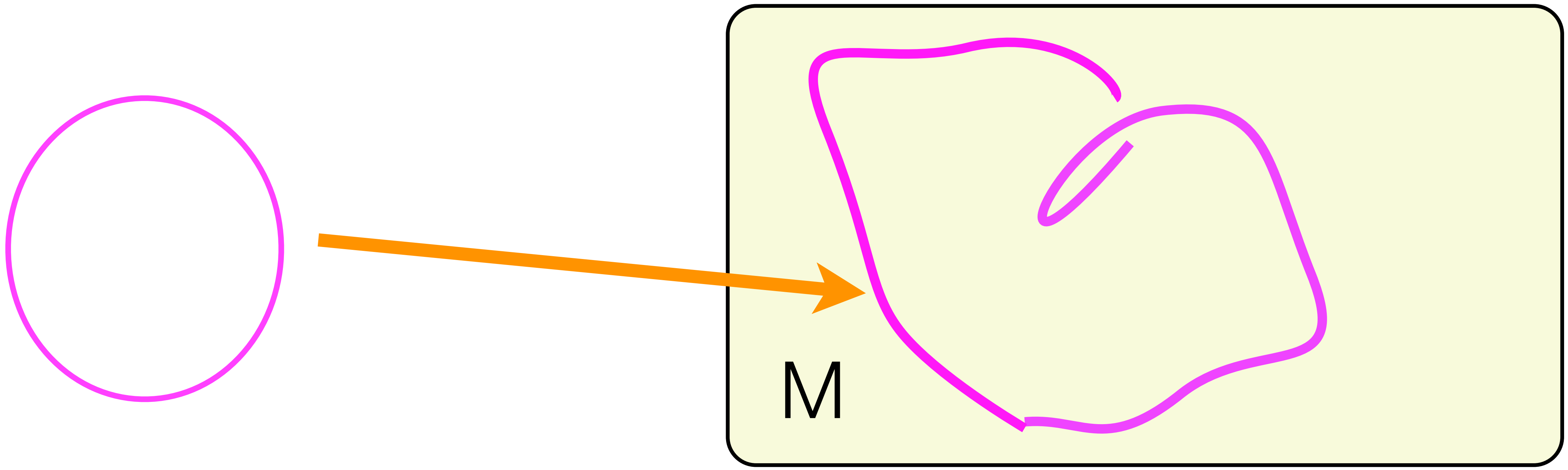

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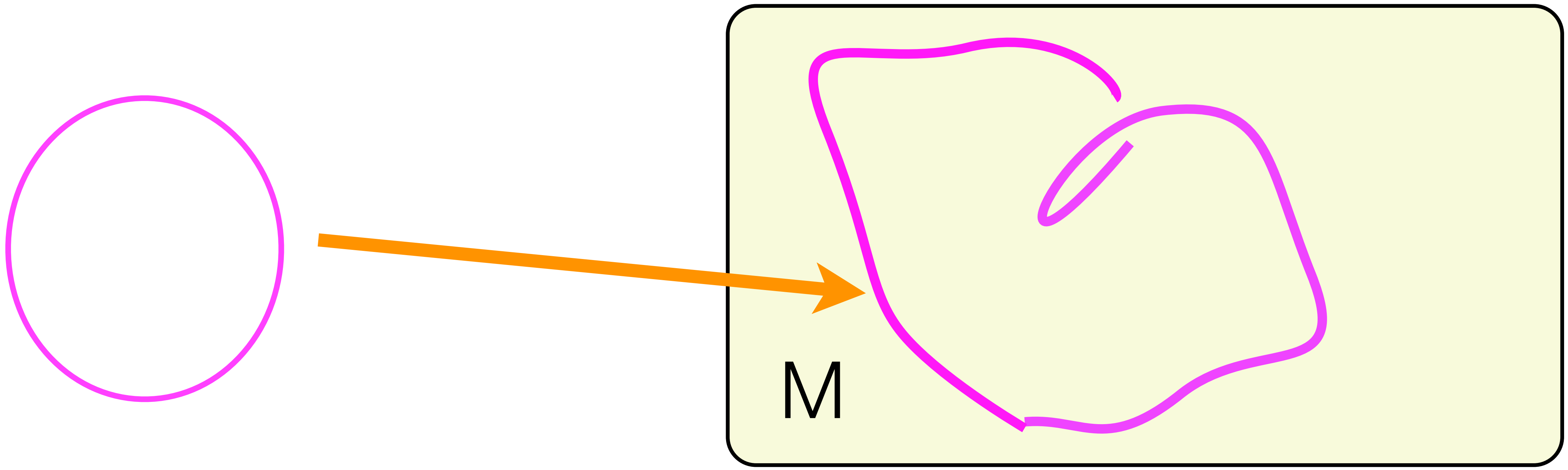
$H_0(LM)$= the zeroth equivariant homology group of LM



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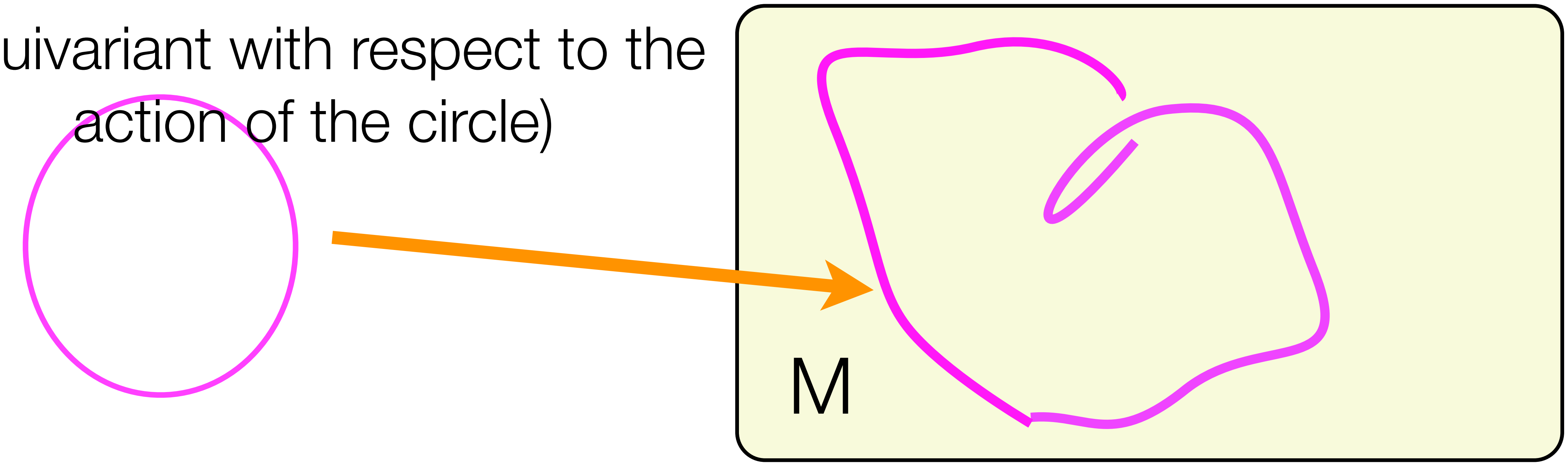

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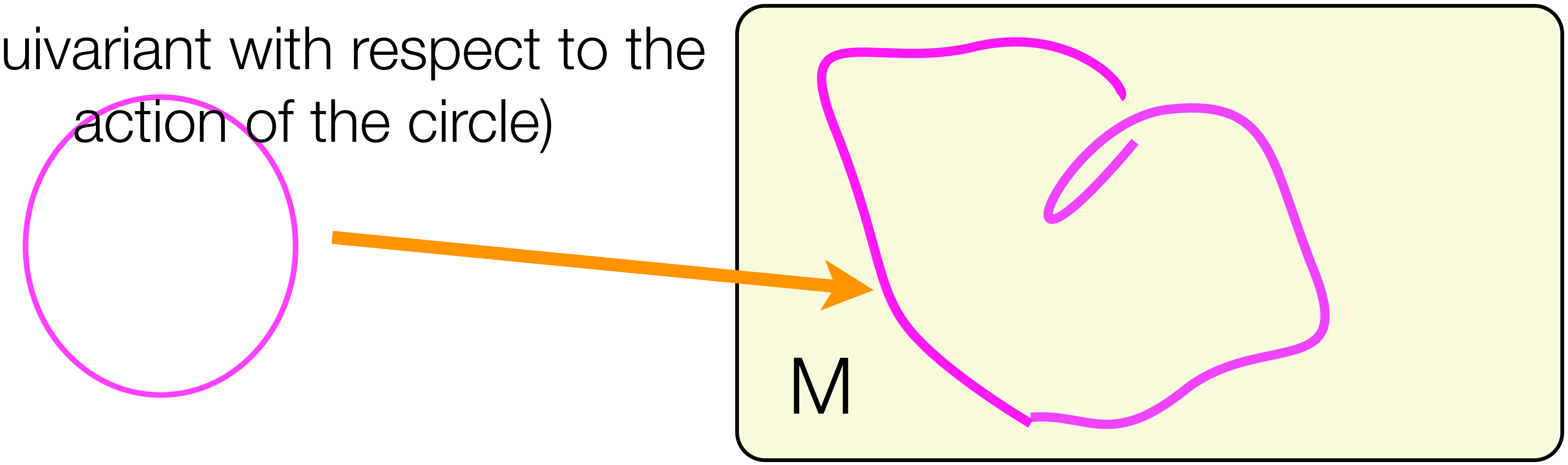

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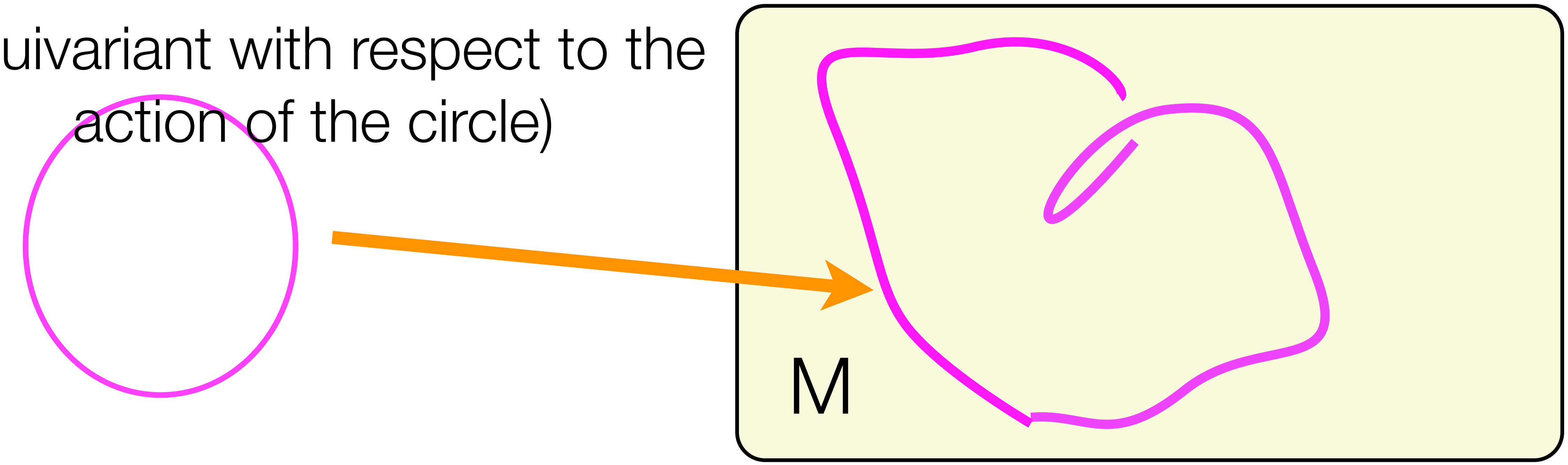

Since the circle is connected,

$H_0(\text{space}) = \oplus_{\{C \text{ is a connected component of space}\}} \mathbb{Z}.C$

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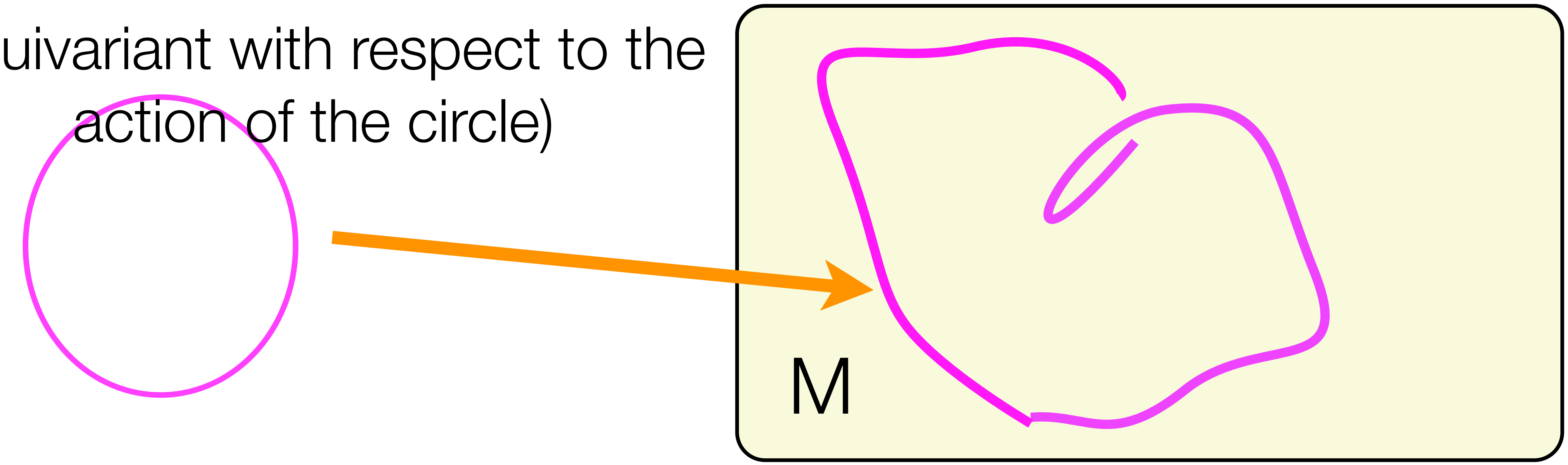

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$H_0(LM) = \oplus_{\{a \text{ in } \pi_0(M)\}}\ \mathbb{Z}.a$

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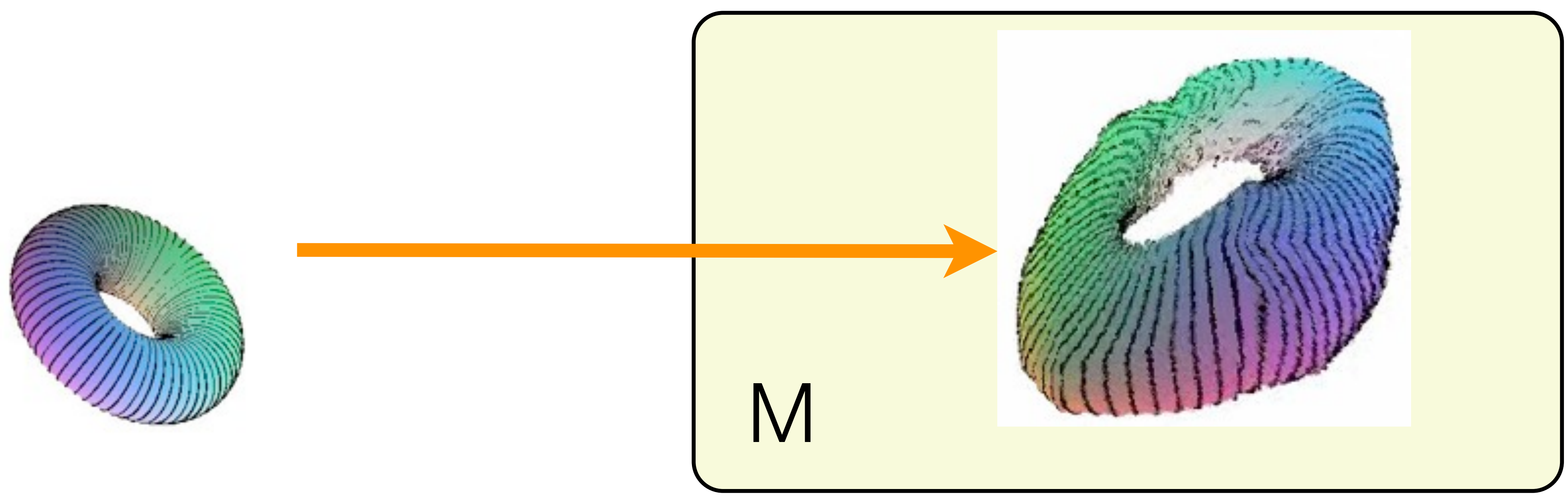

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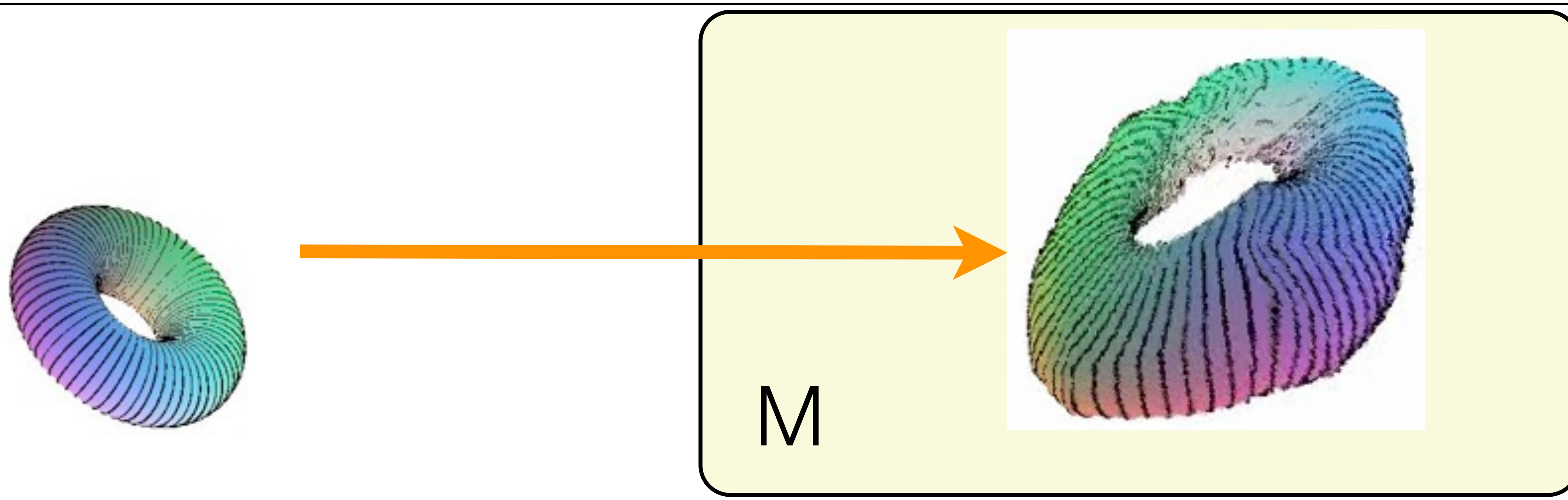
Consider a map f from the circle to LM.

$H_1(LM)$= the first equivariant homology group of LM



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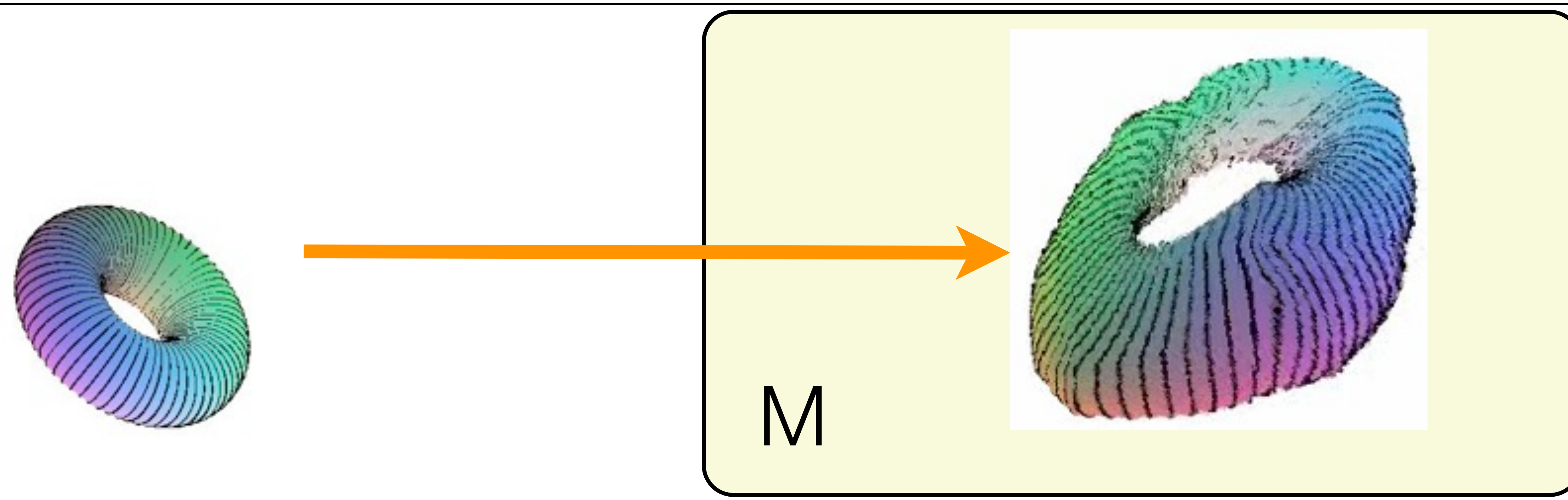

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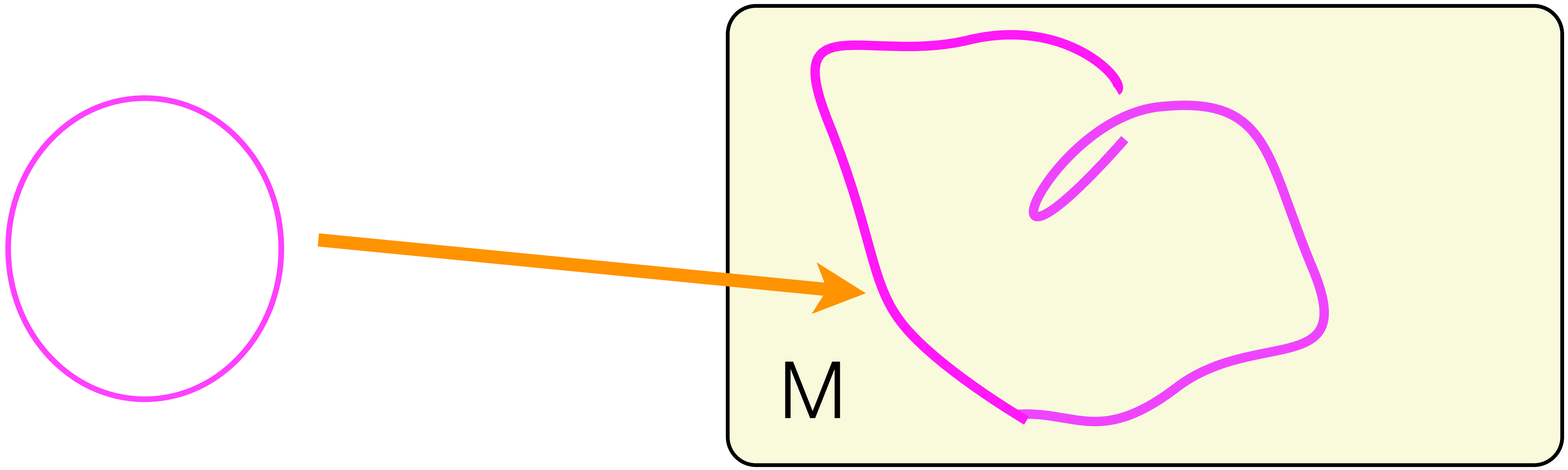
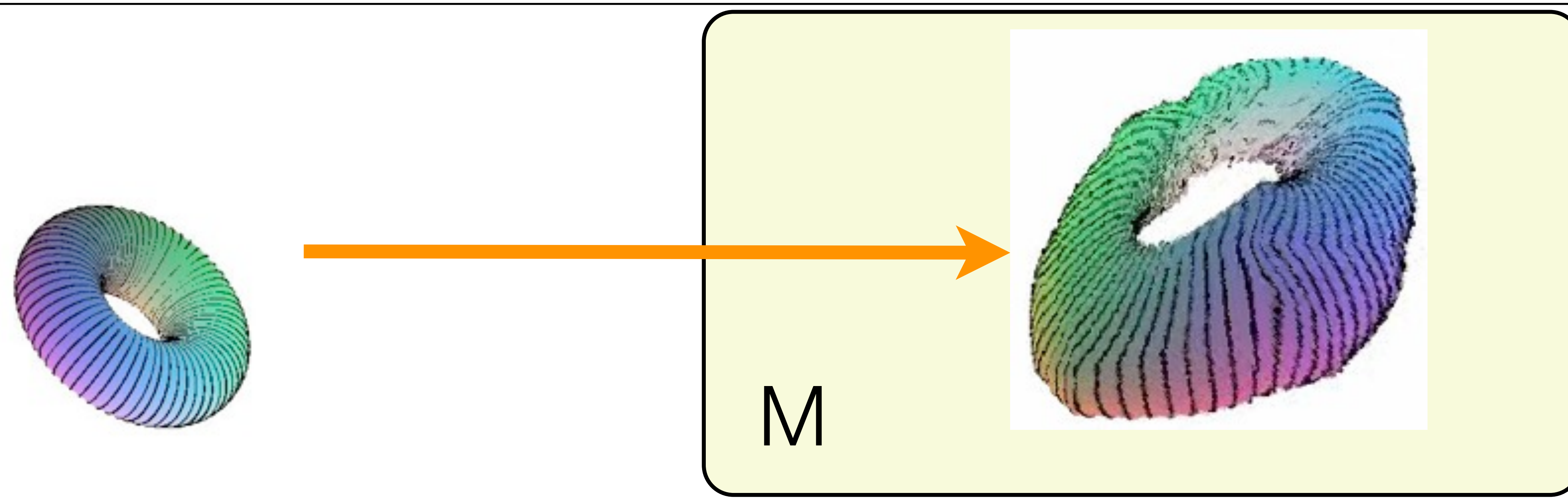
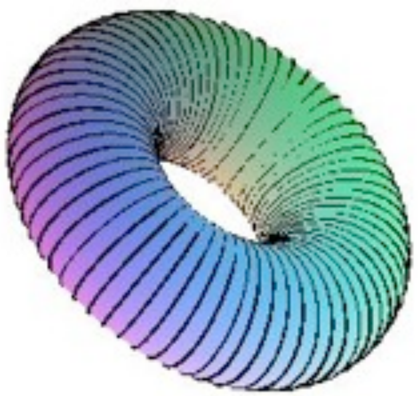


