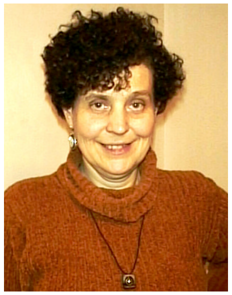
MASS, an immersive undergraduate program

Stony Brook University, February 2015

www.math.psu.edu/mass

History

The program was founded by George Andrews, Svetlana and Anatole Katok in 1996. I joined, as the Director, in 2000, after Penn State received NSF VIGRE grant.

Structure of the program

- Three core custom-made courses (Algebra, Analysis and Geometry, broadly understood), with a research project and an oral final exam, 4 credits each;

- MASS Seminar, 3 credits;

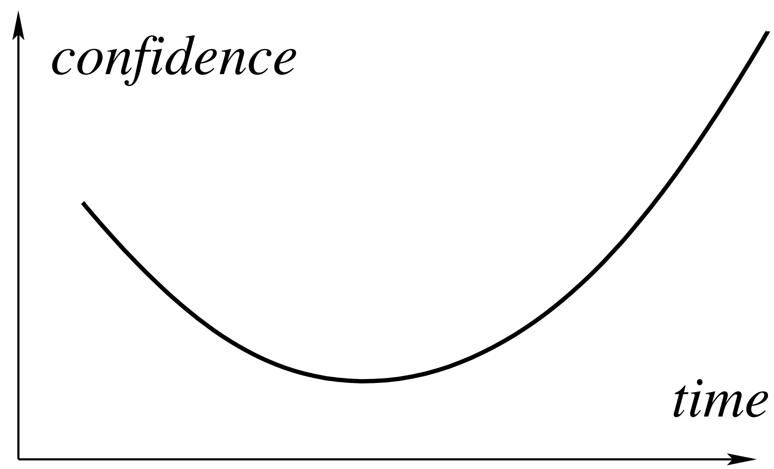
- MASS Colloquium, 1 credit;

- Summer REU, loosely integrated with MASS.

All count as honors courses, totaling 16 credits, transferrable to home institutions.

Core courses: 3 lectures + 1 recitation a week + a written midterm exam.

Each core course has a dedicated TA (sometimes, a MASS graduate).




MASS
2011

Examples of courses taught

- *Arithmetic and geometry of the unimodular group* by S. Katok;

- *Number theory: from Fermats Little Theorem to his Last Theorem* by K. Ono;

- *The exponential universe* by J. Roe;

- *Aspects of symmetry: from representations to Quantum Field Theory* by A. Ocneanu;

- *Mathematical analysis of fluid flow* by A. Belmonte;

- *Geometry and relativity: an introduction* by N. Higson;

- *Topological dynamics* by B. Kra;

- *Integer partitions* by G. Andrews;

- *Mathematical theory of waves* by A. Bressan;

- *Computability, unsolvability and randomness* by S. Simpson;

- *Elliptic curves and applications to cryptography* by K. Eisen-traeger;

- *Explorations in convexity* by S. Tabachnikov;

- *Differential equations from an algebraic perspective* by N. Higson;

- *Introduction to Ramsey Theory* by J. Reimann;

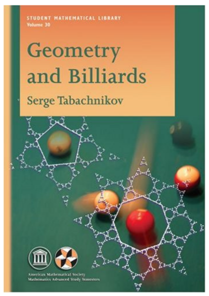
- *From Euclid to Alexandrov: a guided tour* by A. Petrunin;

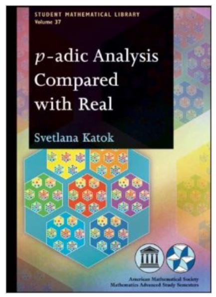
- *Random walk and Brownian motion* by A. Novikov;

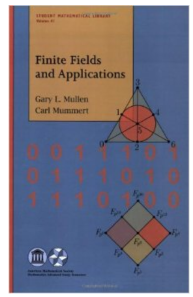
- *Number theory in the spirit of Ramanujan* by G. Andrews.