$H_1(LM)$= the first equivariant homology group of LM

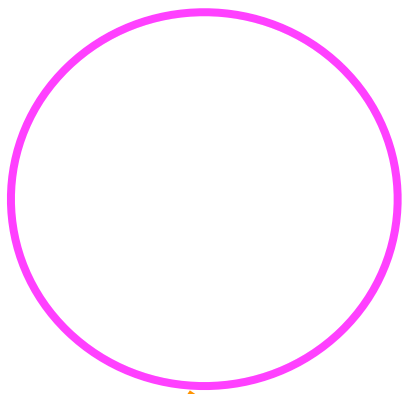


The string bracket

Given a fibered torus



and a closed curve



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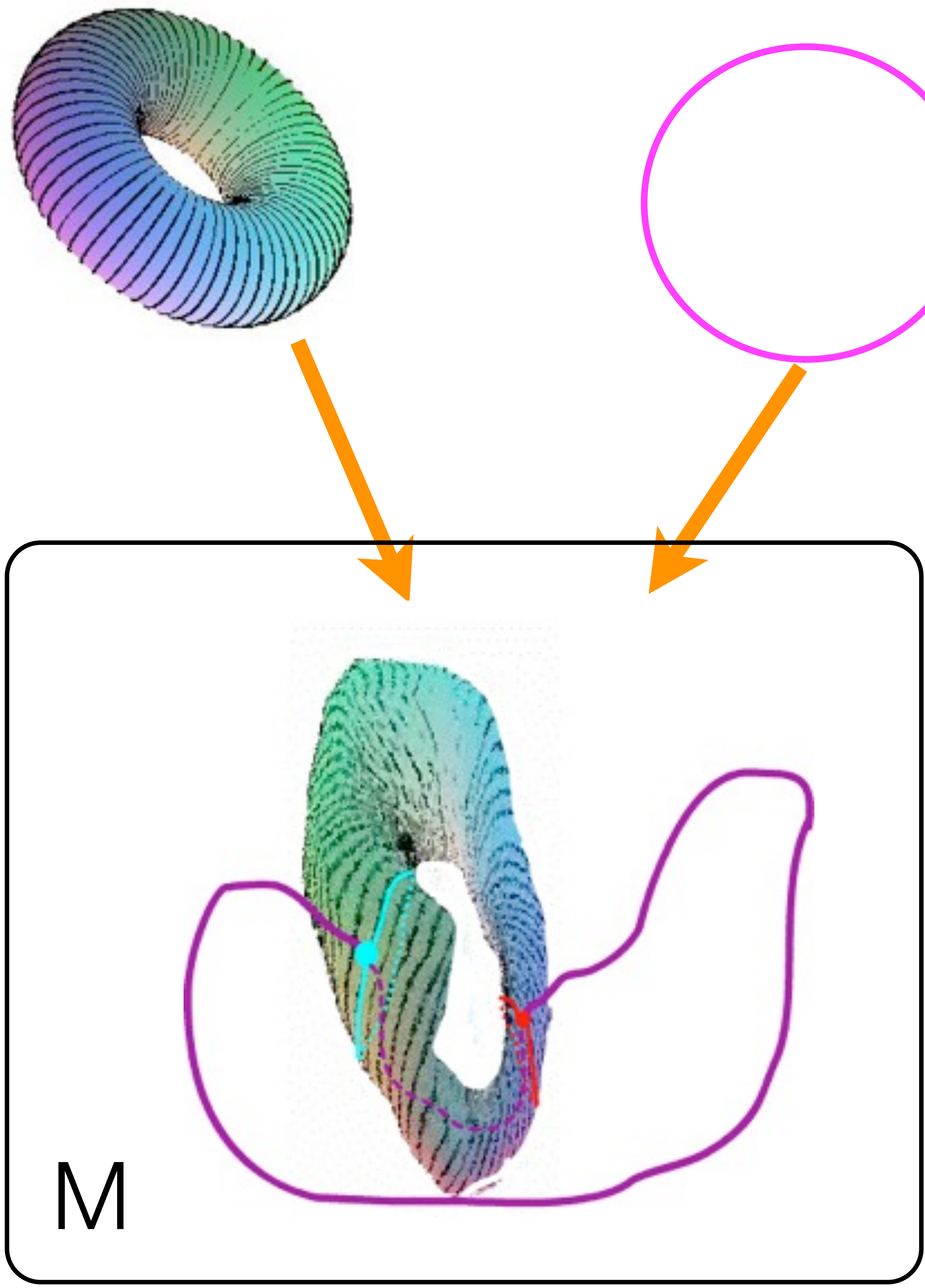

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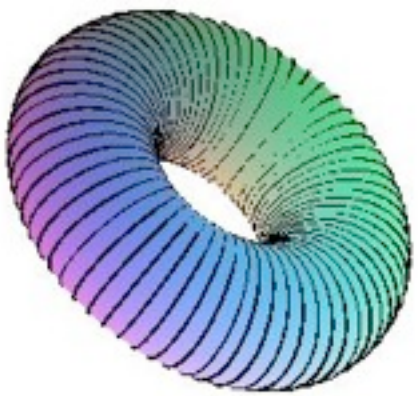

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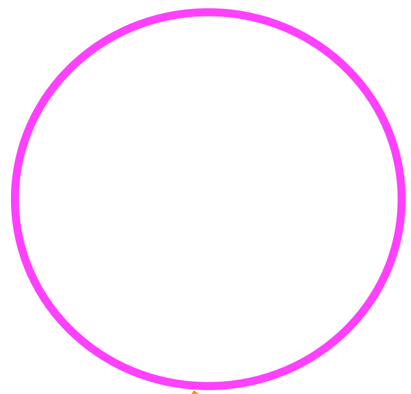
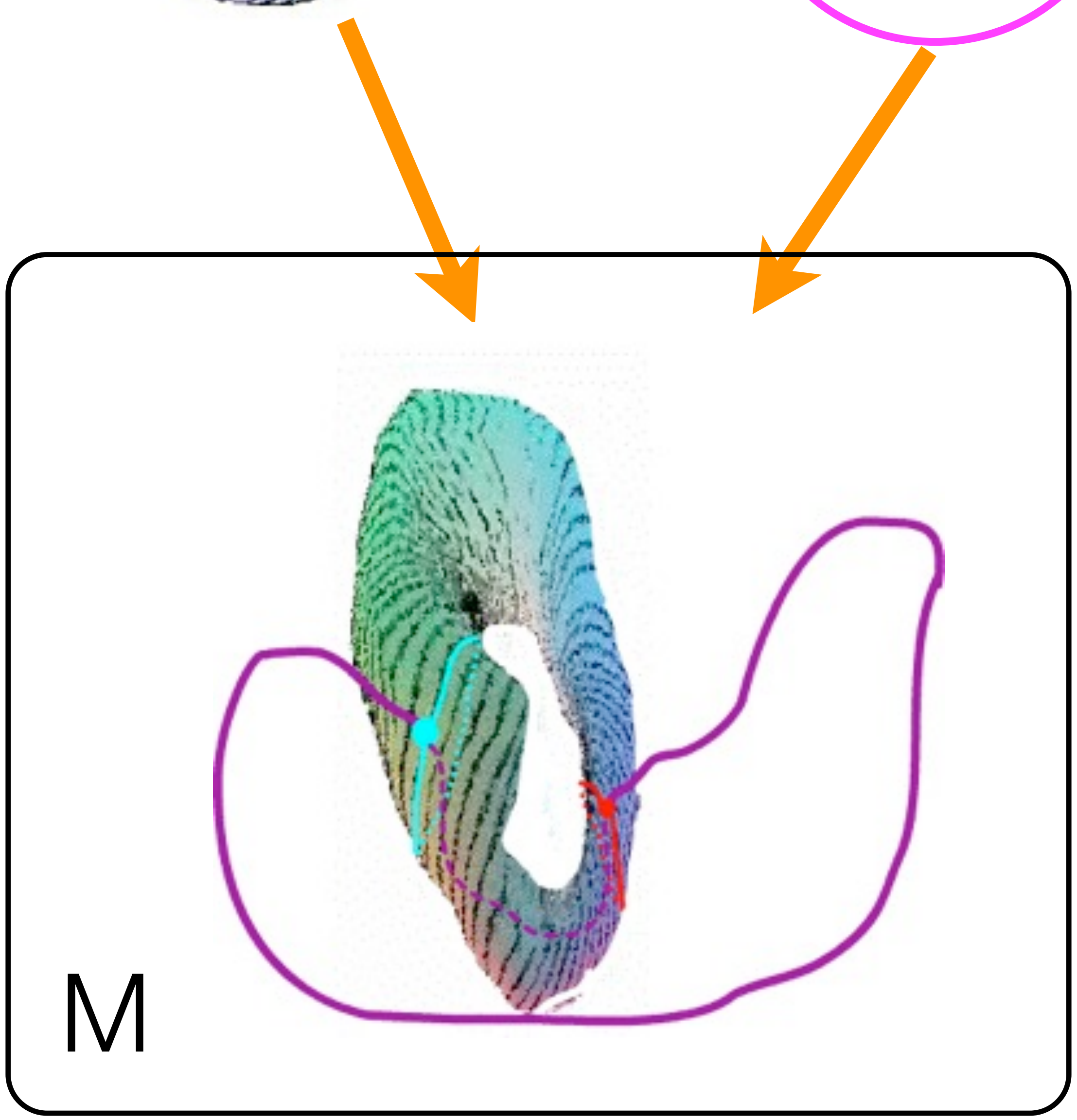
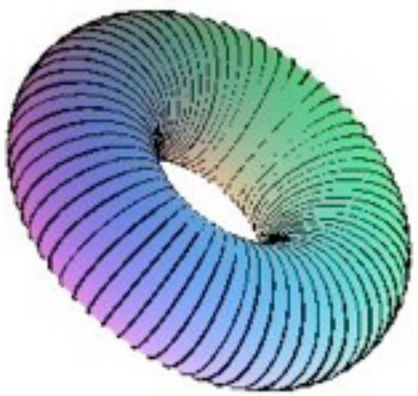




the bracket is defined as follows

The string bracket

Given a fibered torus



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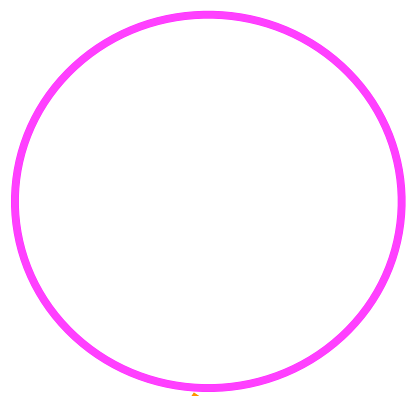

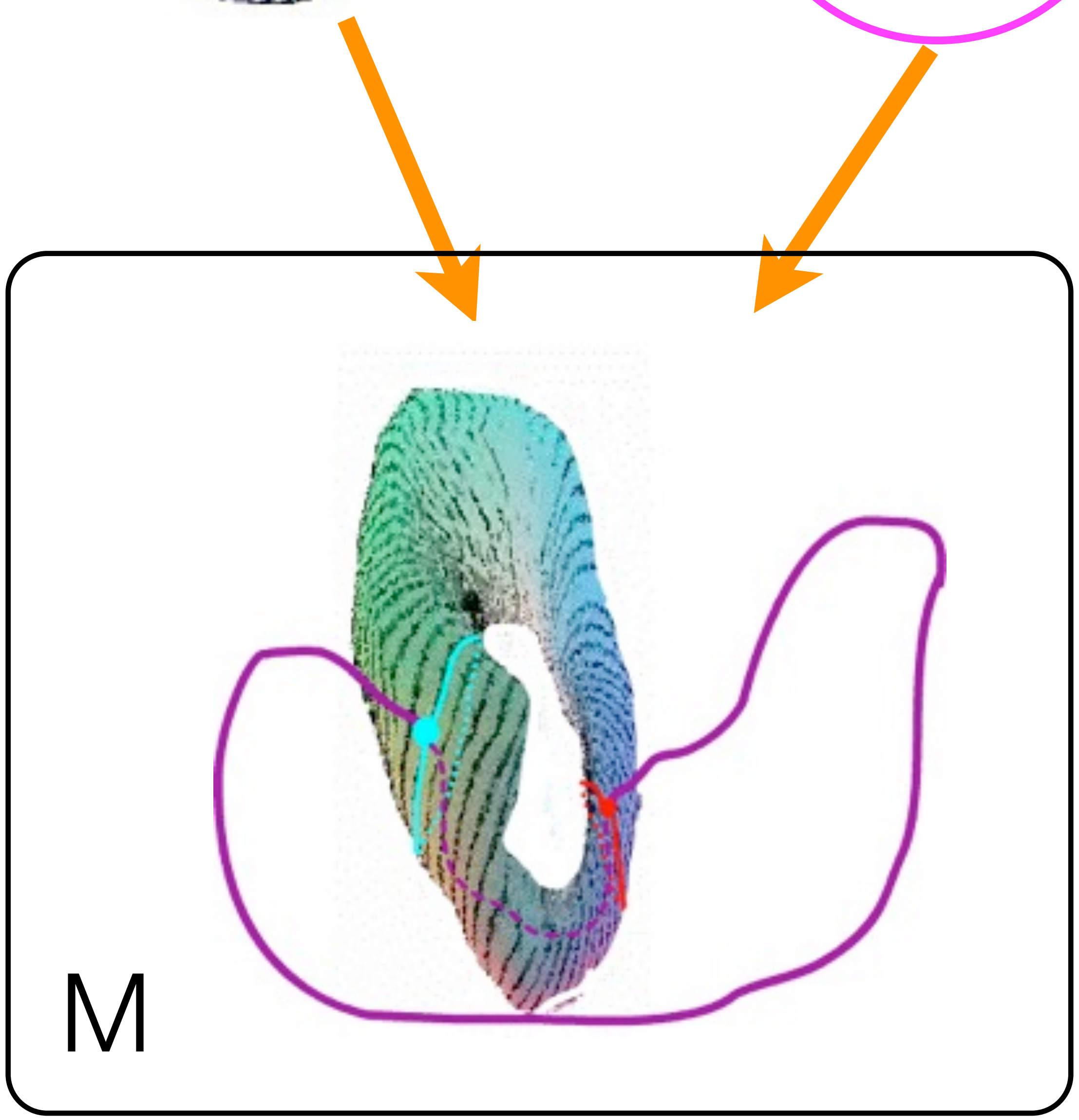

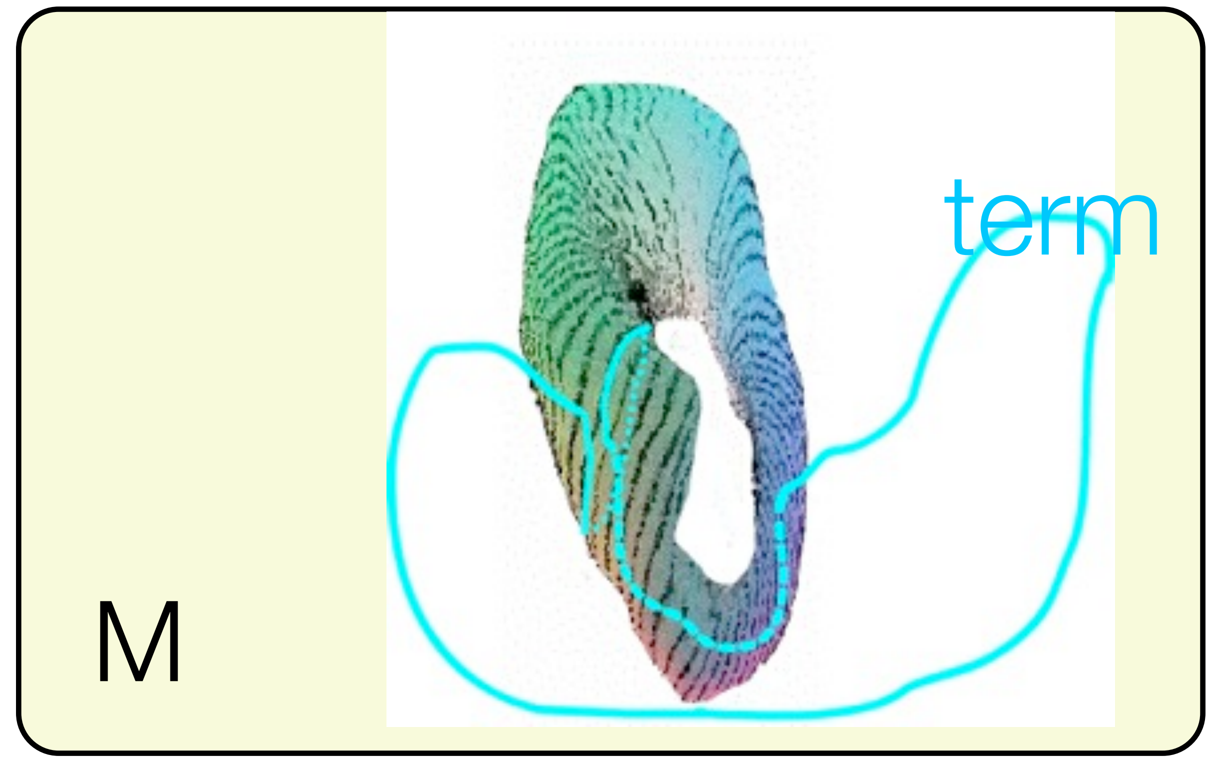

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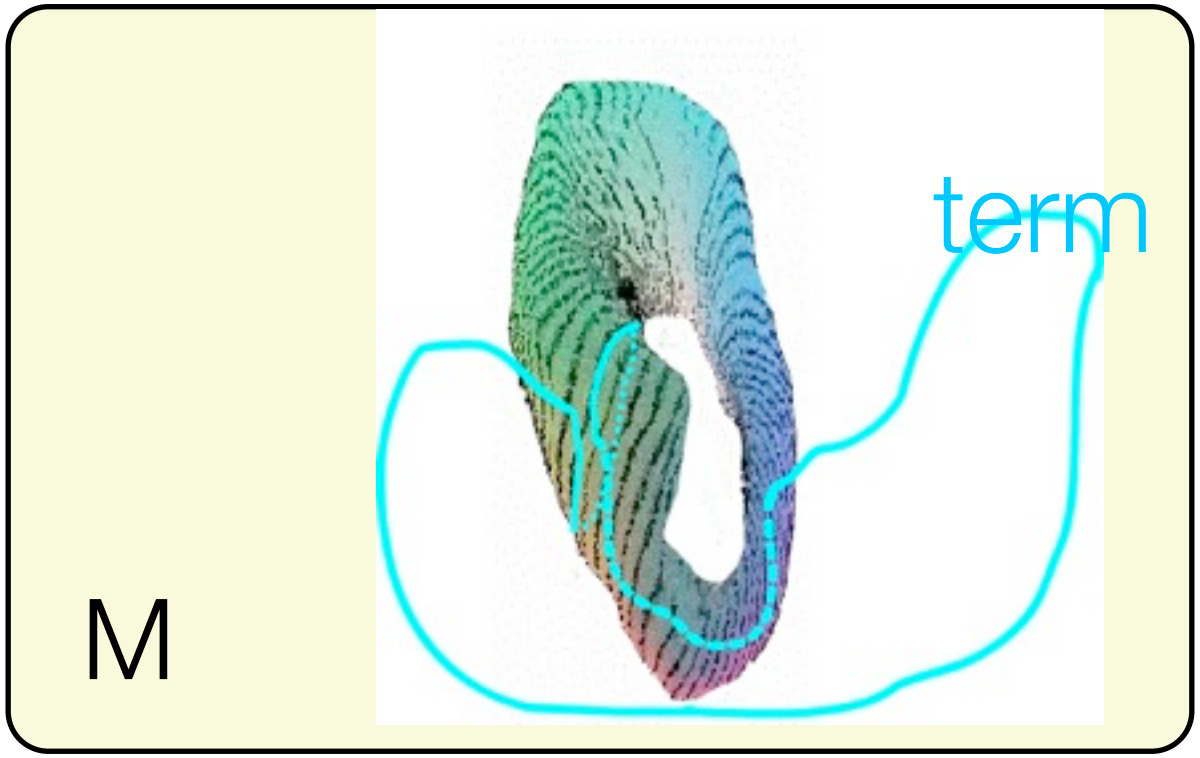

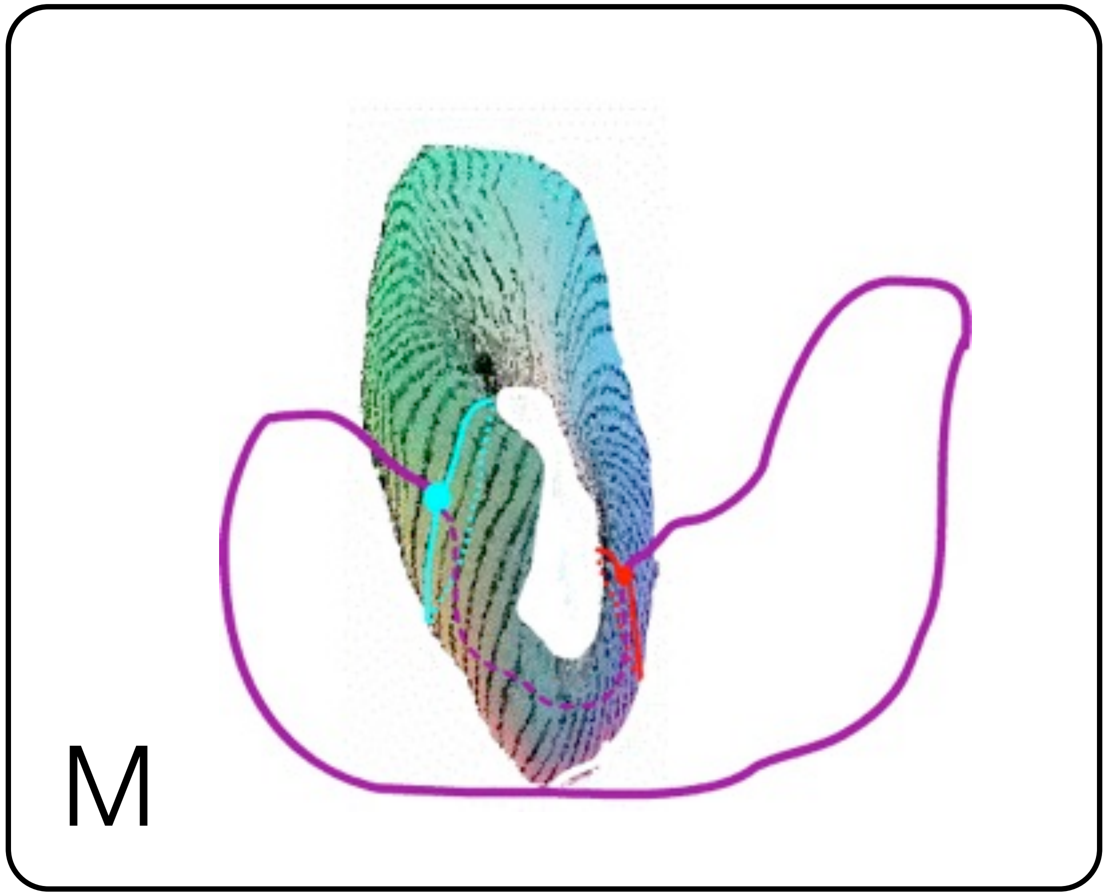

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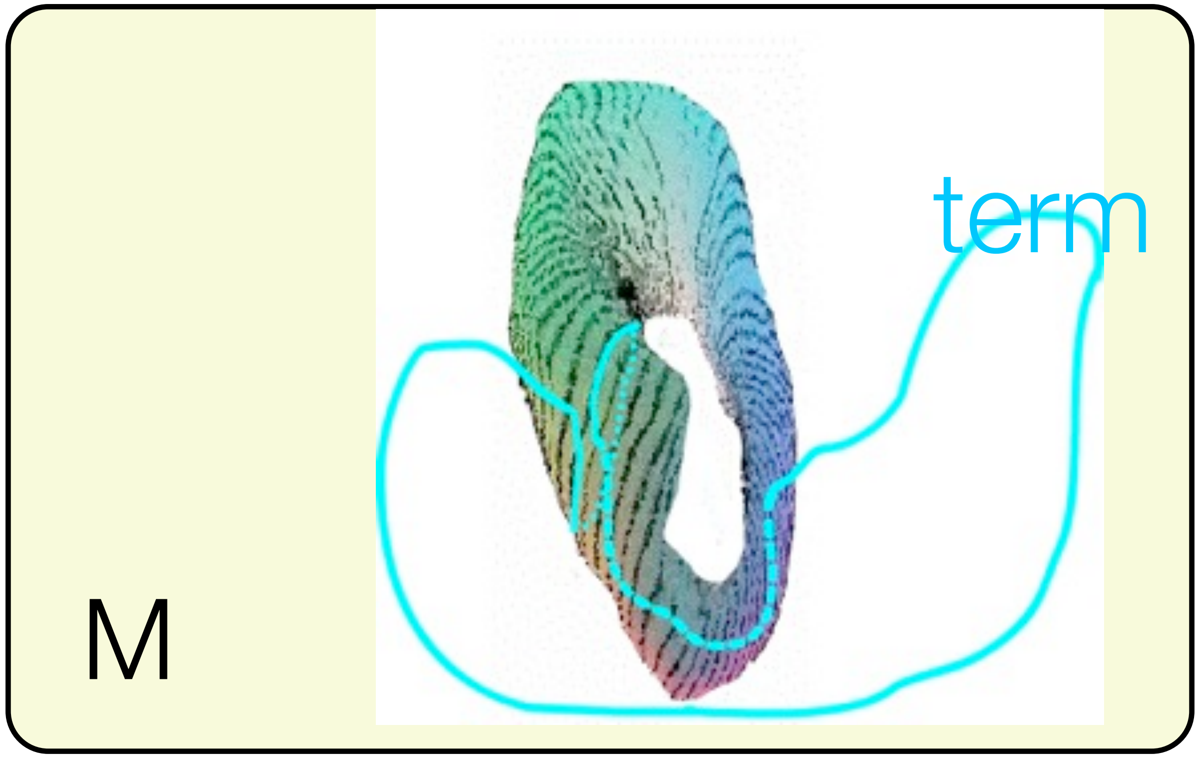