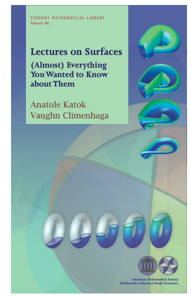
Books based on MASS courses

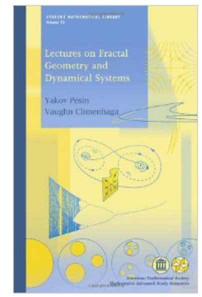
(sometimes co-authored with the TAs)

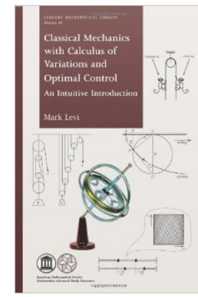












Several more are in the pipeline.

MASS Colloquium (samples)

- *From flapping birds to space telescopes: the mathematics of origami* by R. Lang;
- *About the numbers 12 and 24* by R. Howe;
- *We vote, but do we get what we want?* by D. Saari;
- *Mathematical puzzles that s-t-r-e-t-c-h your intuition* by P. Winkler;
- *Seeing invisible: mathematics of medical imaging* by P. Kuchment;

- *Geometry and analysis on fractals* by A. Kirillov;
- *Tilings with rational polygons* by R. Kenyon;
- *Behind the Hofstadter's butterfly: the competition between order and chaos* by S. Jitomirskaya;
- *Tropical geometry* by I. Itenberg;
- *Topological network topology* by R. Ghrist;
- *Topology and social choice* by S. Weinberger;

- *DNA topology* by De Witt Sumners;

- *Cantor and Sierpinski, Julia and Fatou: crazy topology in complex dynamics* by R. Devane;

- *Inverse problems in arithmetic combinatorics* by A. Razborov;

- *Packing discs in the plane* by R. Graham;

- *How old was Diophantus's son?* by D. Zagier;

- *New solutions to the N-body problem* by R. Montgomery;

- *What does the limit shape mean in geometry and combinatorics?* by A. Vershik;

- *Experiments with triangular billiards* by R. Schwartz;

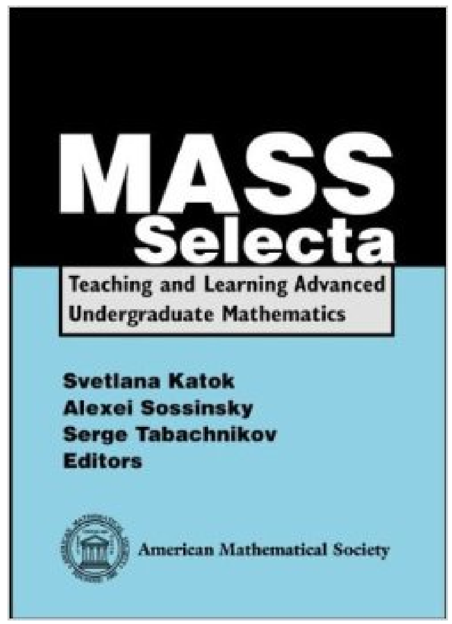
- *Pascal triangles in modular arithmetic, entropy and algebraic functions* by H. Furstenberg;

- *Laurent phenomenon* by A. Zelevinsky;

- *Impossible Crystals* by R. Penrose;

- *Integral lexicographic codes* by J. Conway.

All talks are written down by a dedicated group of students, and some are published:



Final oral exams (much feared):

- Whole week, with 1-2 day breaks;
- One hour preparation (proctored), random ‘ticket’;
- Committee of three: instructor, TA, and a guest;
- Ticket questions, open-ended discussion, presentation of the research project.

Student research (samples)

- R. Vaughan, K. Weis. *On sigma-phi numbers*. Mathematika 48 (2001), 169–189. MASS 1999, supervised by R. Vaughan;

- A. Bressan, M. Burago, A. Friend, J. Jou. *Blocking strategies for a fire control problem*. Anal. Appl. 6 (2008), 229–246. MASS 2006, supervised by A. Bressan;