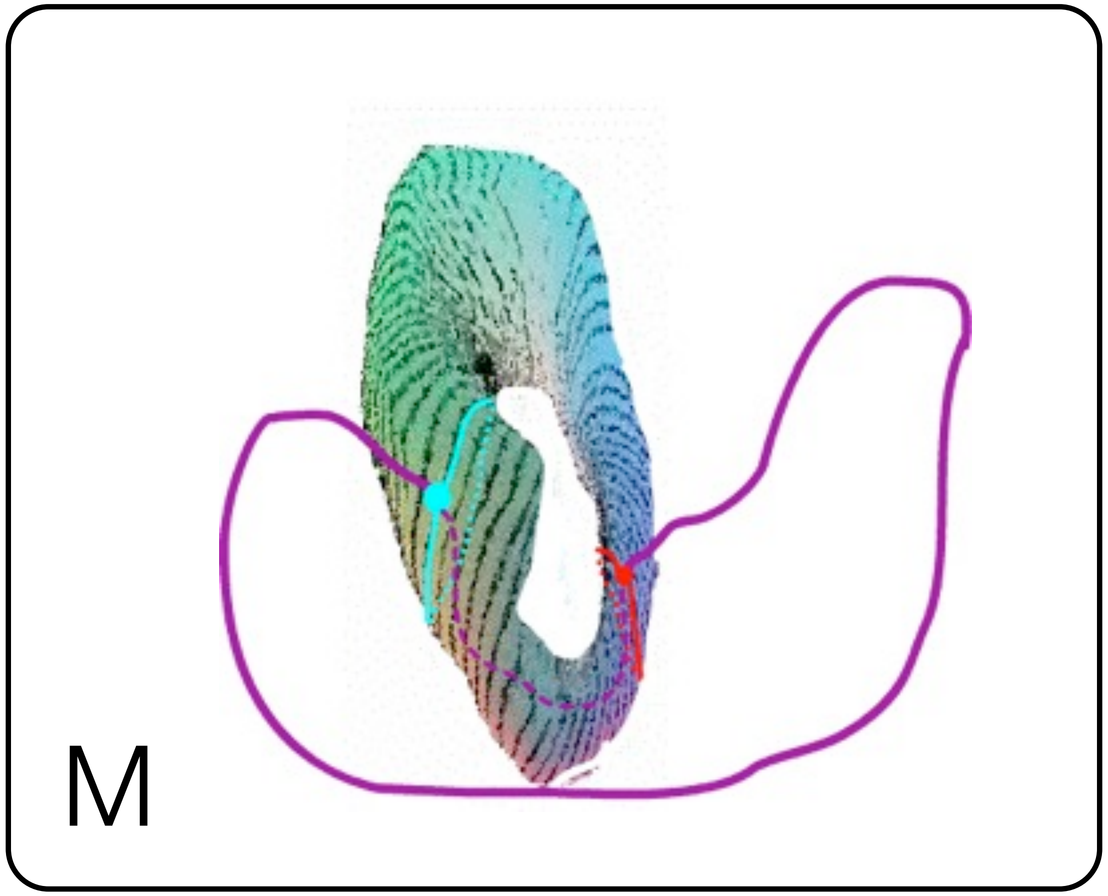

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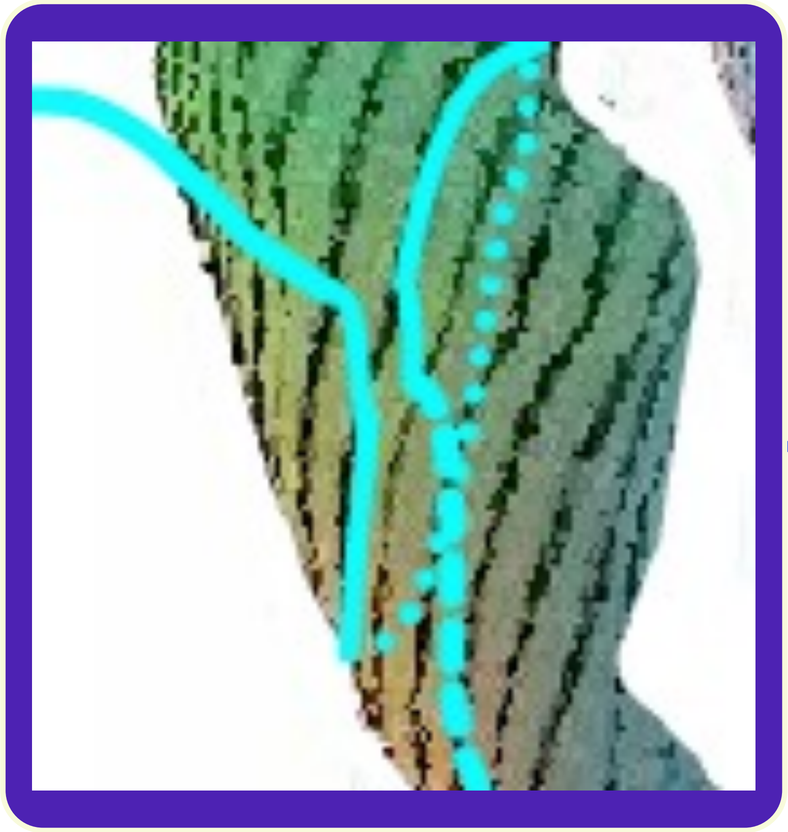

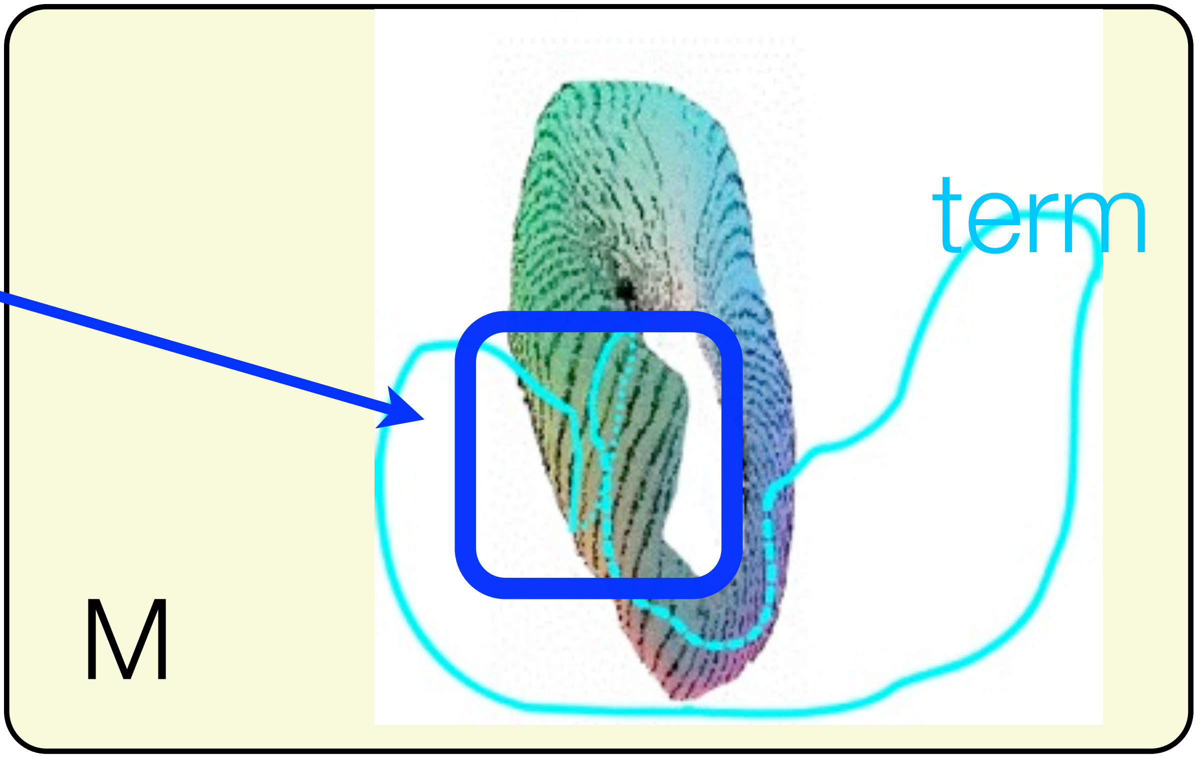

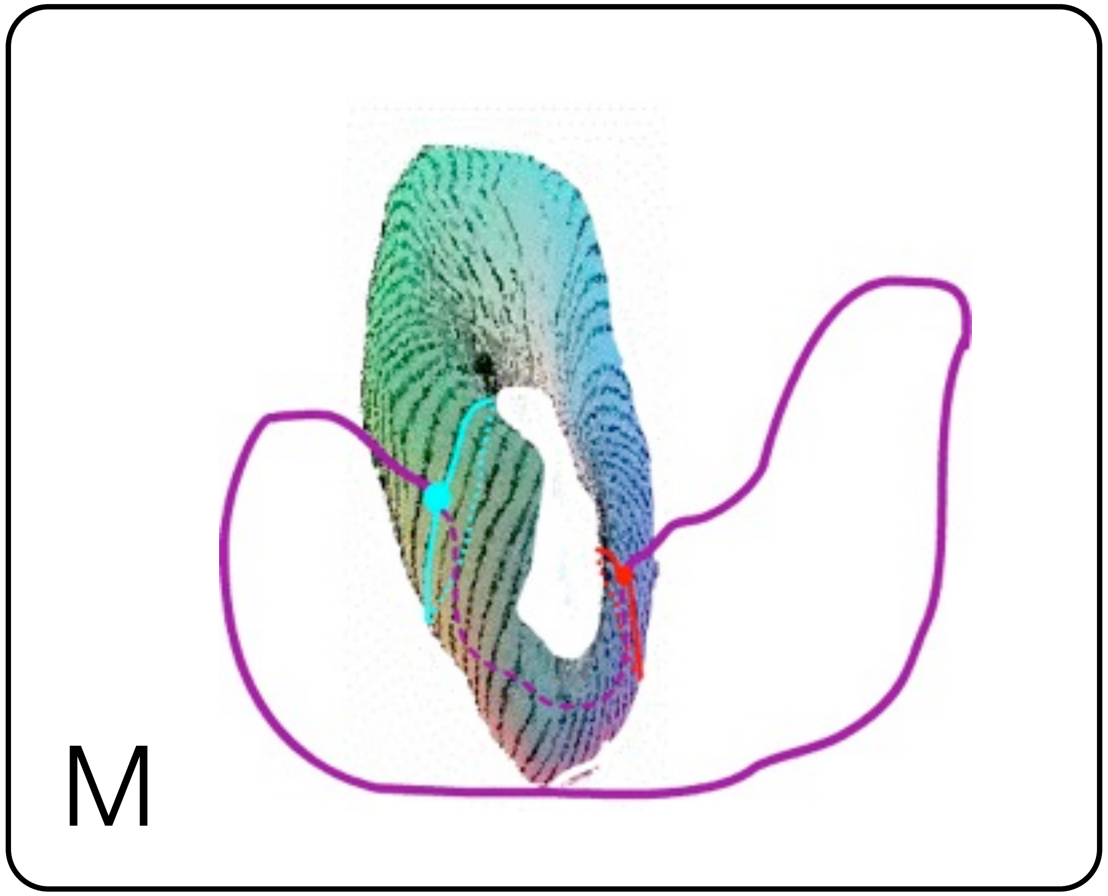

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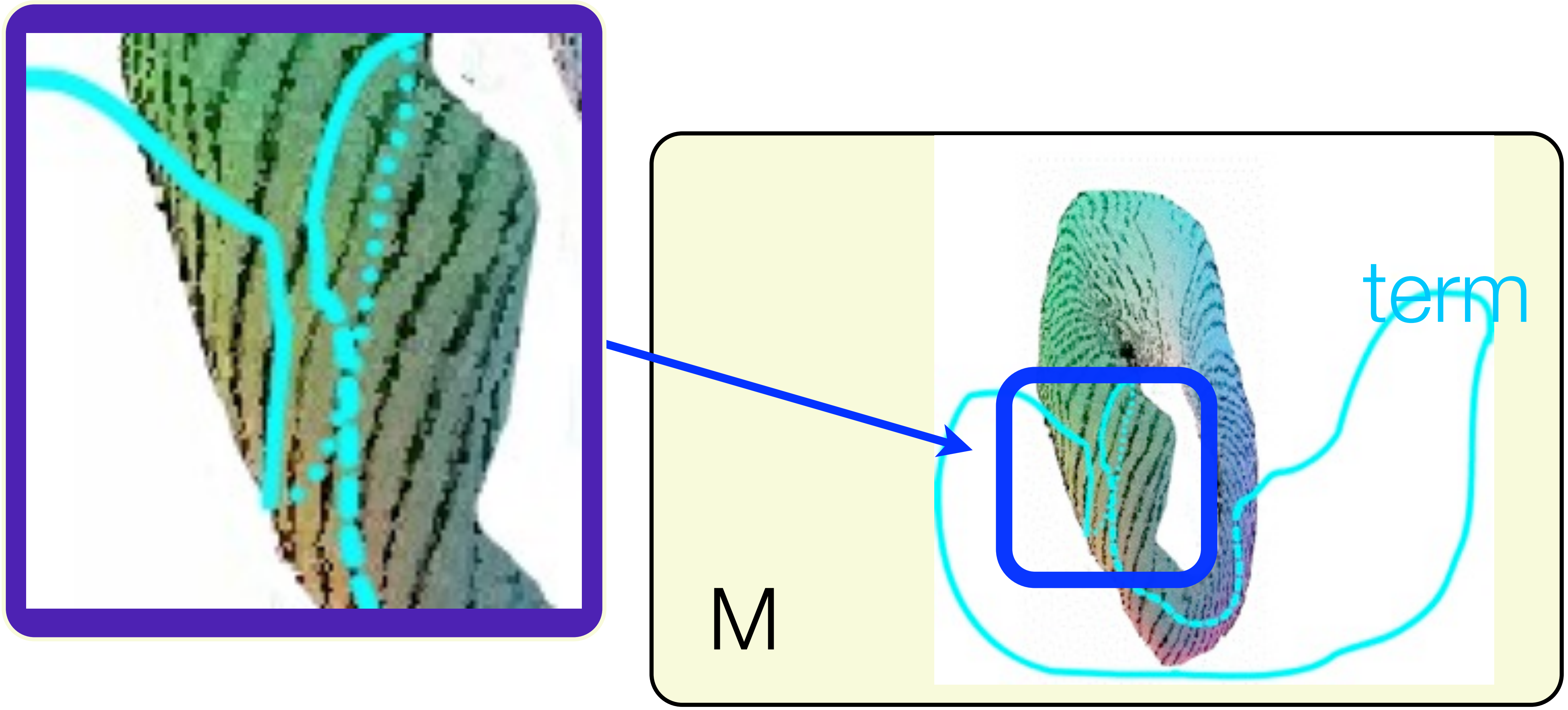
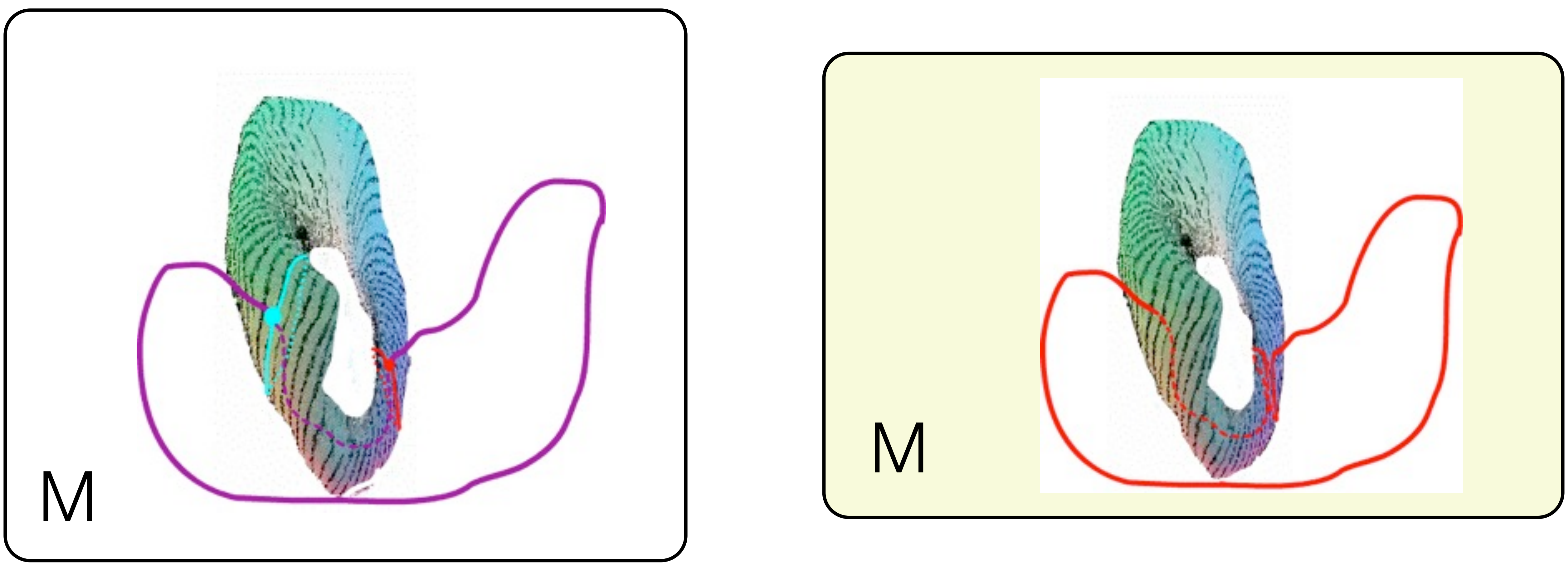
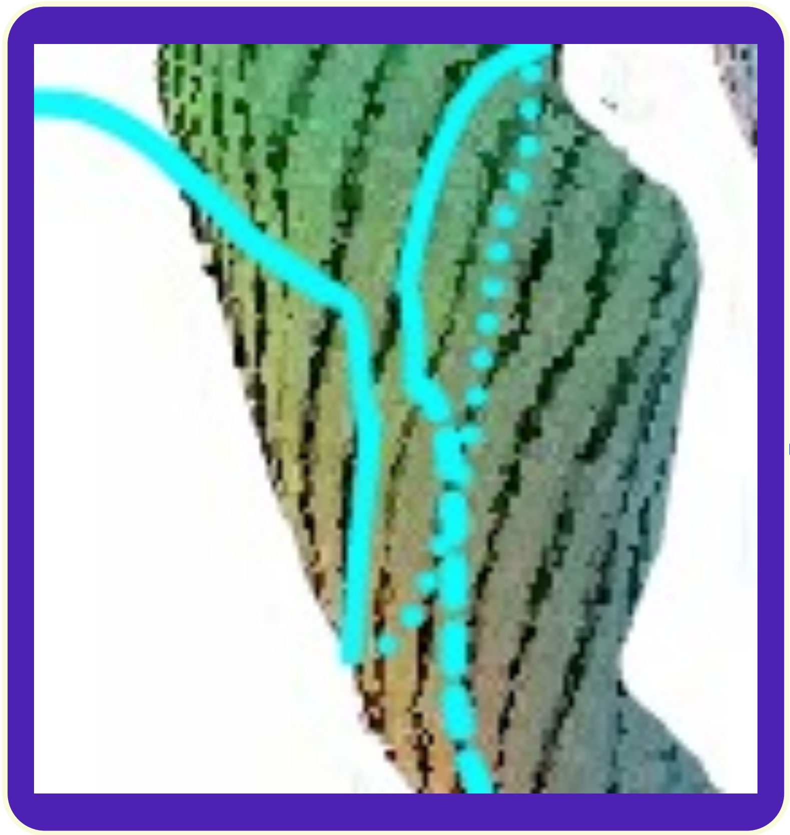
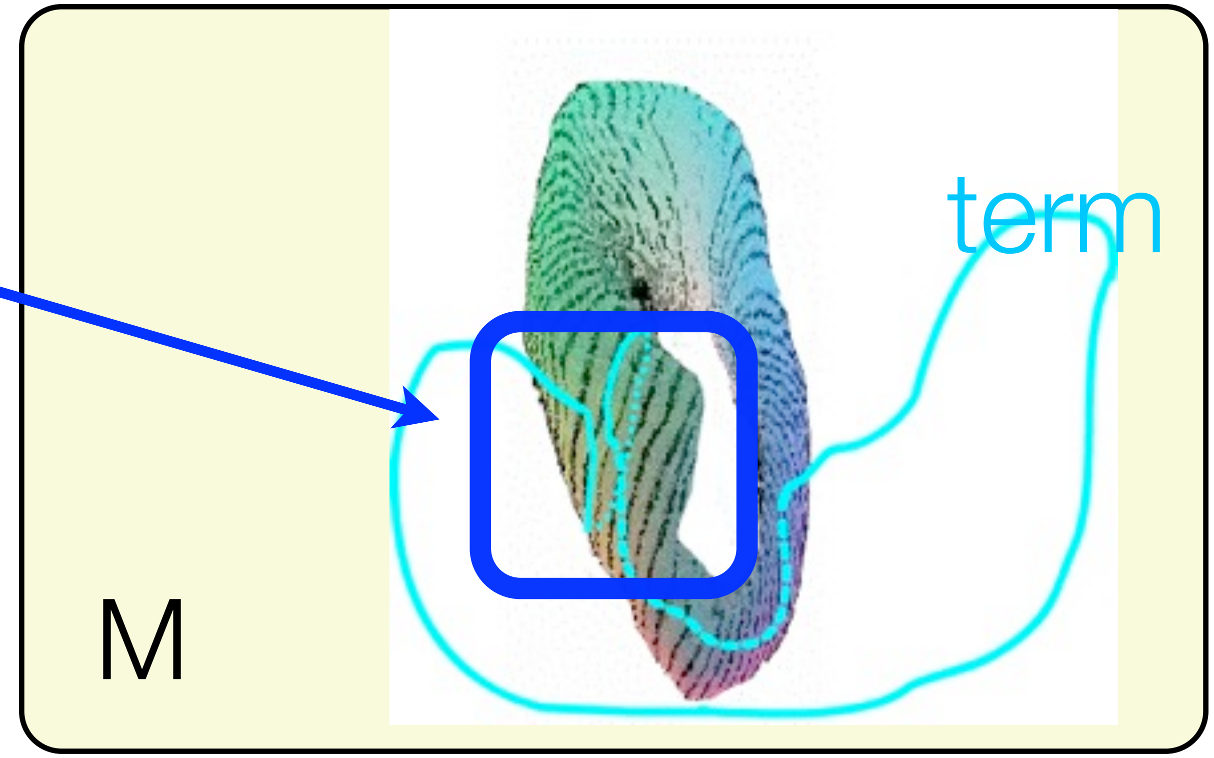


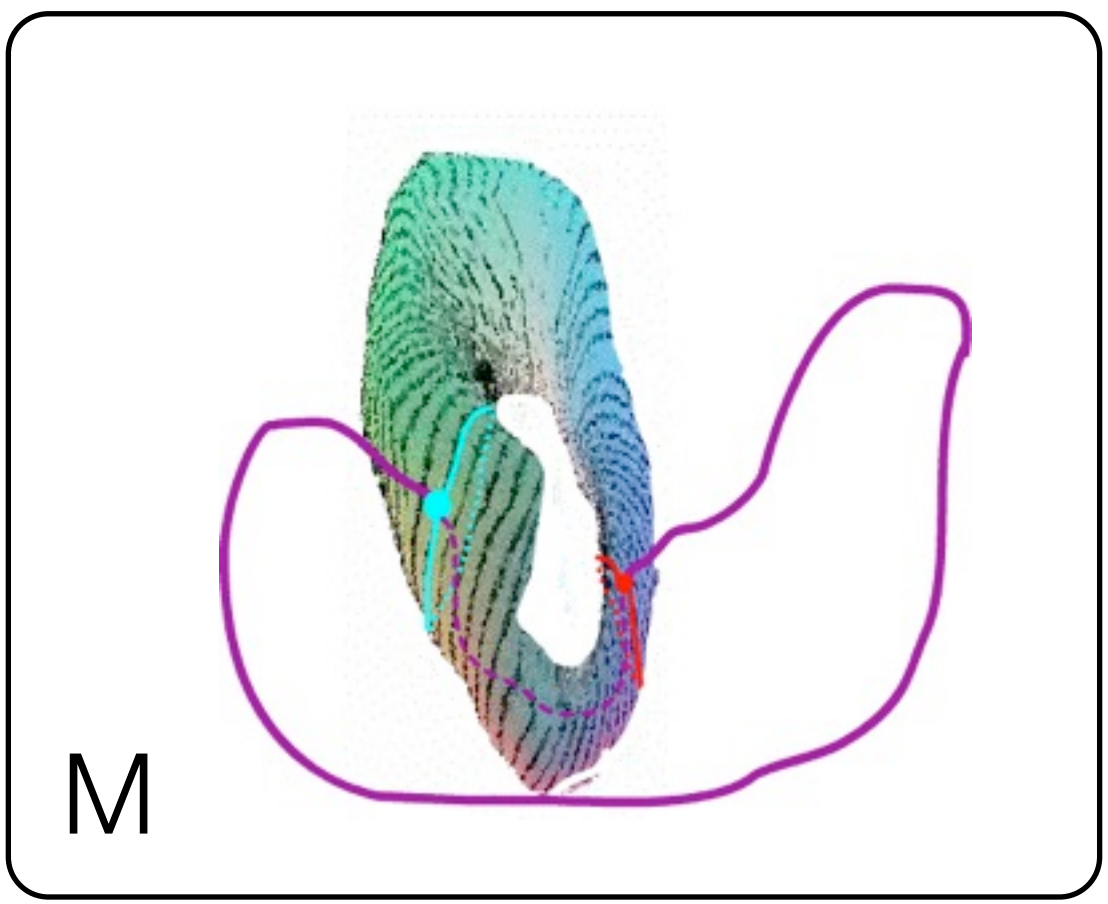


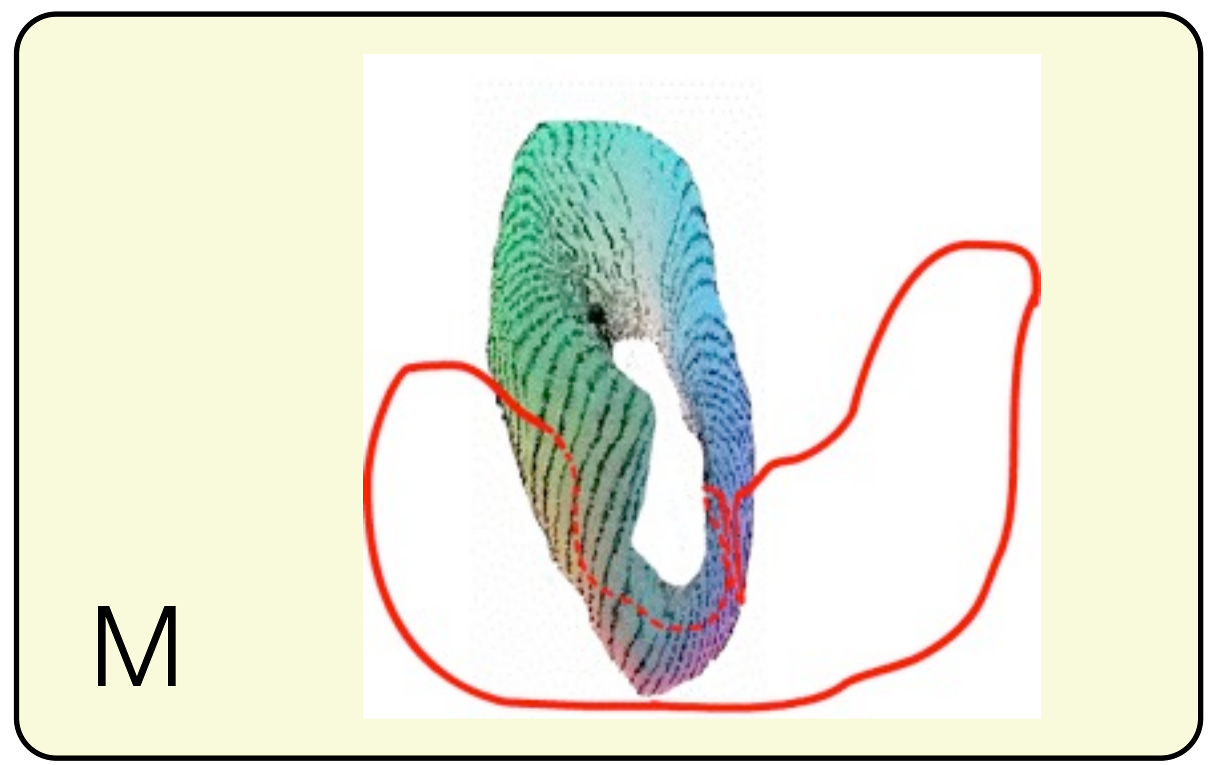
The string bracket

The terms of the bracket are free homotopy classes









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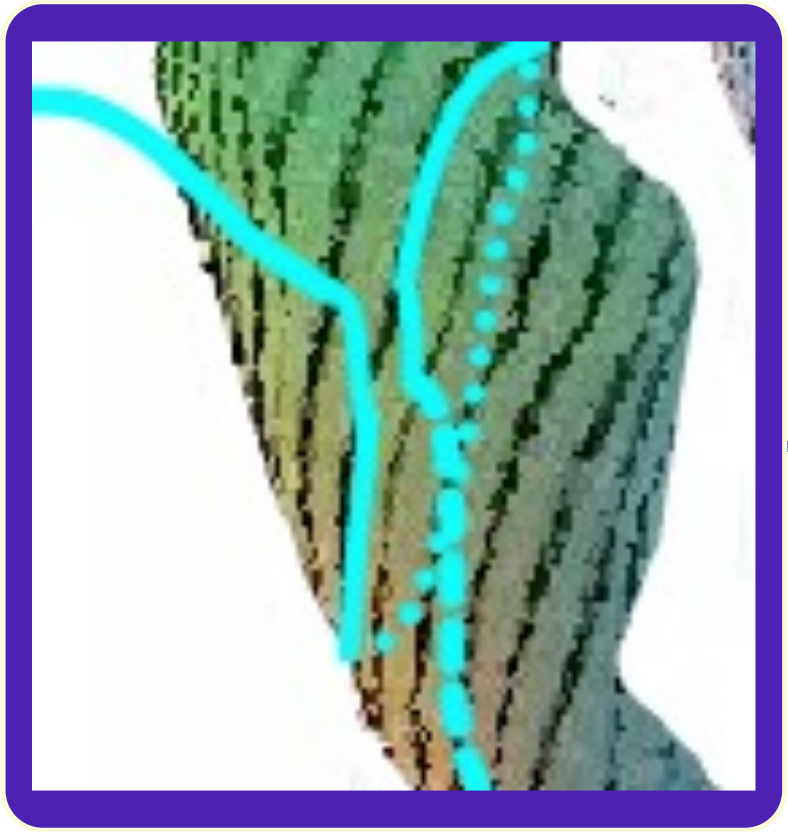