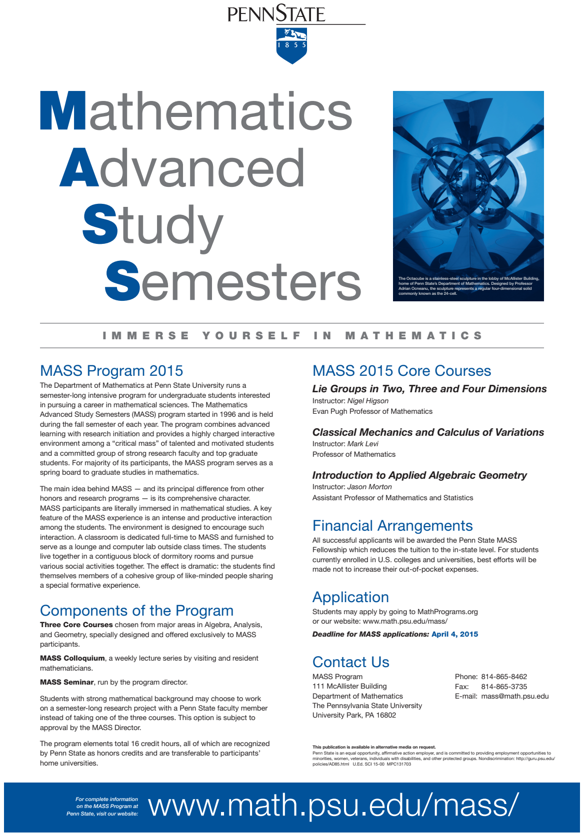
- G. Mullen, B. Vioreanu. *Explicit formulas for permutation polynomials over finite fields*. Bull. Inst. Combin. Appl. 57 (2009), 99–106. MASS 2006, supervised by G. Mullen;

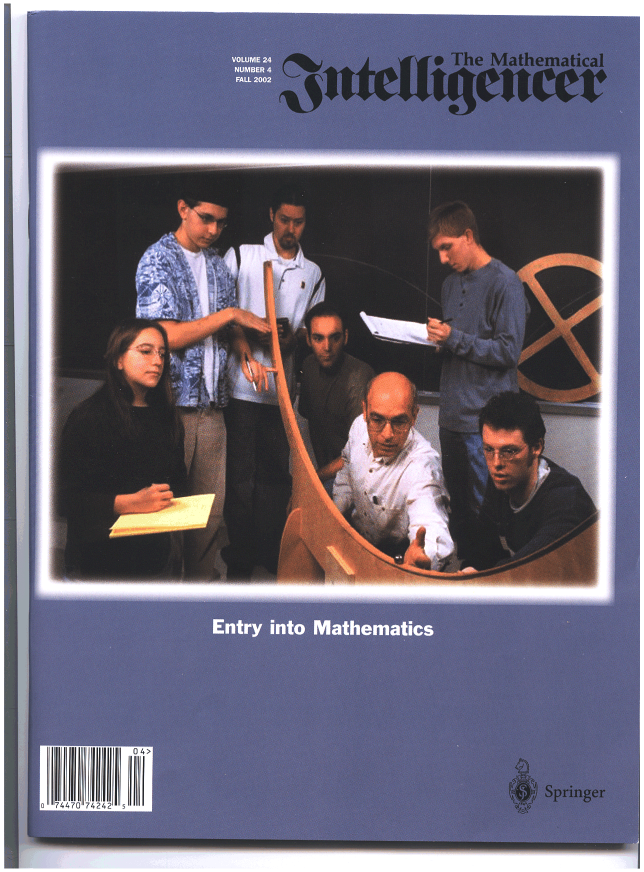
- S. Howe, M. Pancia, V. Zakharevich. *Isoperimetric inequalities for wave fronts and a generalization of Menzins conjecture for bicycle monodromy on surfaces of constant curvature*. Adv. Geom. 11 (2011), 273–292. REU and MASS 2008, supervised by S. Tabachnikov;

- D. Rudenko. *On equidissection of balanced polygons*. J. Math. Sci. 190 (2013), 486–495. MASS 2011, supervised by S. Tabachnikov;

- M. Chao, D. Levenstein, V. Nitica, R. Sharp. *A coloring invariant for ribbon L-tetrominos*. Discrete Math. 313 (2013), 611–621. REU and MASS 2012, supervised by V. Nitica.

Advertising





Bringing Eastern European Mathematical Traditions to North American Students

Paul D. Humke, Yulij Ilyashenko, and Serge Tabachnikov

Three Programs for Undergraduate Mathematics Majors Focus on Discovery and Exploration

This article presents descriptions of three noteworthy programs designed to expose undergraduate mathematics majors to a wider variety of mathematical experiences than is typically found in college courses. The programs share a common philosophical core, in that they all draw on the mathematical and intellectual traditions of Eastern Europe.

The courses offered in the programs are quite different from the usual undergraduate fare, as the emphasis is on discovery and depth of understanding rather than on "covering the material" and preparing for examinations. These are not summer programs; rather, they offer semester-long courses that run during the regular academic year, and credits can be transferred to the students' home institutions. By encouraging independent thinking and exploration, all three programs give students a taste of what it is like to do research in mathematics.

Further information about the programs is available on their respective websites. A list of such programs, together with other information of interest to undergraduate mathematics majors, may be found on the AMS website at `http://www.ams.org/employment/undergrad.html`.

—Allyn Jackson

Budapest Semesters in Mathematics

`http://www.stolaf.edu/depts/math/budapest/`

Paul D. Humke

Initiated by Paul Erdős, László Lovász, and Vera T. Sós in 1984, the Budapest Semesters in Mathematics (BSM) program offers a unique opportunity to North American undergraduates for a semester or a year of study in one of the world's hubs of mathematical activity. A wide variety of courses in all areas of mathematics are offered under the tutelage of eminent Hungarian teacher-scholars, most of whom have had years of teaching experience in North America. Classes are small and taught in English, and credits are transferable to the student's home institution. In keeping with the Hungarian tradition, professors closely monitor each individual student's progress. Considerable time is devoted to *problem solving* and encouraging *student creativity*. Emphasis is on depth of understanding rather than on the quantity of material. The imprint of the Hungarian tradition is particularly prominent in the lively regular BSM colloquium talks and in some particular courses.

Paul D. Humke is the North American director for the Budapest Semesters in Mathematics program. He is also professor of mathematics at St. Olaf College and at Washington and Lee University. His email address is `humke@stolaf.edu`.

Administration and finances

Administration: MASS Advisory Board, Director, Administrative assistant.

Financial arrangements: tuition reduced to the in-state level; R&B and travel; MASS stipend.

Size of the program: 15–20 students, recruited from across the USA (some are from Penn State).

Finding: NSF (VIGRE, MCTP) and a small endowment.

Participants (case studies)

Justin Holmer, MASS 1996 (undergraduate at Stony Brook, graduate student at University of Chicago, postdoc at UC Berkeley, now: Associate Professor at Brown).

> The MASS program definitely helped me through a critical point in my education and career development. In addition to the actual material I leaned, it taught me about the learning process itself... I was forced to understand that the linear model of learning math from the elementary to the advanced was not accurate, but instead one acquires knowledge by seeing the same material on several separate passes from different perspectives, and on each pass more of material seeps into your brain.

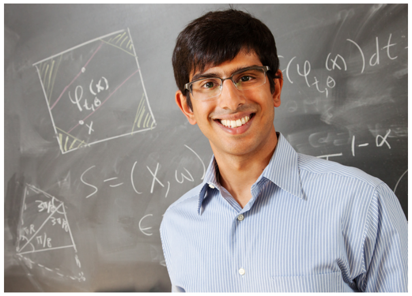
Jayadev Athreya, MASS 1999 (undergraduate at Iowa State, graduate student at the University of Chicago, postdoc at Yale and Princeton, now: Assistant Professor at UIUC).

MASS was a crucial period in my mathematical formative years. The intense nature of the coursework and the mathematical social interaction really helped me prepare for graduate school, and also to develop long-lasting friendships. The year I took MASS, the courses were in Dynamical Systems (taught by Mark Levi), Geometry (Anatole Katok), and Analytic Number Theory (Robert Vaughan). My current research interests now lie somewhere at the intersection of these three areas, so the influence of these wonderful teachers, and in particular the emphasis on solving difficult problems, cannot be overemphasized. I would not be what I am now without MASS.