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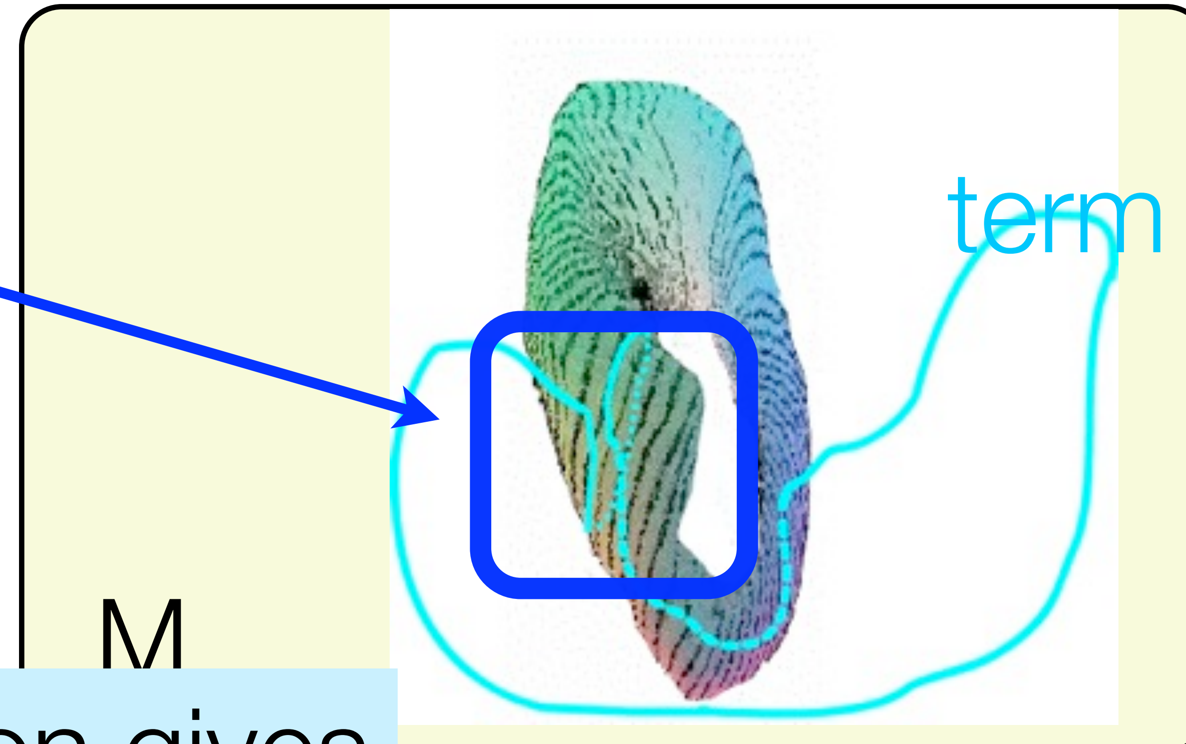
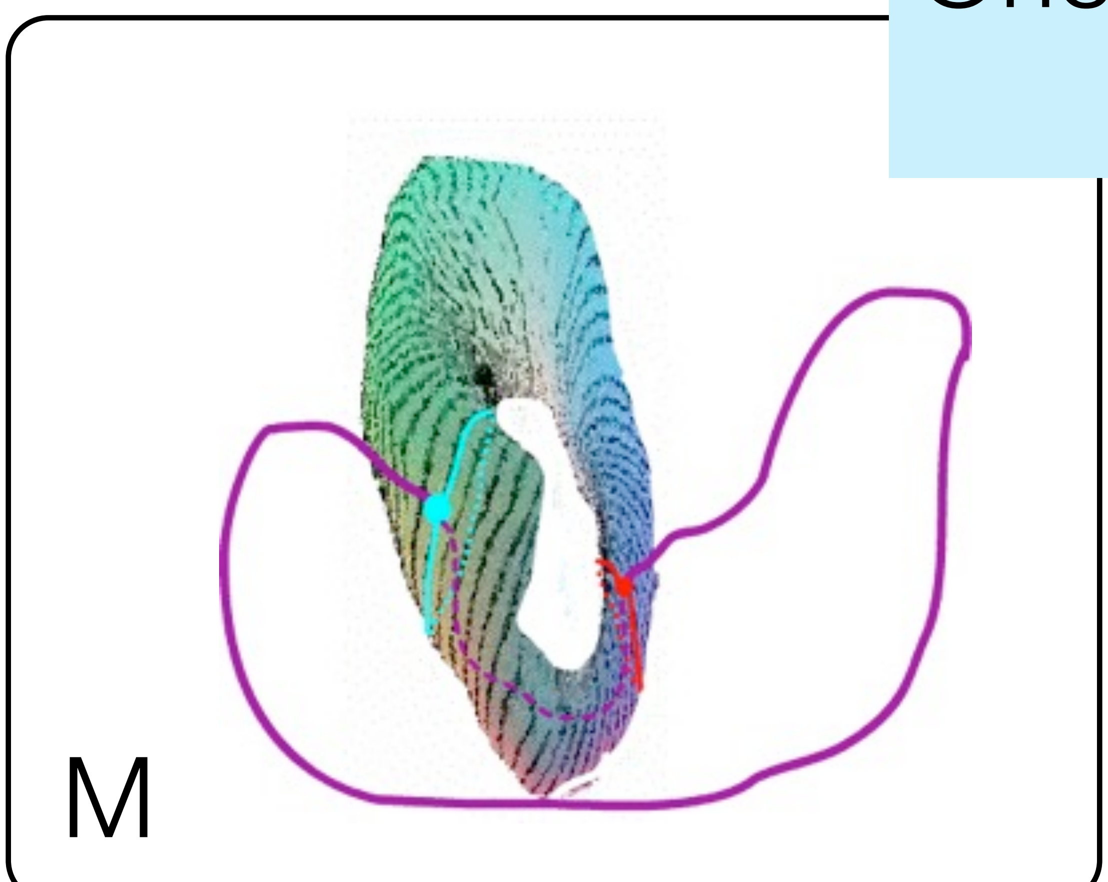
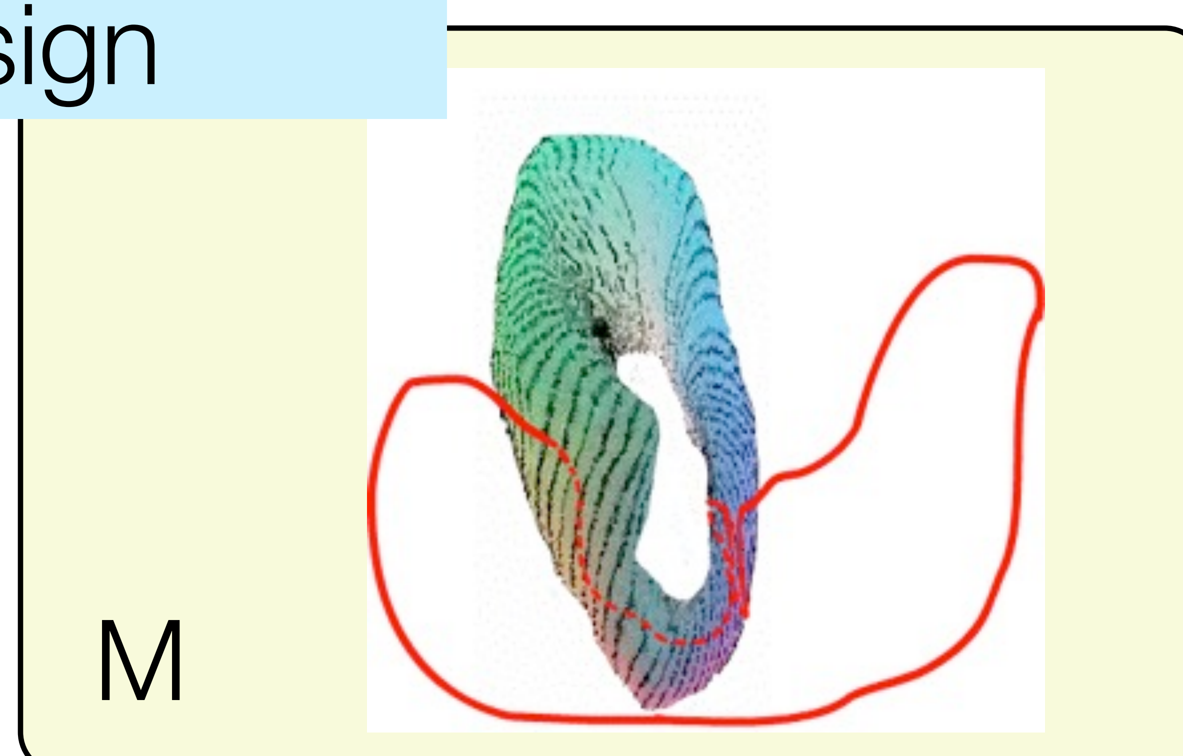


Orientation gives a sign





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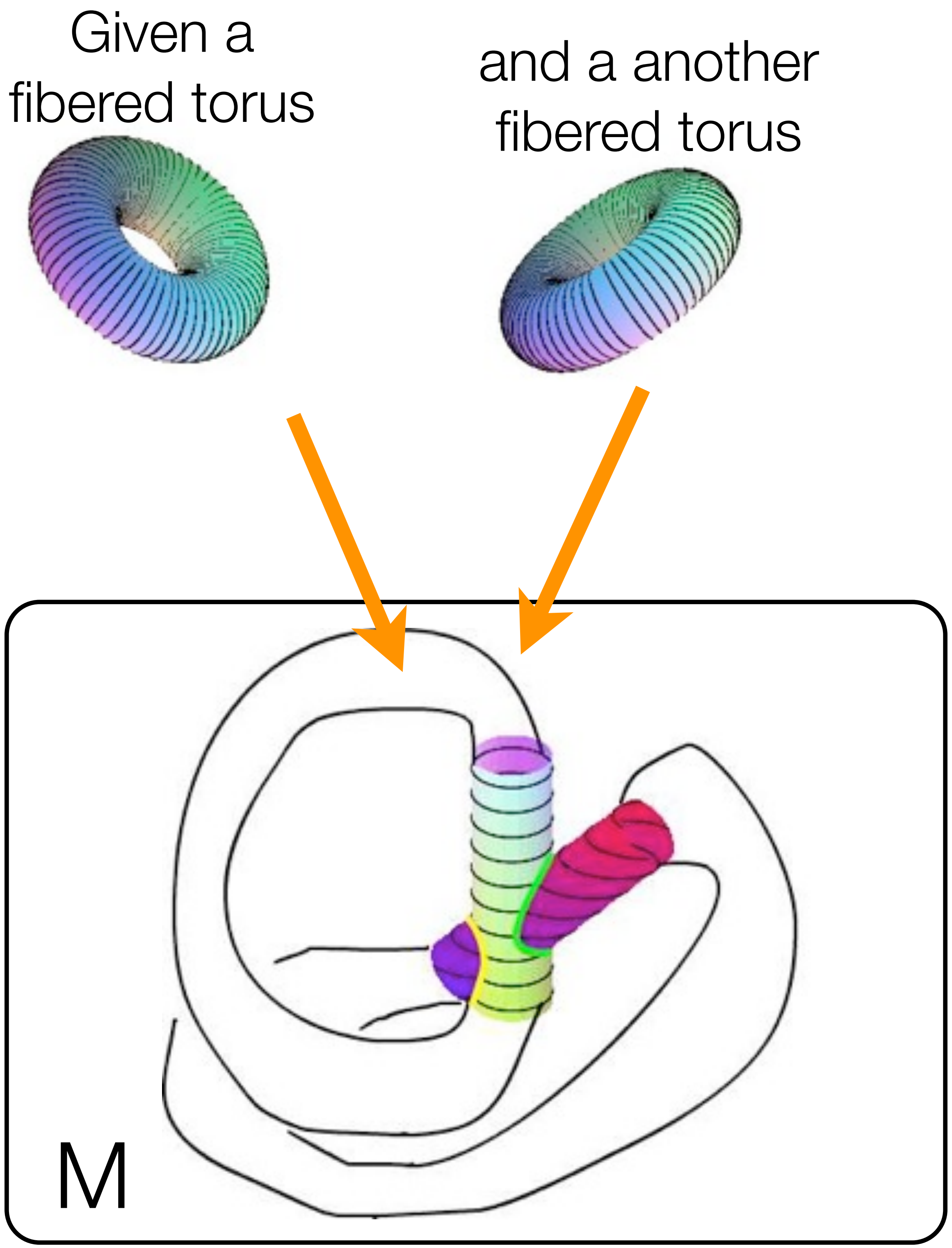
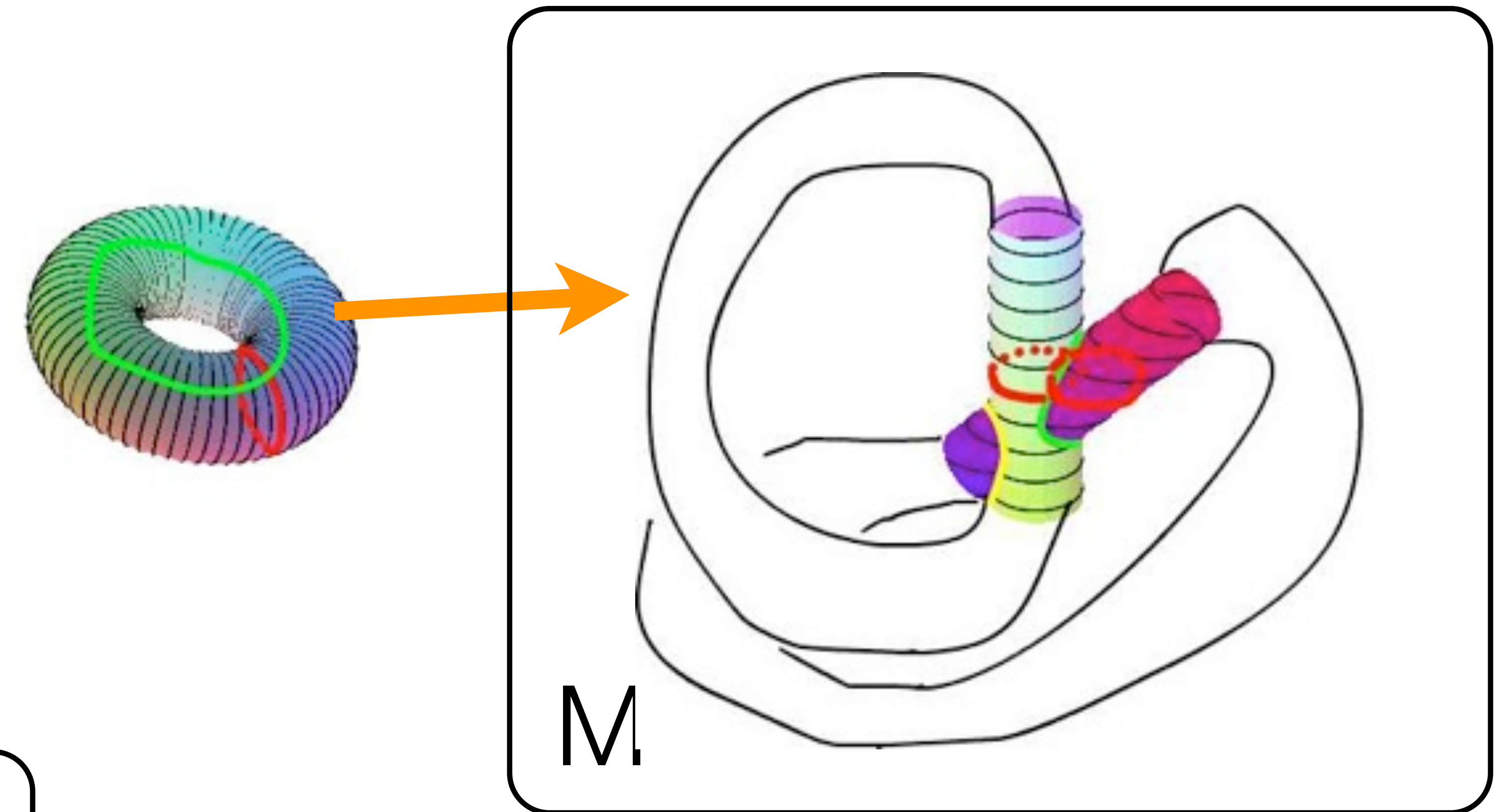
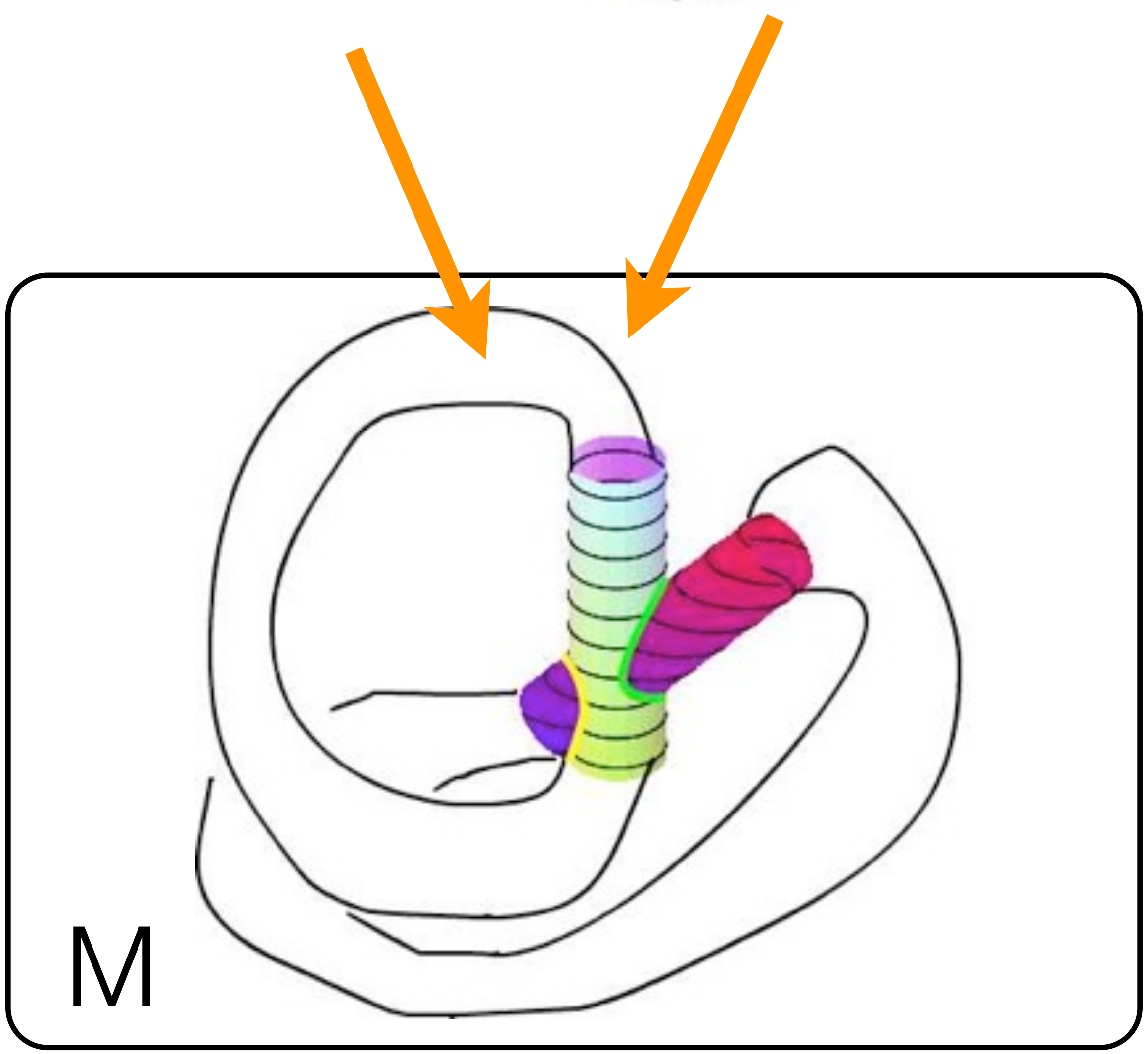


The string bracket

Given a
fibered torus

and a another
fibered torus





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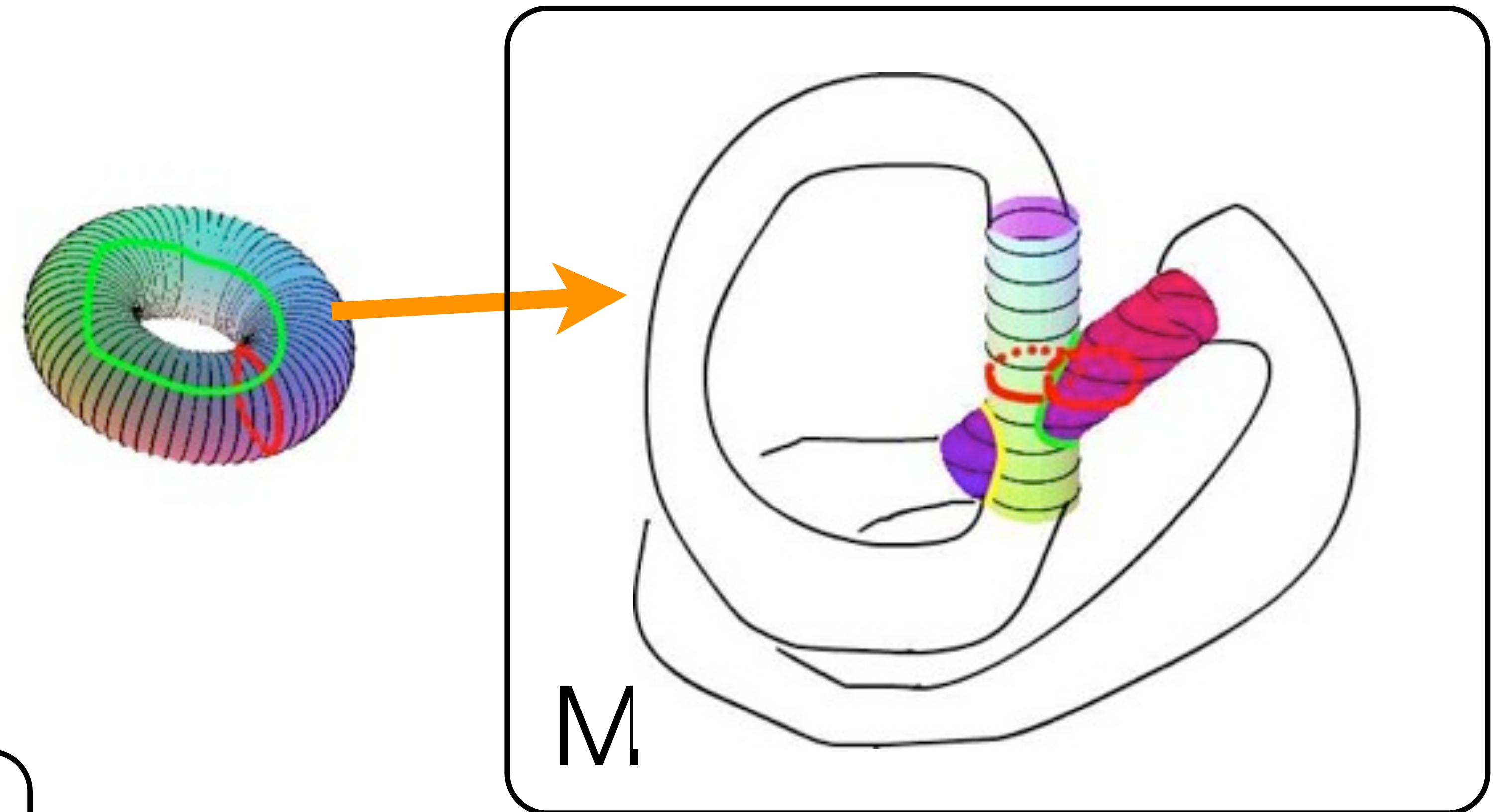

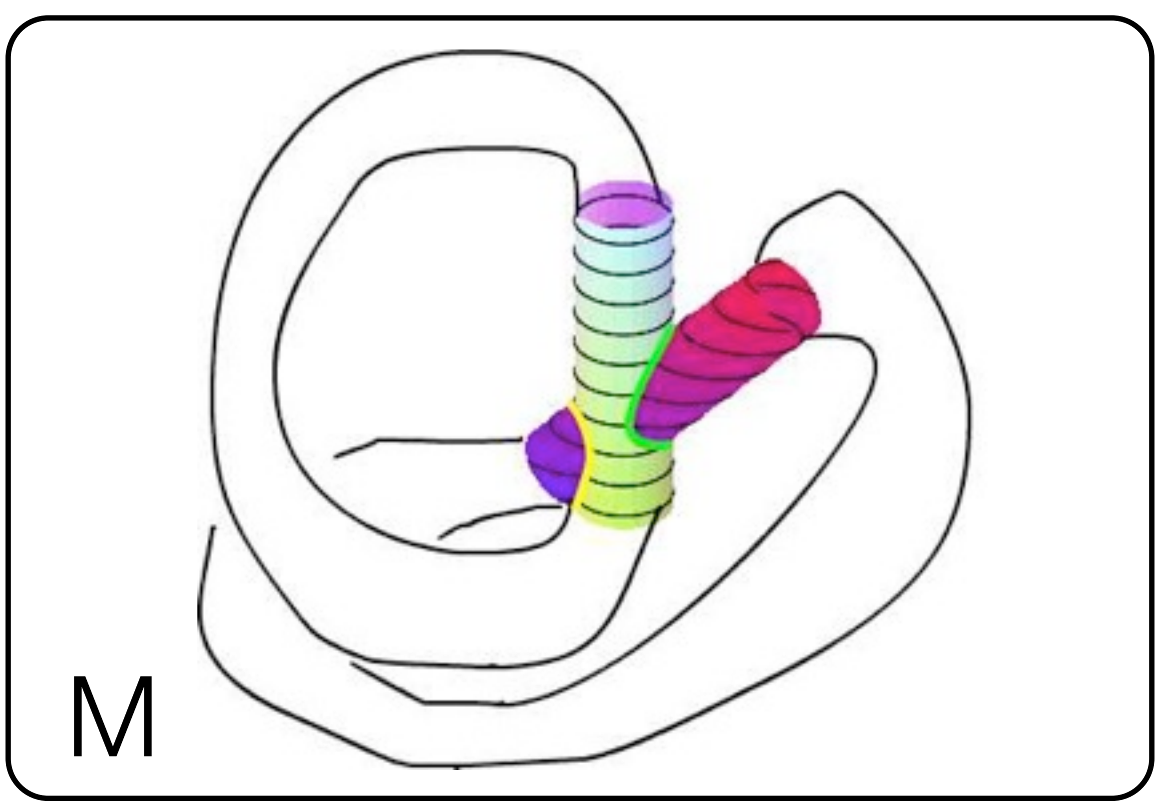

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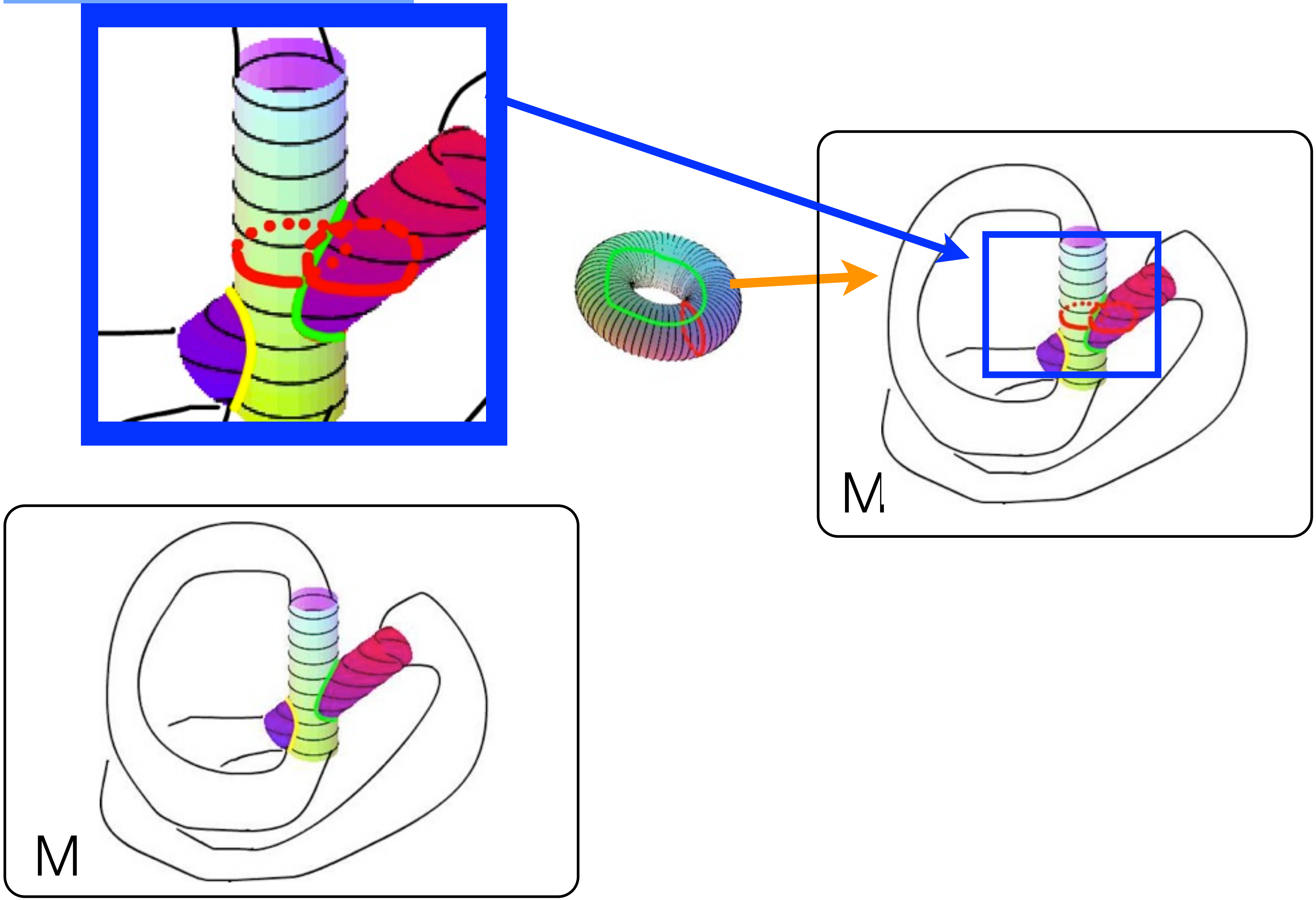

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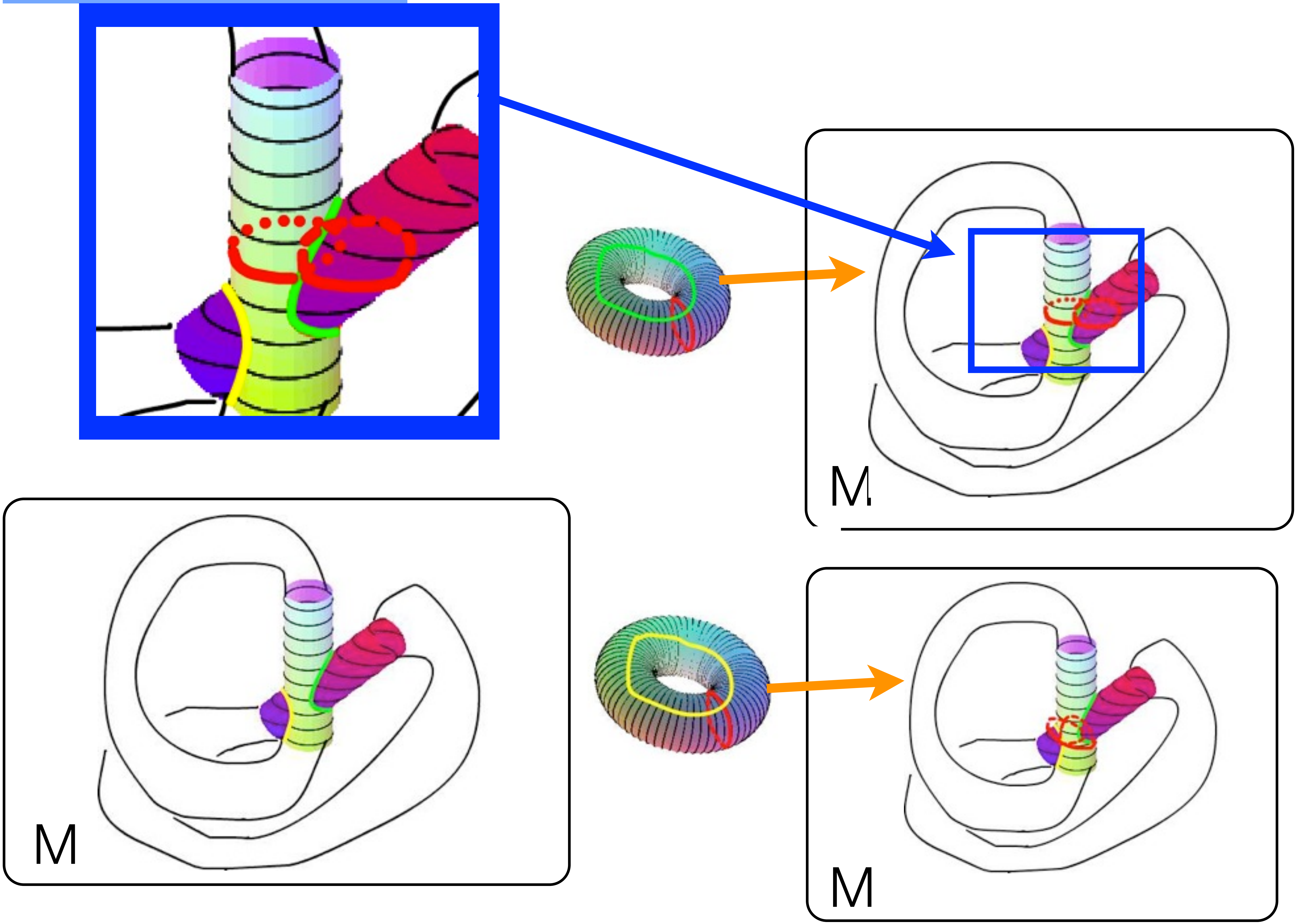

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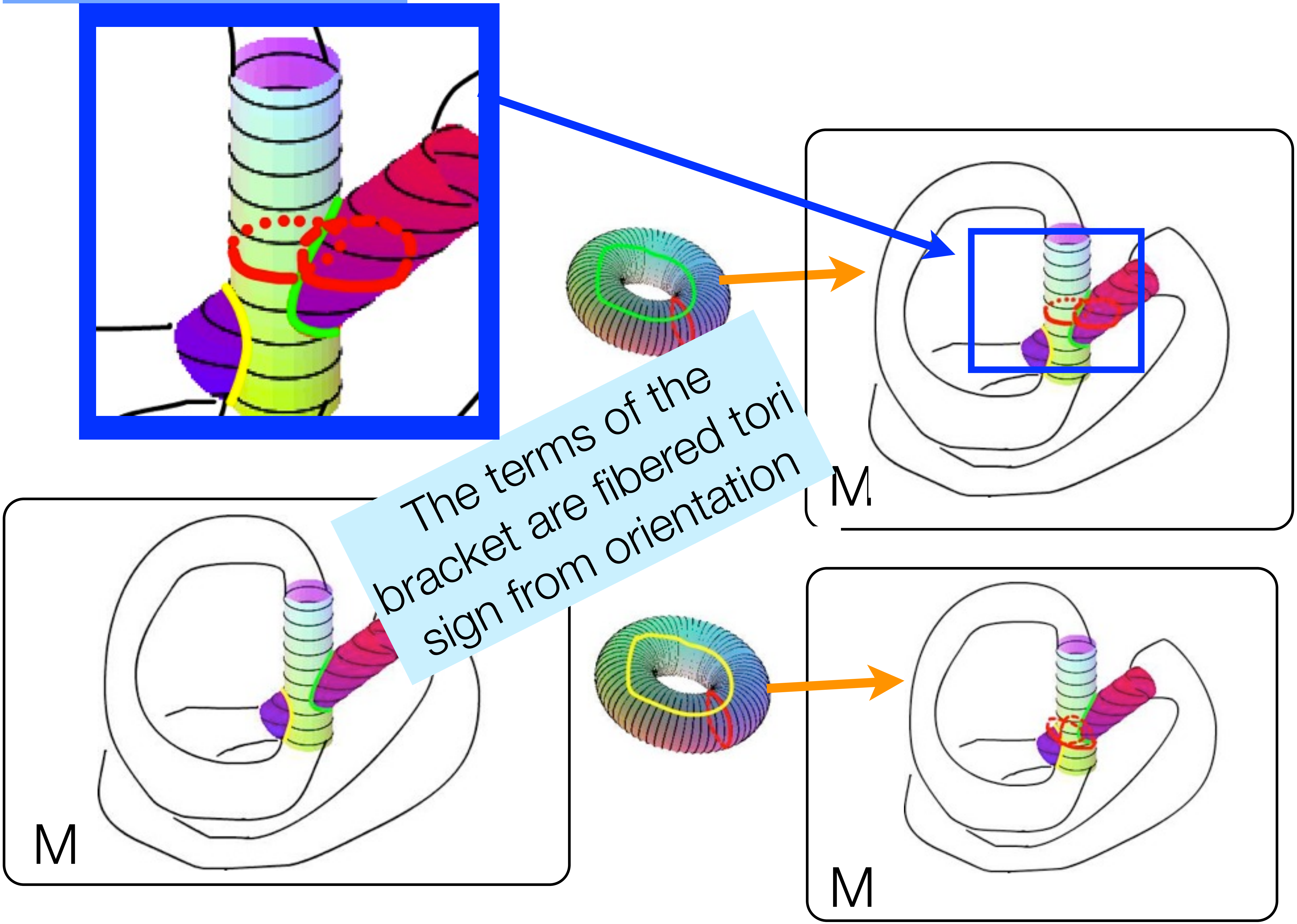
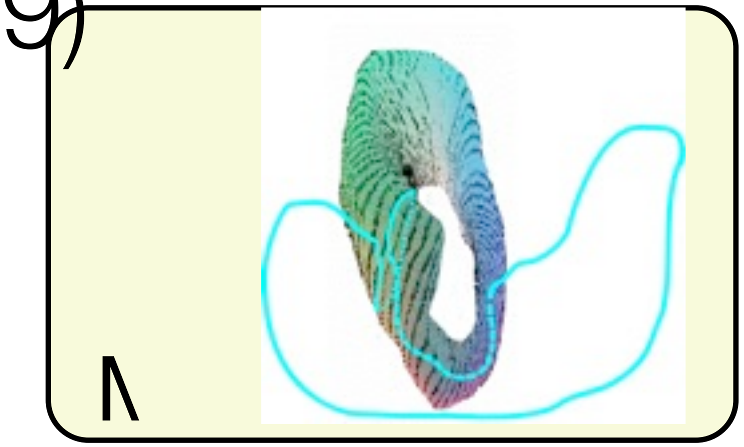
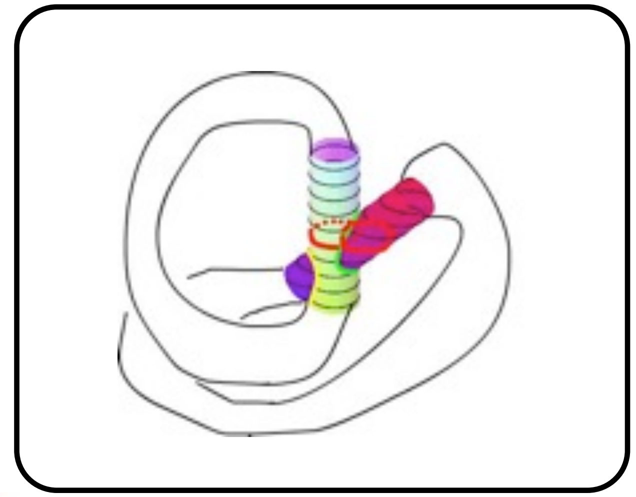


Monday, May 30, 2011

Theorem (Sullivan, C, 1999)

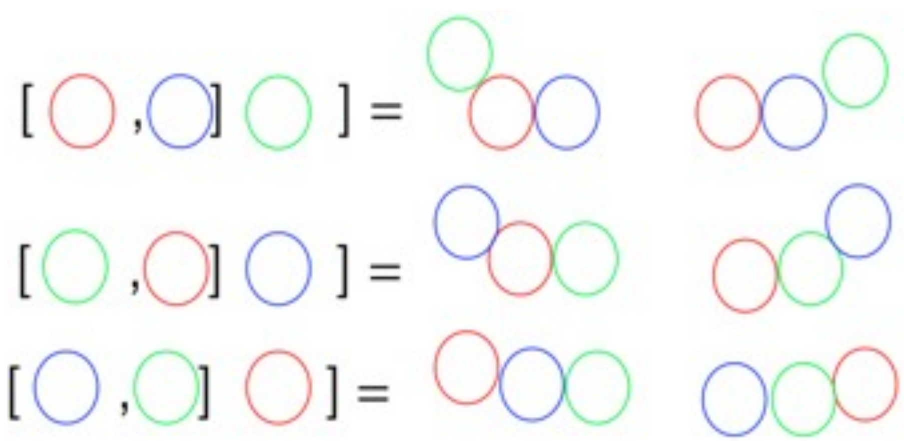


- $H_0 \otimes H_1 \to H_0$ is a Lie module.



- $H_1 \otimes H_1 \to H_1$ is a Lie algebra.

- Jacobi



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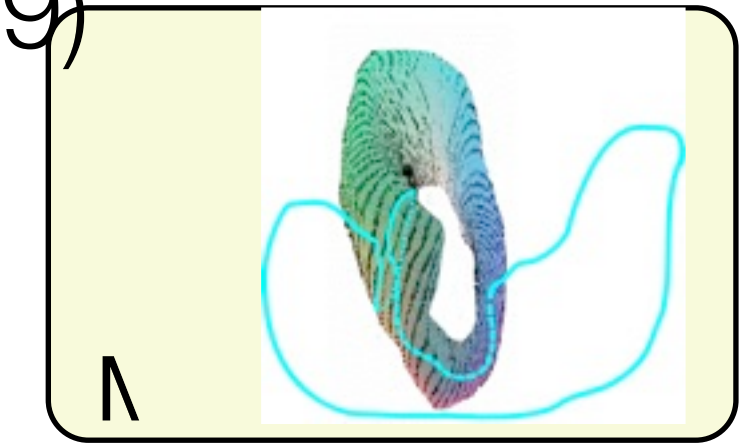

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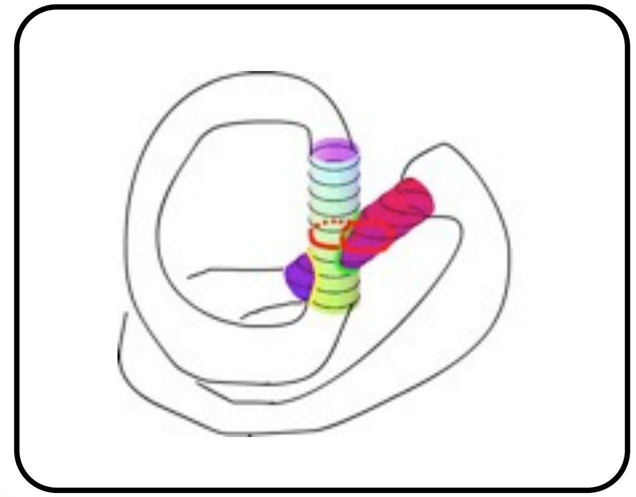

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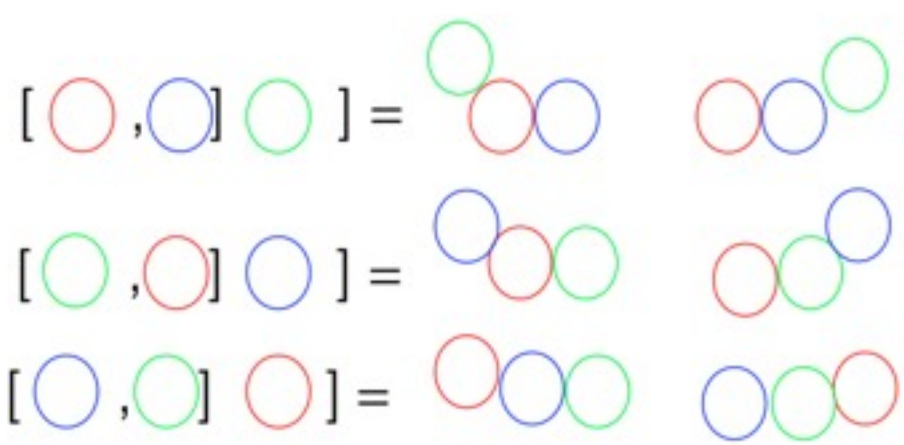

- *(In general, $H \otimes H \to H$ is a Lie algebra of degree 2-d, d is the dimension of the manifold. When d=2, we get the Goldman bracket $H_0 \otimes H_0 \to H_0$).*

Recall that in a surface, if X has an embedded representative then the M[X,Y]= i(X,Y) and

$$\pm[\, w_1 w_2 w_3 w_4,\ X\,] =$$

$$w_1 X w_2 w_3 w_4 - w_1 w_2 X w_3 w_4 + w_1 w_2 w_3\, X\, w_4 - w_1 w_2 w_3 w_4\, X$$

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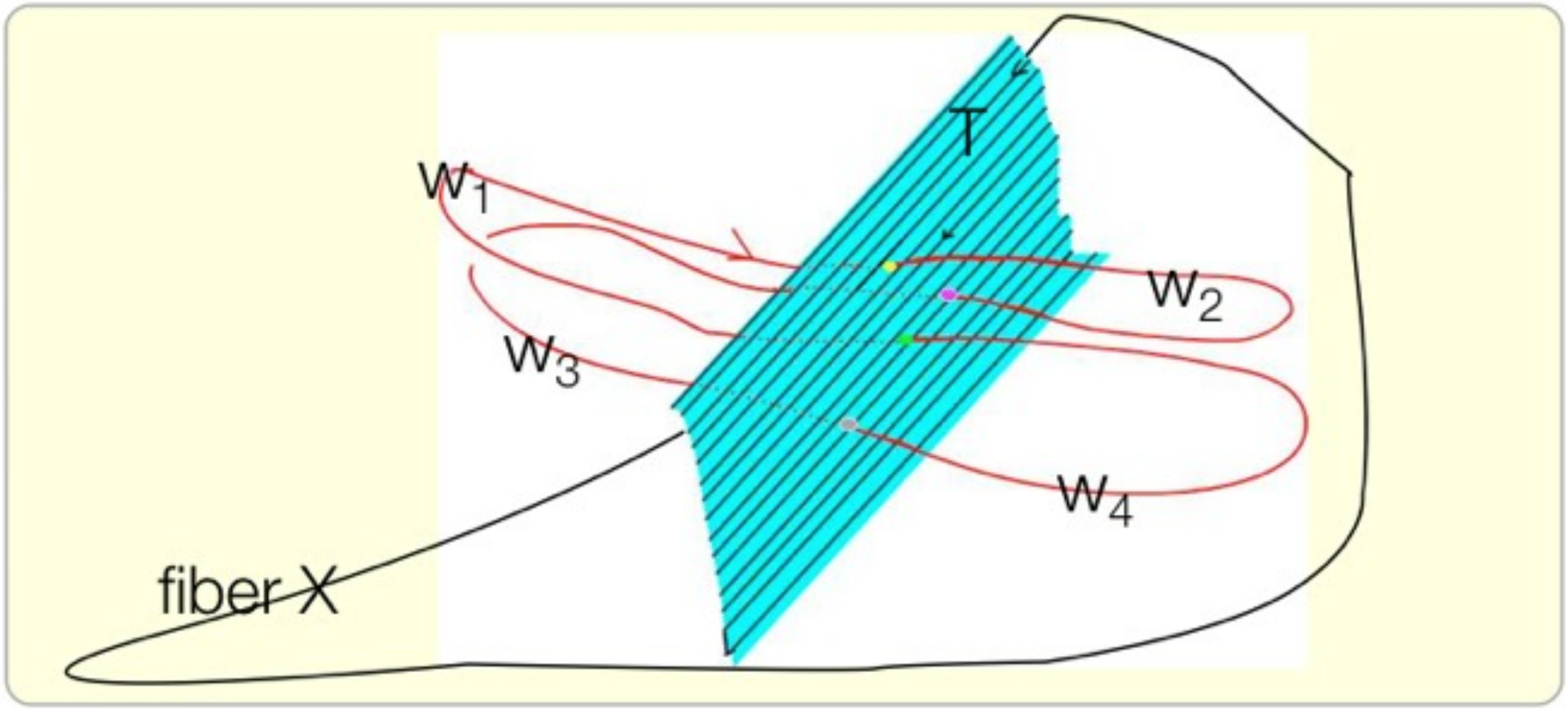
If T is fibered torus and W is a free homotopy class W then M[T,W] $\leq$ i(T,W)

Does M[T, W]= i(T,W) hold, possibly assuming T embedded?

IT is a separating, embedded fibered torus with fiber X



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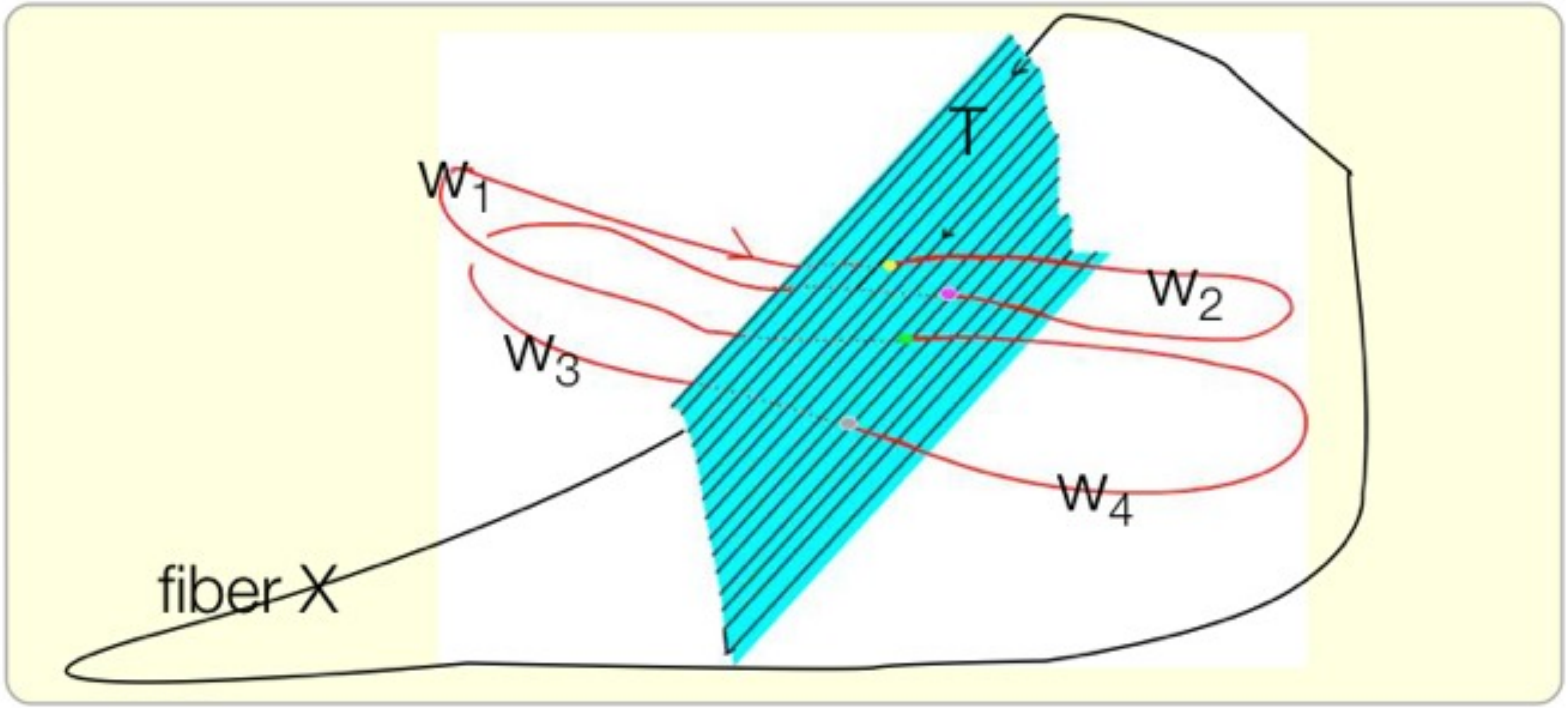

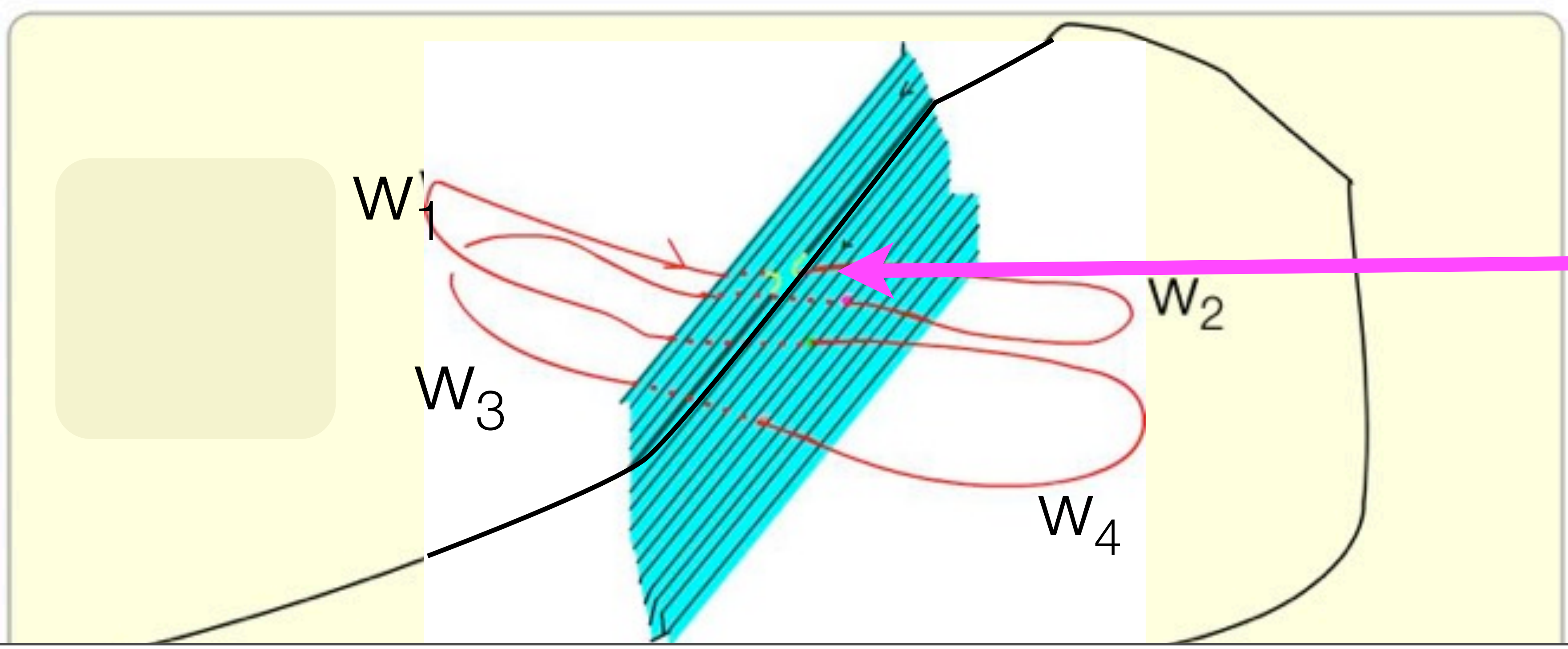

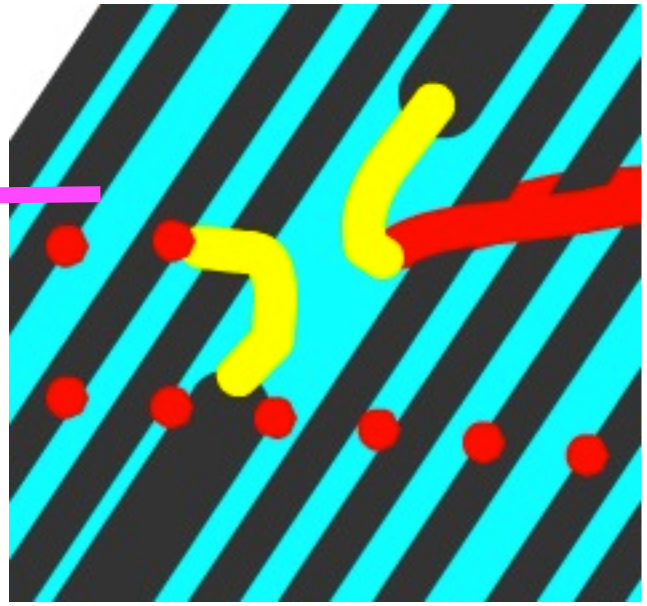

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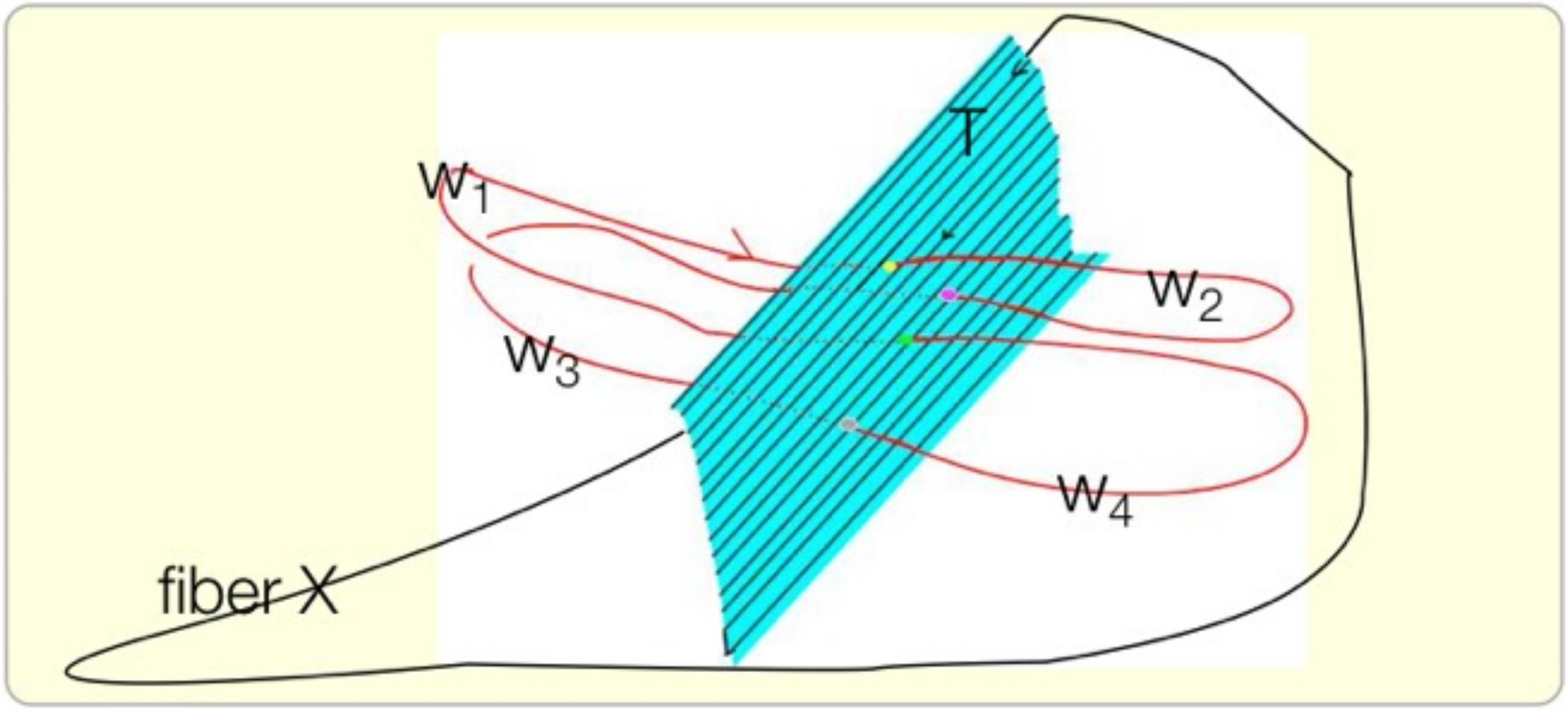
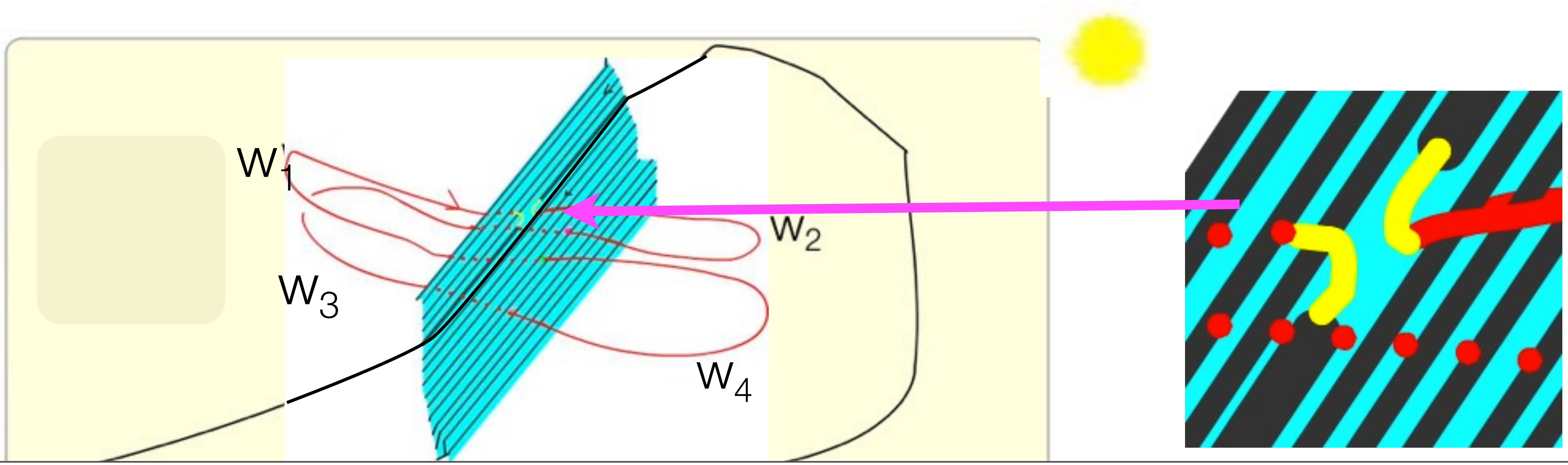


$w_1 X\, w_2 w_3 w_4$



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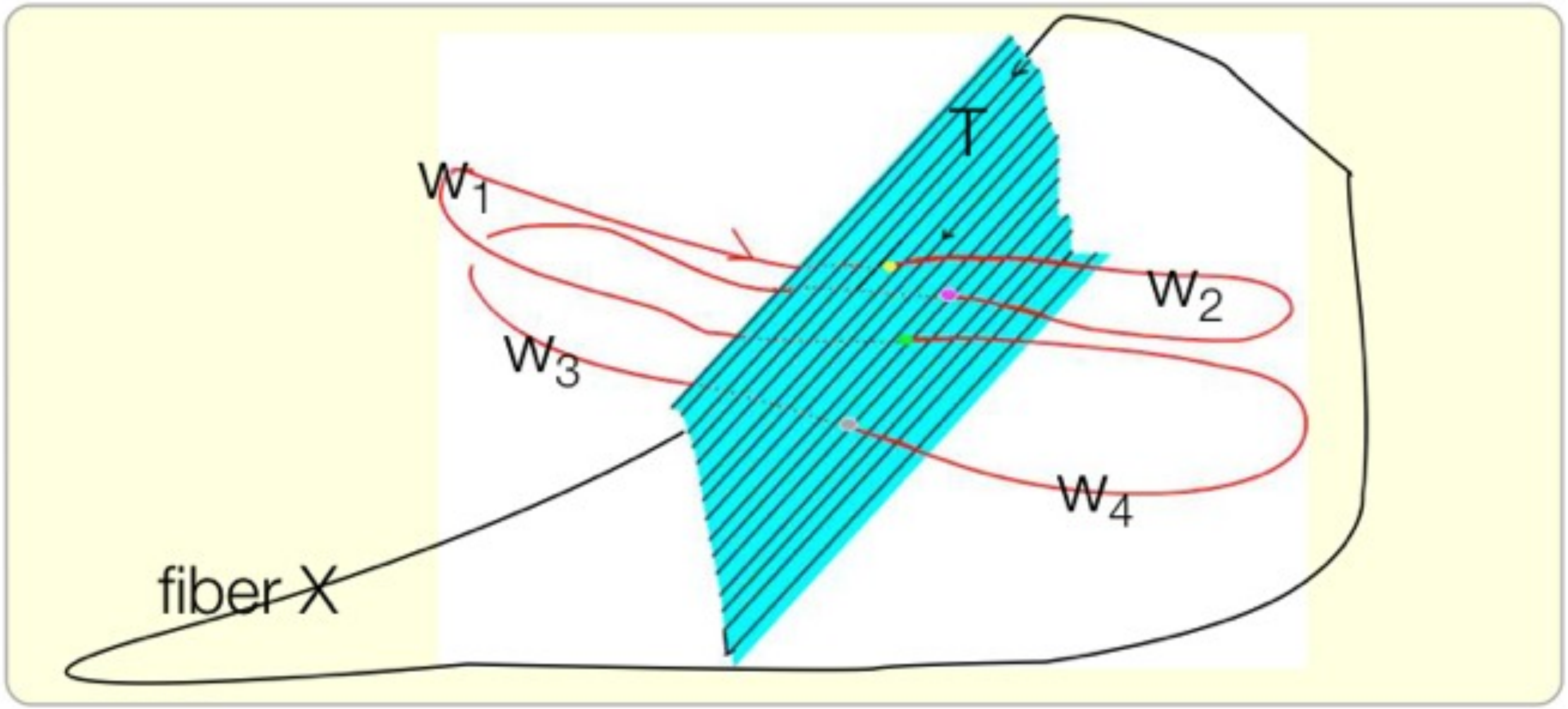

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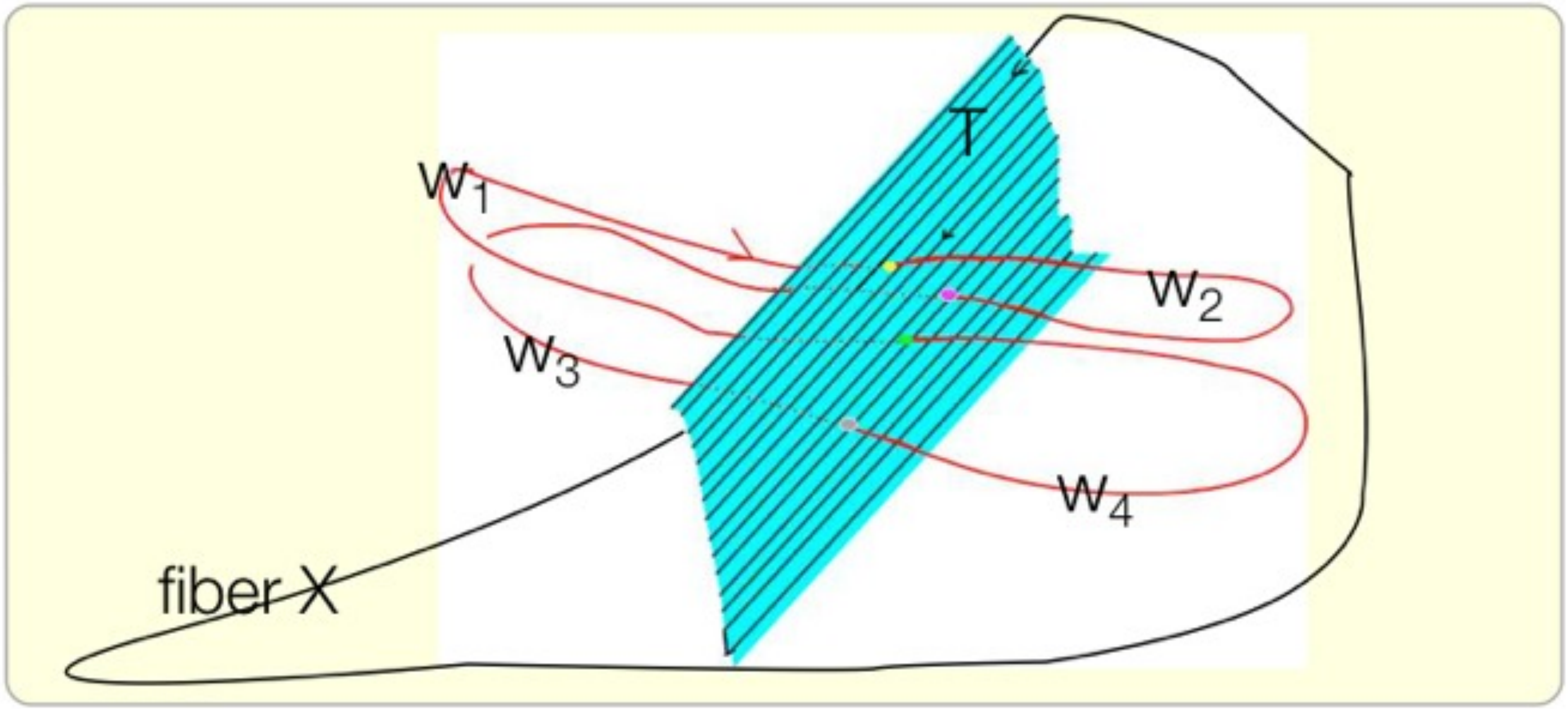

$w_1X\ w_2w_3w_4$

$\pm[w_1w_2w_3w_4,<T,X>]=$

$w_1Xw_2w_3w_4 - w_1w_2X\ w_3w_4 + w_1w_2w_3Xw_4 - w_1w_2w_3w_4X$

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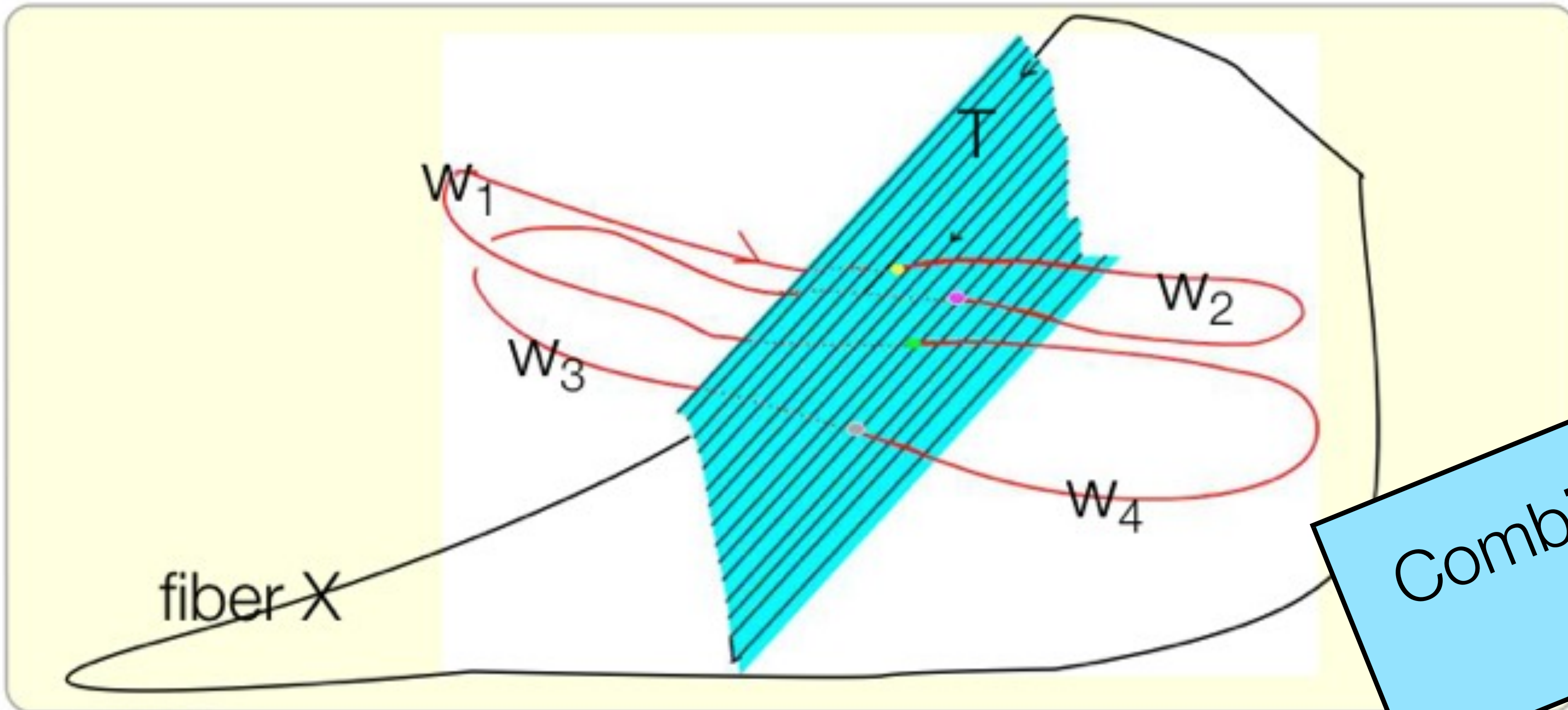

Combinatorial presentation of the string bracket

$w_1X\, w_2w_3w_4$

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Recall: A Seifert fibered manifold is a manifold that is a disjoint union of circles organized in a particular way.

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Except for finitely many, the circles are freely homotopic to a curve h.

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The center of the fundamental group of a Seifert manifold is typically generated by h.

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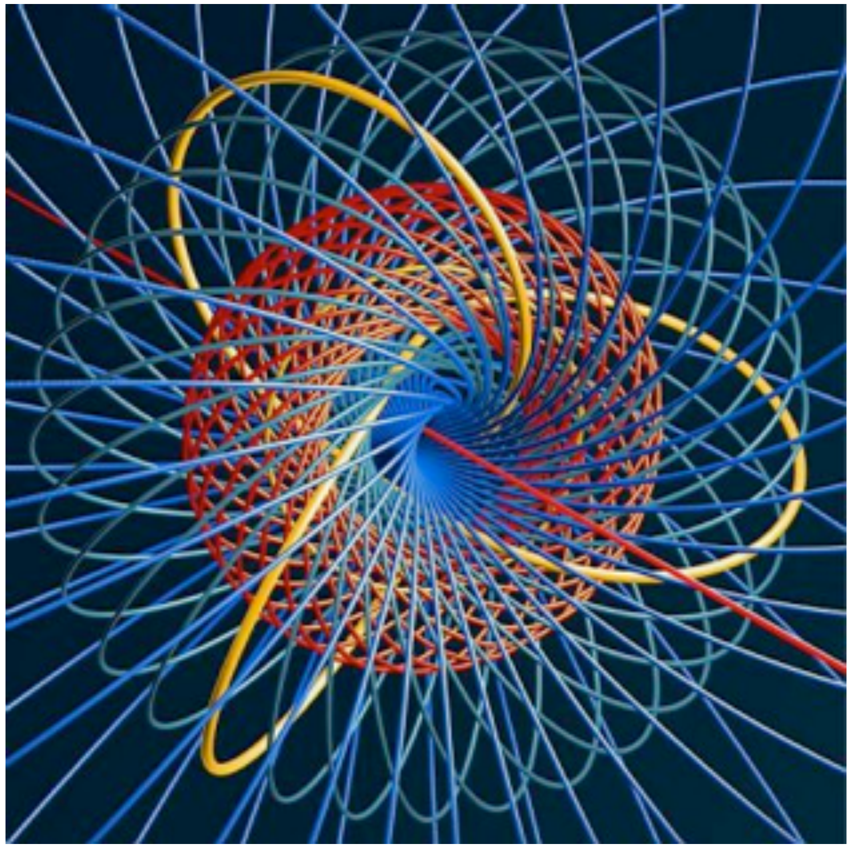

Image by Jos Leys

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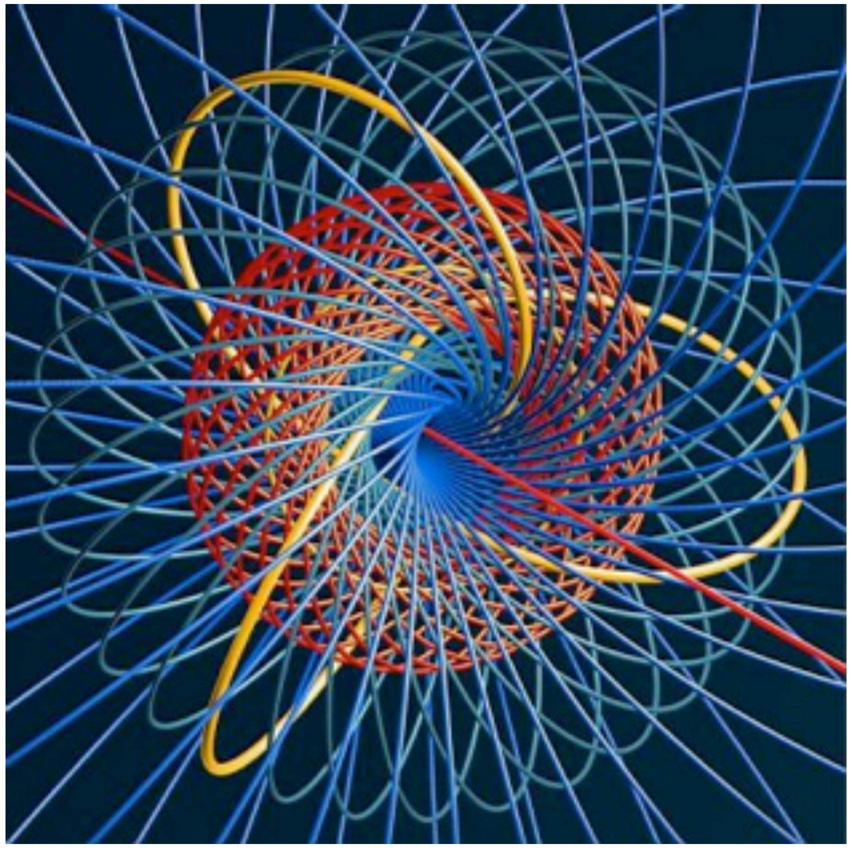

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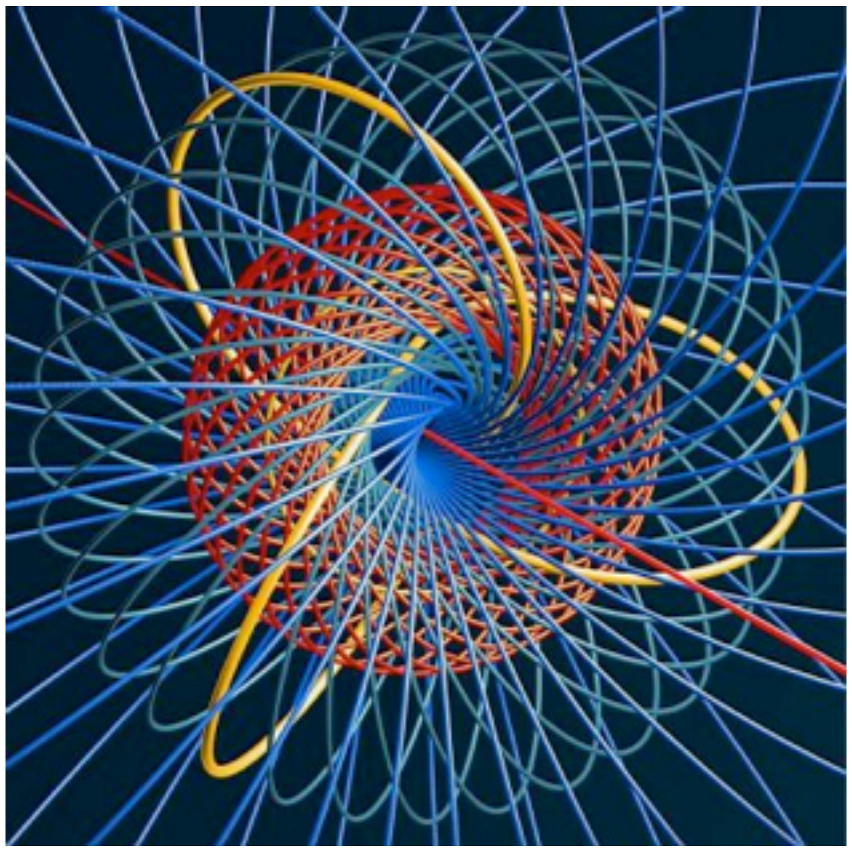

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$[w_1w_2w_3w_4,\langle T,h\rangle]$

$=w_2w_3w_4w_1h - w_3w_4w_1w_2h + w_4w_1w_2w_3h - w_1w_2w_3w_4h$

$= 0$

Theorem (Gadgil, C)

Let T be (the homology class corresponding to) an embedded fibered torus whose fiber is not the generic fiber of a Seifert piece.

Let A be (free homotopy class of) a closed curve.

Then $M[T, A^2] = 2\, i(T, A)$

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Why A^2 ?

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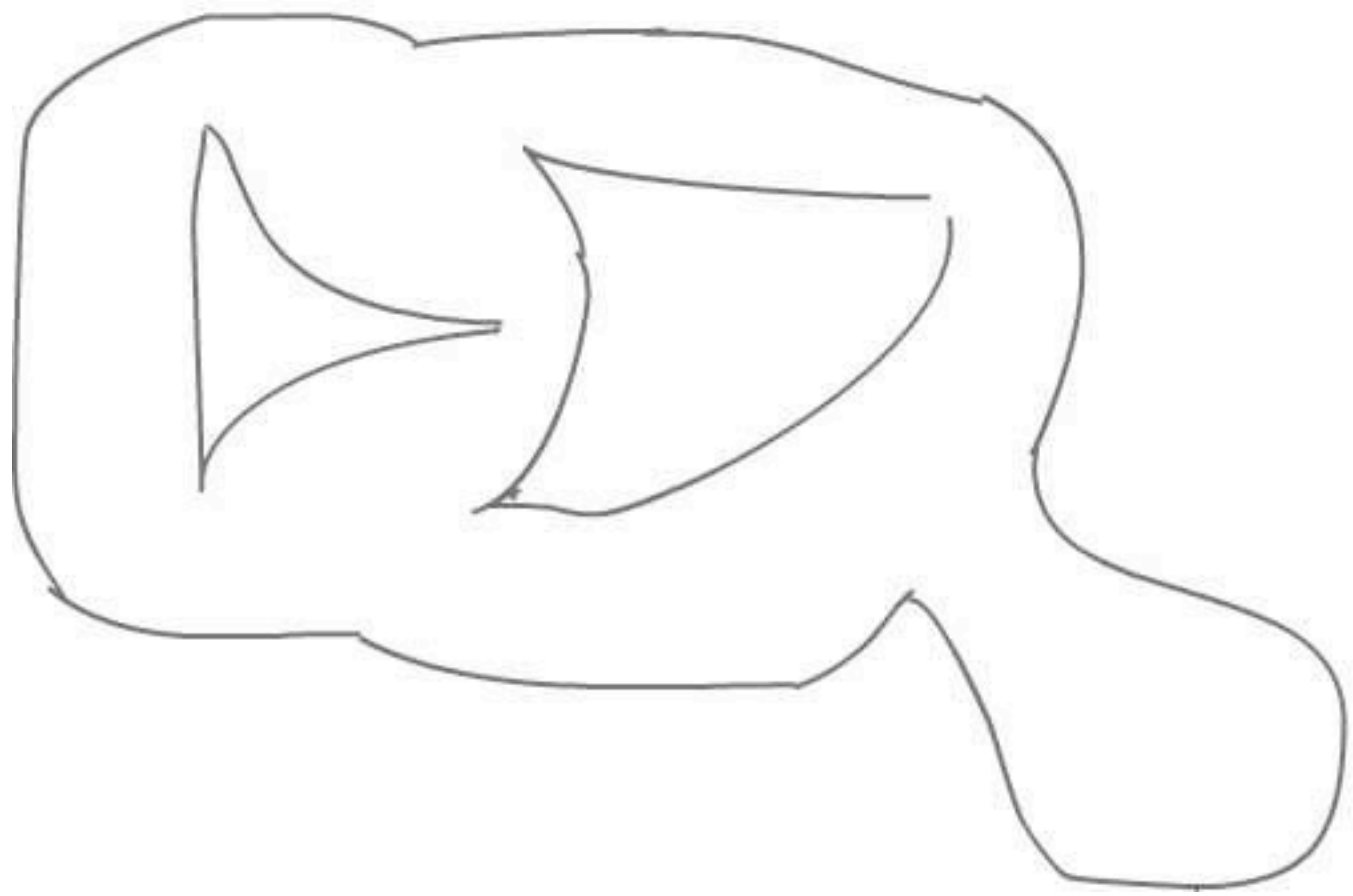

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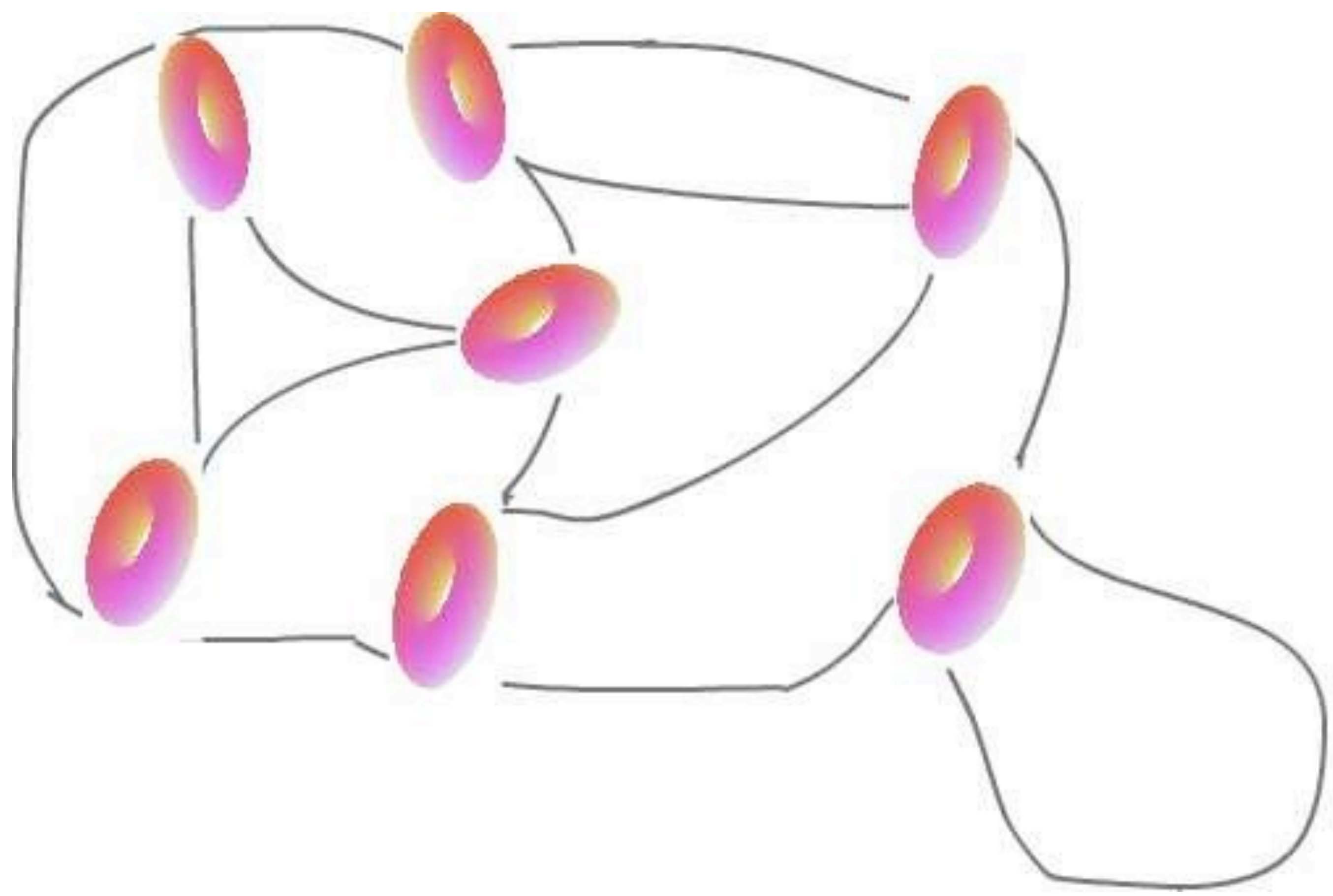
There exists a minimal collection of π_1 injective tori such that



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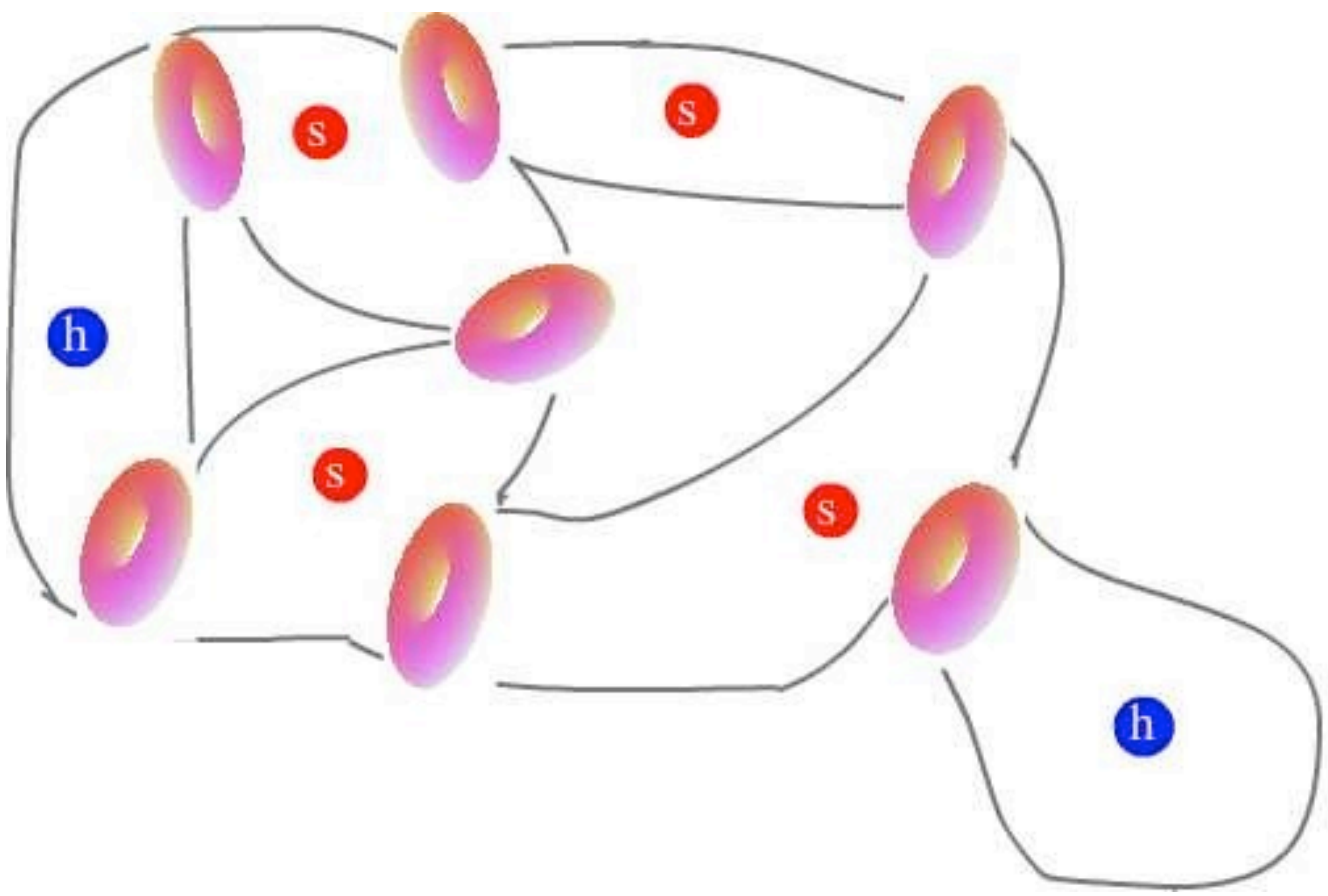

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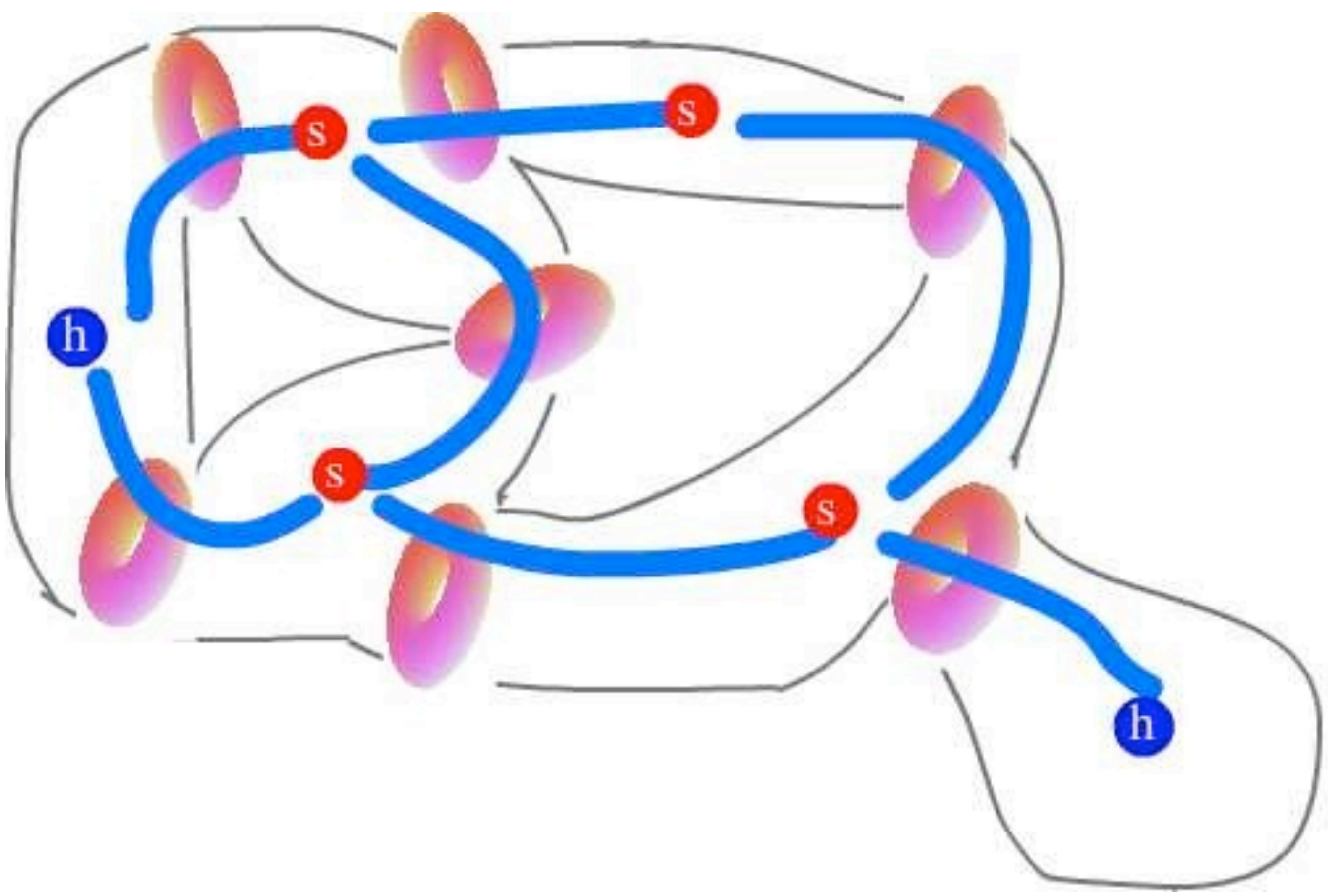

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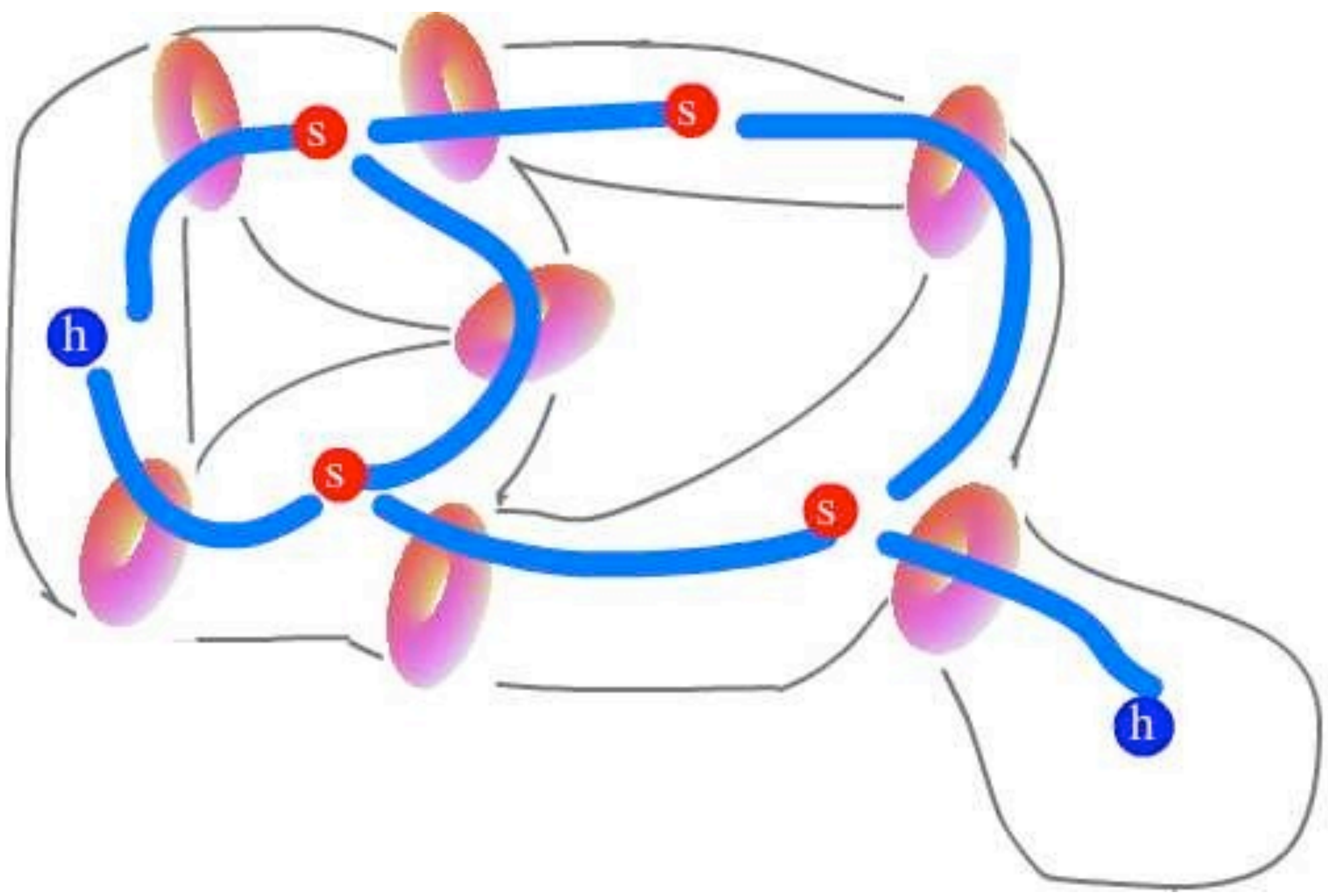
Thm (Gadgil, C) String topology gives the H-S colored graph of the graph of group of M. Also, genus and number of boundary components of Seifert pieces.



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Step 1 of the proof. Use [T, T'] to "classify" tori

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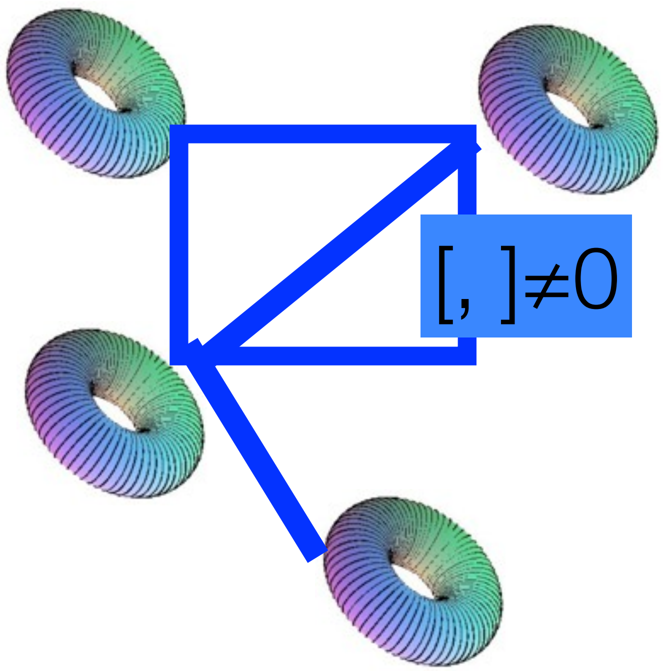

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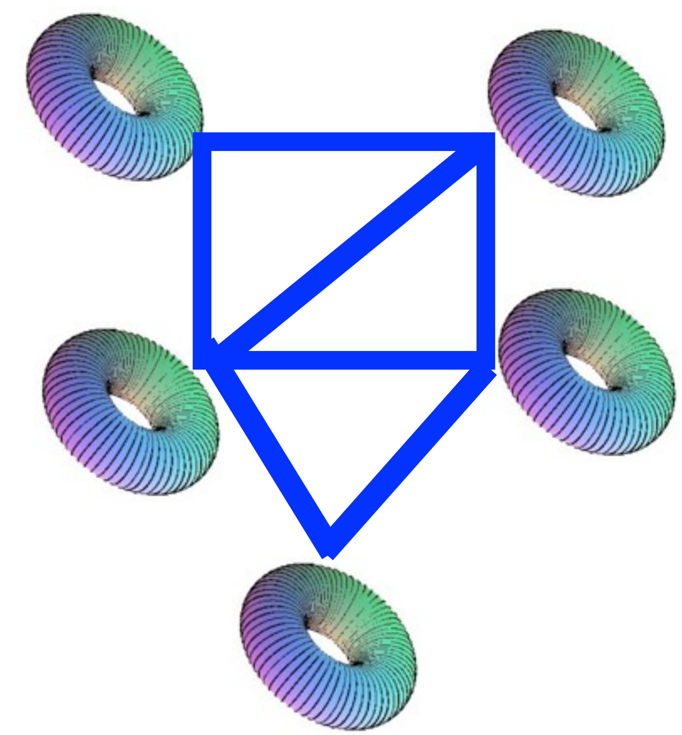
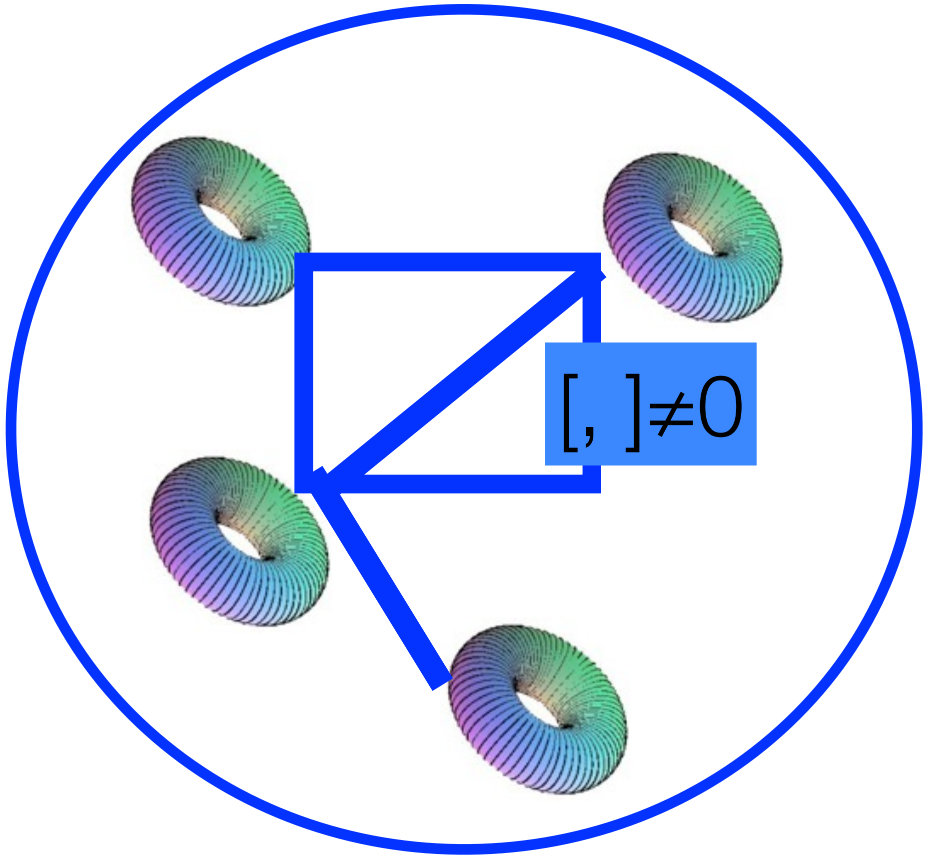


Consider the graph with vertices all fibered tori, with an edge between two tori if [,]≠0



Theorem (Gadgil, C) String topology gives the H-S colored graph of the graph of group of M and the genus and number of boundary components of Seifert pieces.

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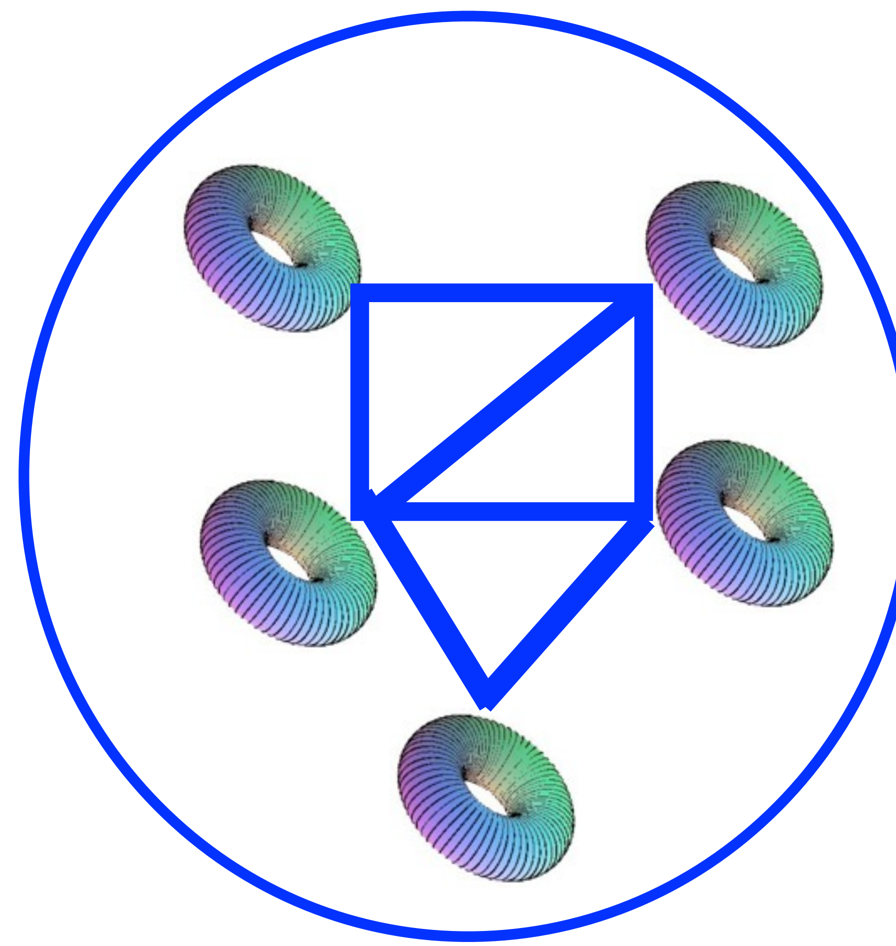

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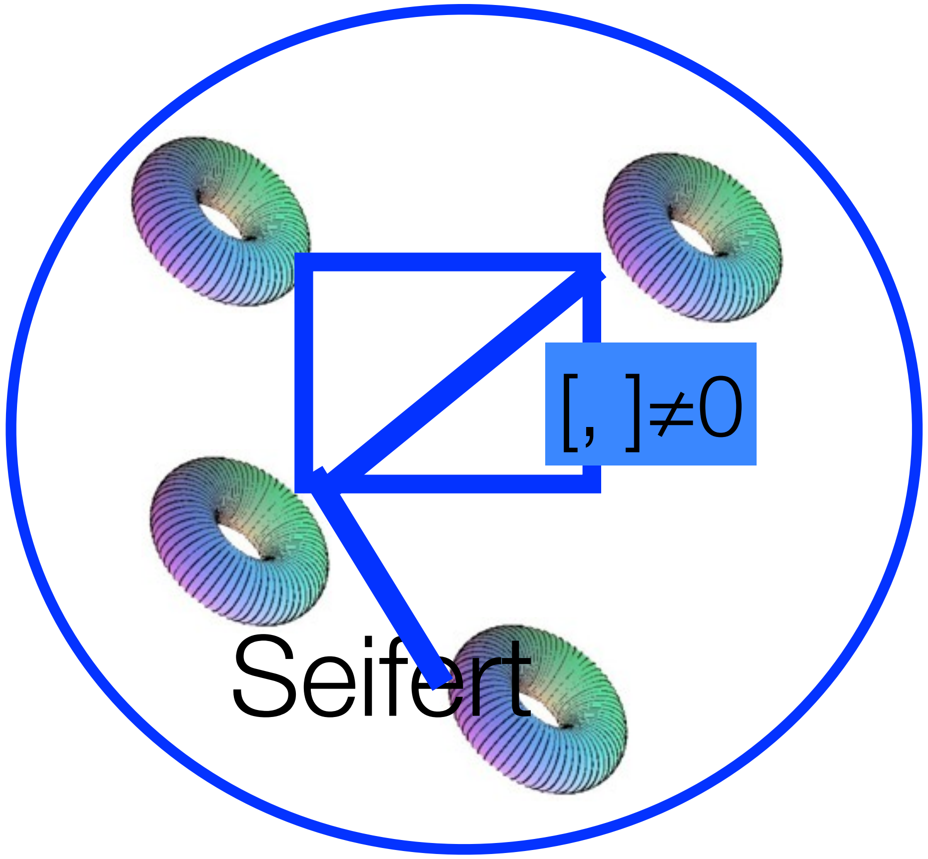

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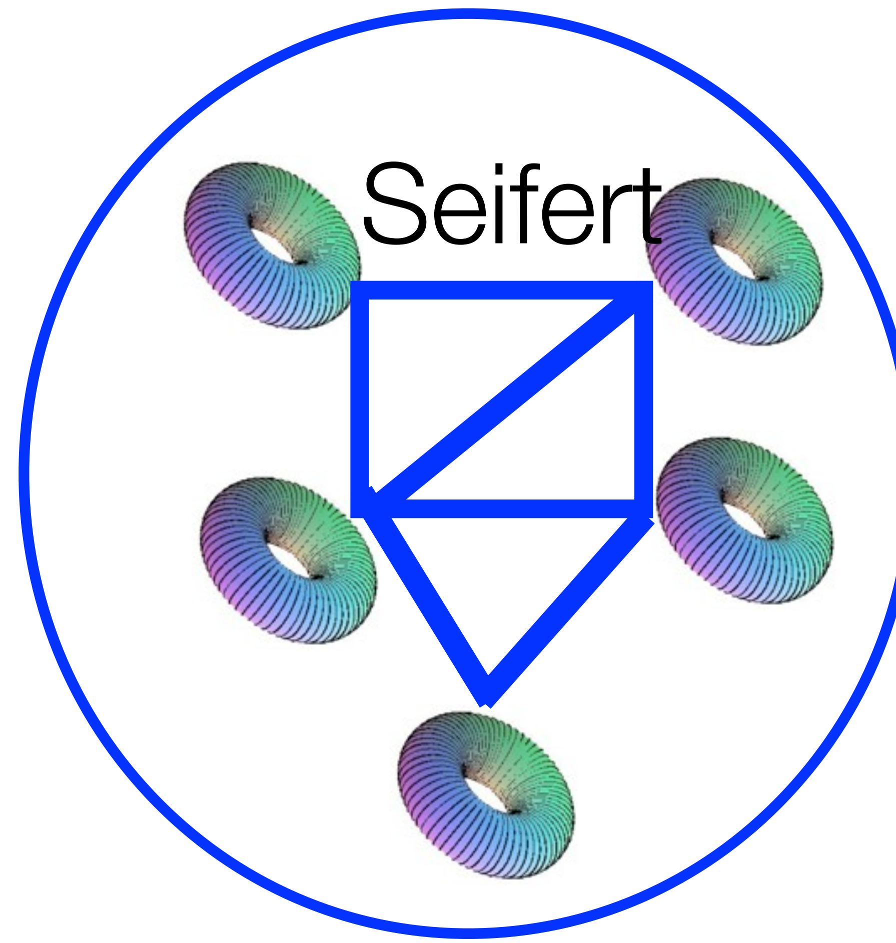

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Theorem (Gadgil, C) String topology gives the H-S colored graph of the graph of group of M and the genus and number of boundary components of Seifert pieces.

Step 2 of the proof. Say two fibered tori T, T' are equivalent if $M[T,A^2]=M[T',A^2]$

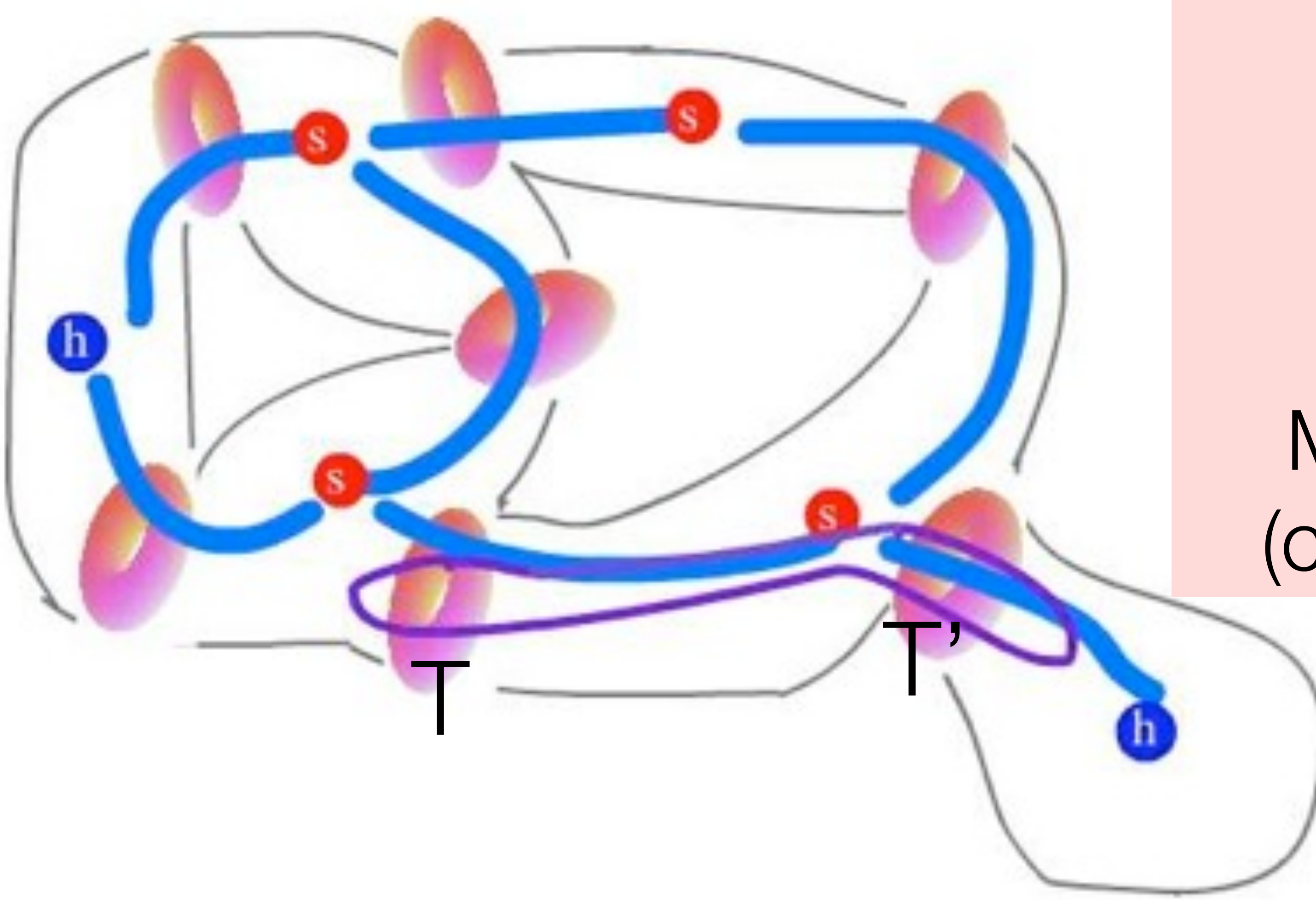
Thus (for most tori) T and T' are equivalent if and only if they are the same torus, with different fiber.

Theorem (Gadgil, C) String topology gives the H-S colored graph of the graph of group of M and the genus and number of boundary components of Seifert pieces.

$M[\ T,\ 0^2\ ] \neq 0$

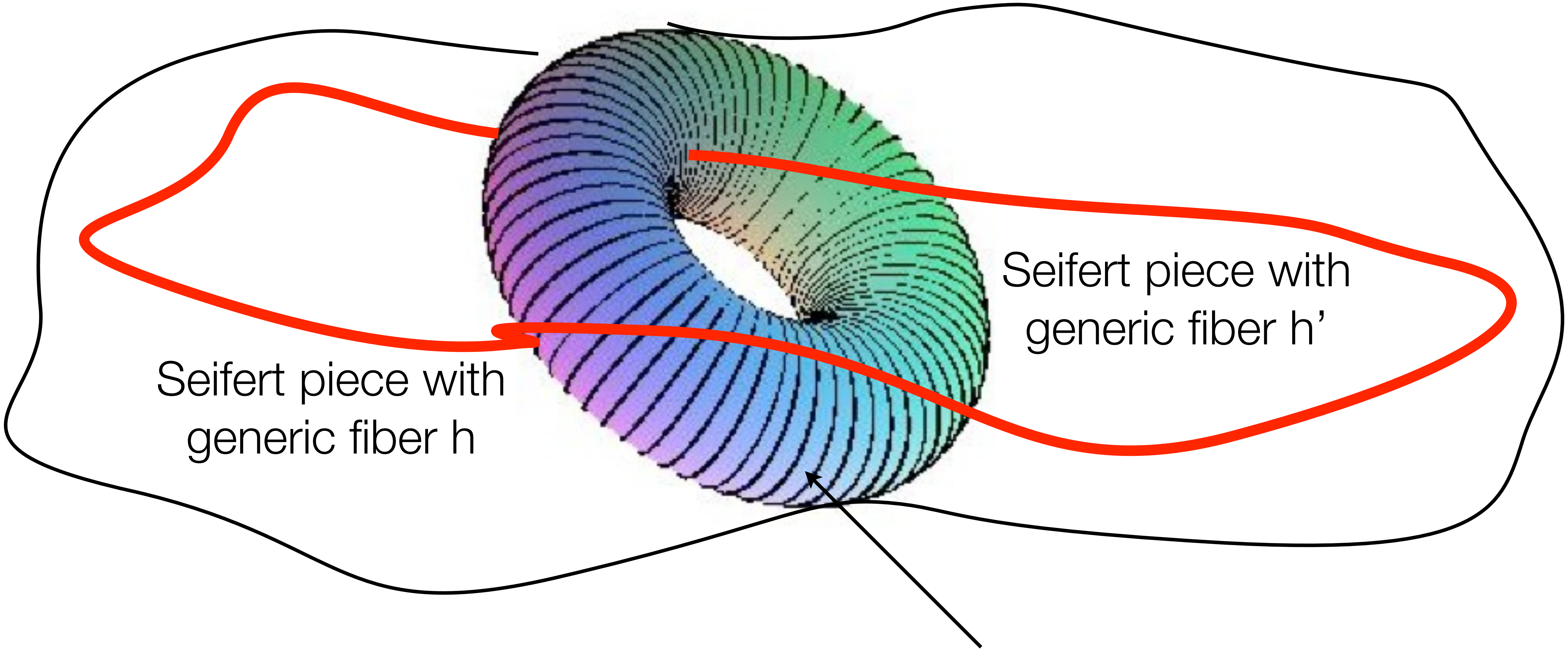
$M[\ T',\ 0^2\ ] \neq 0$

$M[T'',\ 0^2\ ] = 0$ for all other (classes of) peripheral tori



Step 3. Use $M\ [\ T, A^2] = 2\ i(T, A)$ to “reconstruct’ the graph and Seifert pieces genus and number of boundary components.

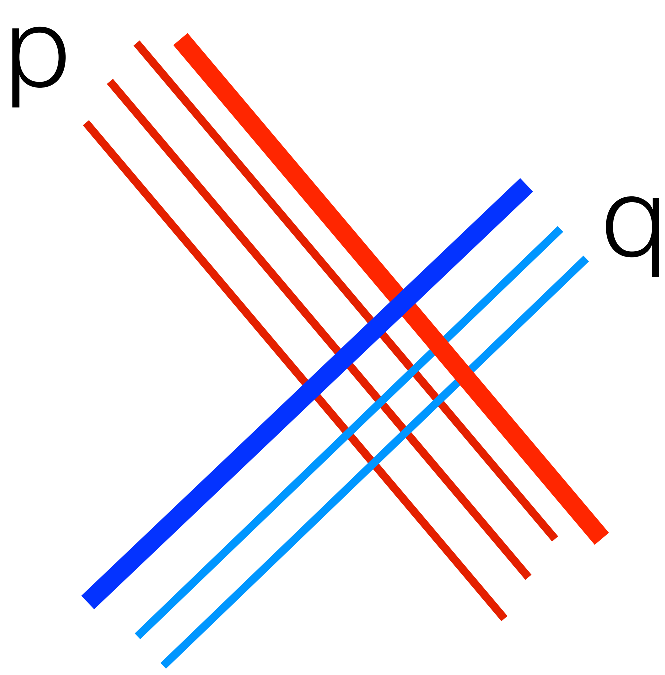
Why A^2 ?



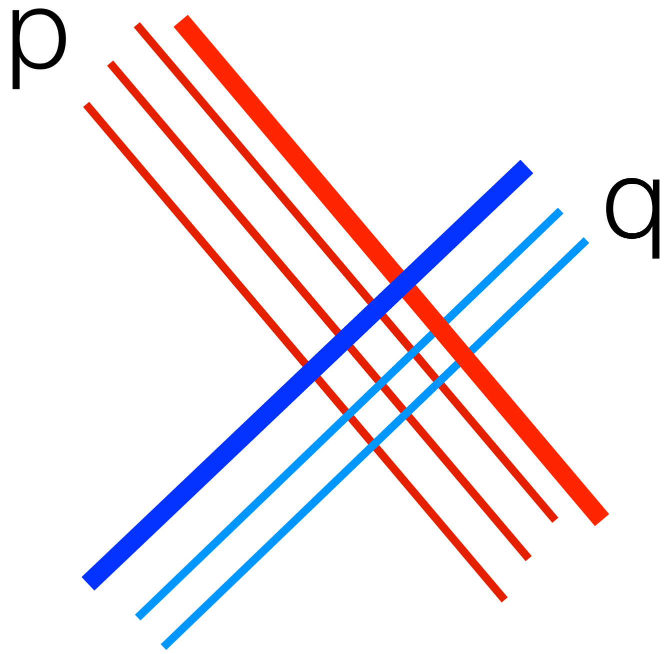
$$[w_1w_2, \langle T, h.h'\rangle] = w_1w_2\ h.h' - w_1 h.h' w_2 = 0$$

Detailed study of tori

<table>
<tr><th>T torus</th><th>peripheral</th><th>interior</th></tr>
<tr><td>generically fibered</td><td>vertex isolated</td><td>non-isolated</td></tr>
<tr><td rowspan="4">upright T(h,a)</td><td>p(a) simple and separates</td><td>p(a) simple and separates</td></tr>
<tr><td>vertex isolated
M always even
T(h,a) in C and there exists A in π_0 such that
• $M[\langle T,a\rangle,A^2]\neq 0$ for all $\langle T,a\rangle$ in C
• $M[\langle T,a\rangle,A^2] = 0$ for all $\langle T,a\rangle$ not in C</td><td>vertex isolated
M always even
M=0 Seifert clump
M even outside Seifert clump</td></tr>
<tr><td>p(a) simple non-separating</td><td>p(a) non-simple or non-separating</td></tr>
<tr><td>vertex isolated
M even and odd</td><td>non-isolated
M even and odd</td></tr>
</table>


p
q

Consider X and Y are free homotopy classes of closed curves, such



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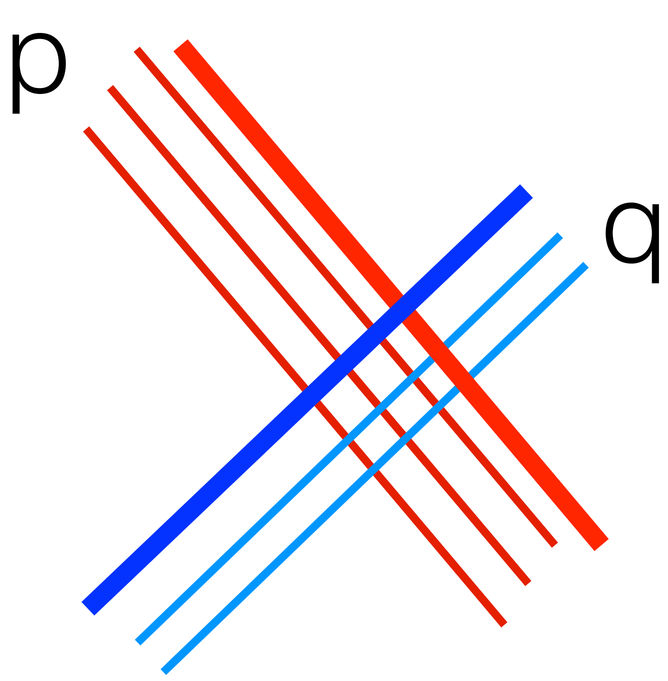
Counting Theorem (Gadgil,C.) If p and q are large enough,



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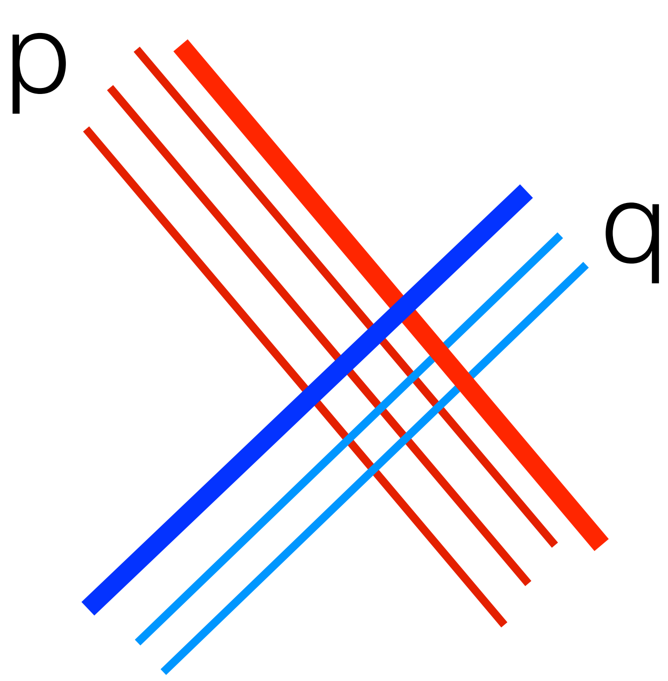

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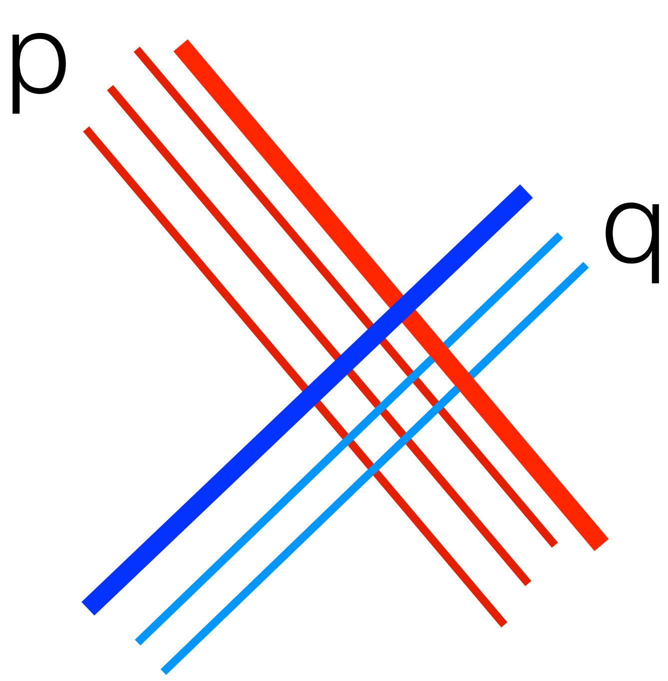

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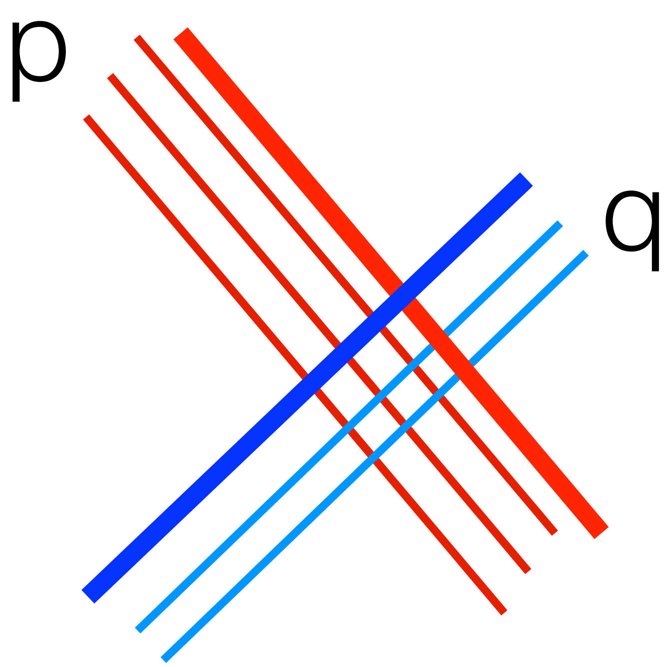

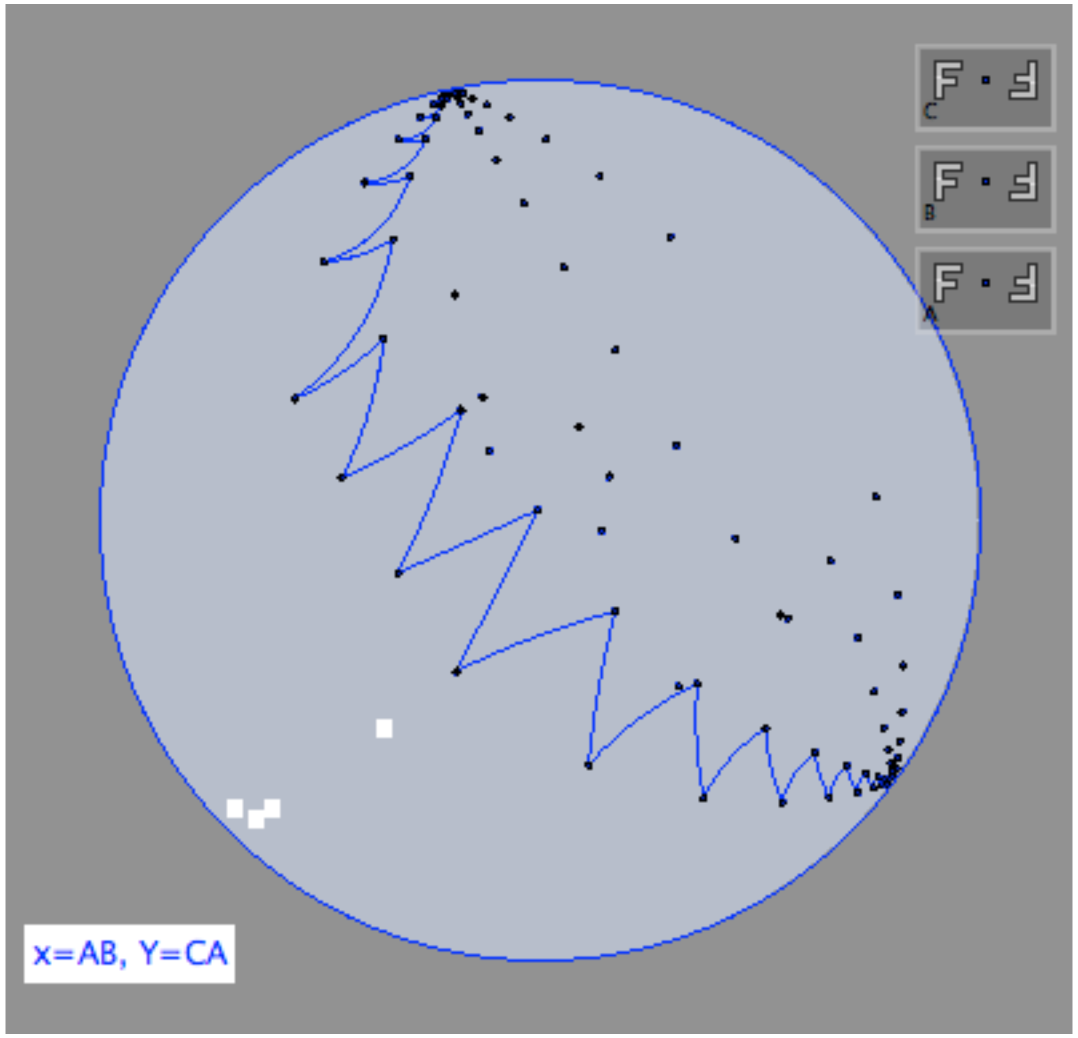

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