Alice Medvedev, MASS 2000 (undergraduate at Caltech, graduate student at UC Berkeley, postdoc at UIC and UC Berkeley, now: Assistant Professor at CUNY).

> In MASS courses, I saw for the first time how different branches of mathematics are brought to bear on the same object, to great effect... Most importantly, it was the first time I interacted with mathematicians as a colleague rather than as a student. I learned to write mathematics by watching a grown-up mathematician trying to read my over-concise first drafts. I learned to pull ideas from different branches of mathematics together to solve problems. I began to think of myself as a mathematician.

Aaron Naber, MASS 2004 (undergraduate at Penn State, graduate student at Princeton, postdoc at MIT, now: Associate Professor at Northwestern), gave an invited sectional talk at ICM 2014 in Seoul: *The structure and meaning of Ricci curvature*.

Some statistics

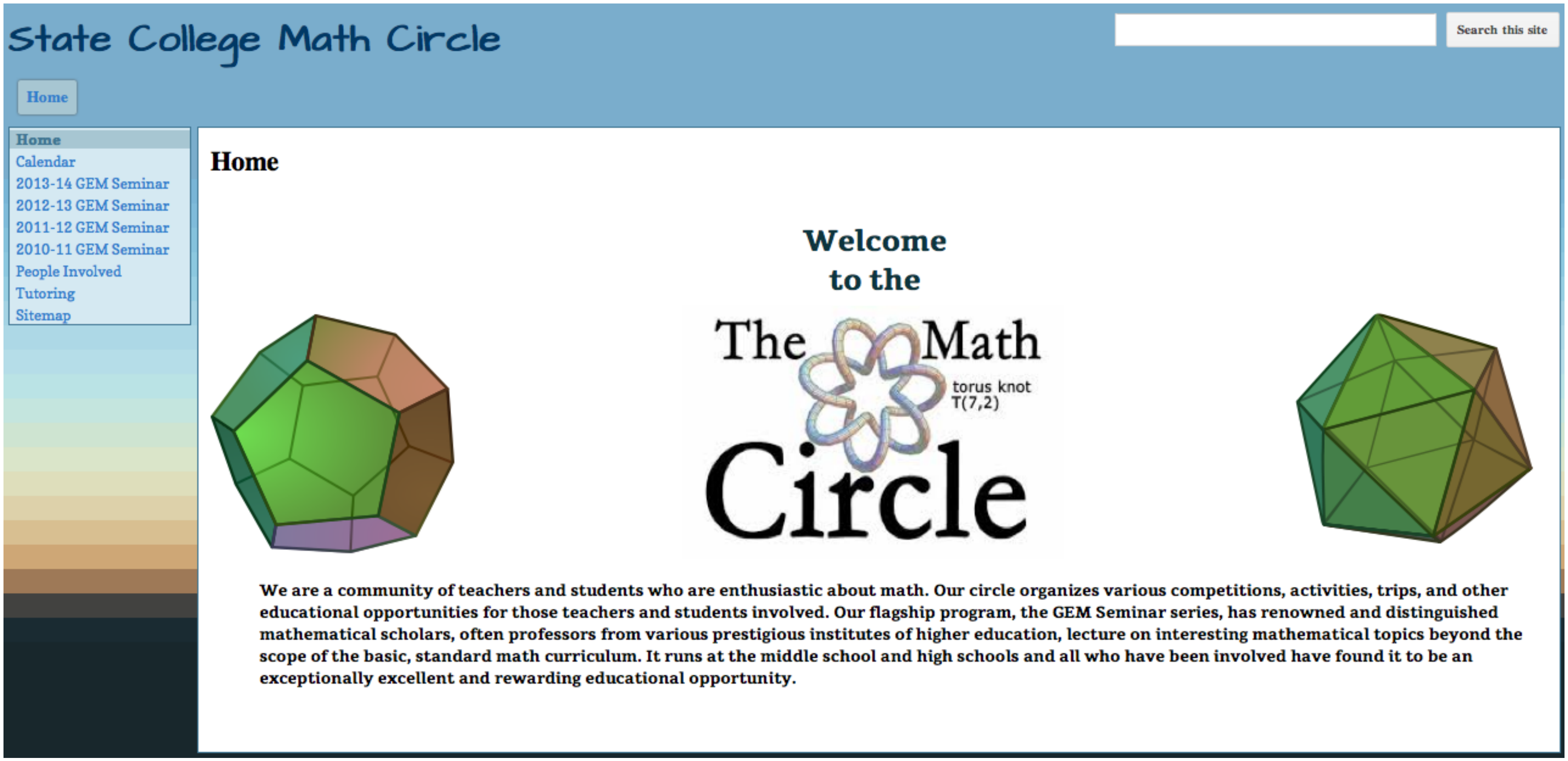
Of those whom we have been able to track down and who have entered workforce, the career choices are, approximately, as follows:

- Tenure-line faculty at universities: 25%;
- Middle and high school teachers: 11%;
- Research staff at national labs, institutes and universities: 10%;
- Industry (mostly, mathematics-intense): 40%;
- Postdocs: 14%.

Auxiliary activity: Math Circles at State College

State College Area School District: nine elementary, two middle, and one high school; altogether, about 7,000 students.

Working with a number of teachers, coordinated by Greg Somers (Presidential Scholar Teacher of 2006).



State College Math Circle

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Home

We are a community of teachers and students who are enthusiastic about math. Our circle organizes various competitions, activities, trips, and other educational opportunities for those teachers and students involved. Our flagship program, the GEM Seminar series, has renowned and distinguished mathematical scholars, often professors from various prestigious institutes of higher education, lecture on interesting mathematical topics beyond the scope of the basic, standard math curriculum. It runs at the middle school and high schools and all who have been involved have found it to be an exceptionally excellent and rewarding educational opportunity.

A typical schedule:

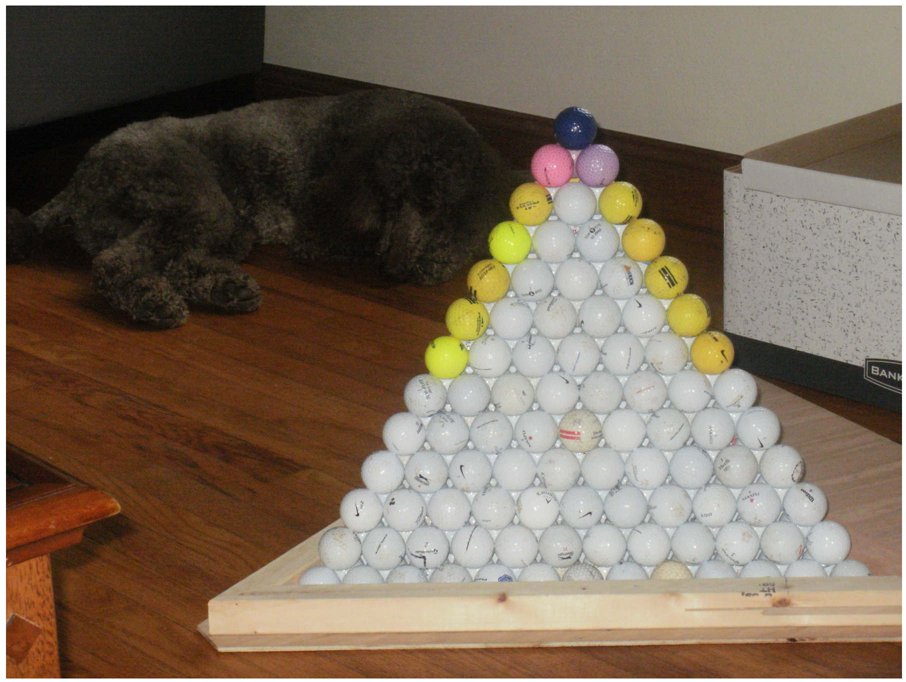
2011-12 GEM Seminar

State College Area High School

Date	Topic	Speaker
Friday, October 14	Organizational Meeting	
Tuesday, October 25	Cross-sections / 4D Objects	Dr. Levi / Dr. Tabachnikov, PSU
Friday, November 18	ARML Power Contest	5th place nationwide :)
Tuesday, November 29	Fibonacci Numbers and More	Dr. James Sellers, PSU
Friday, December 9	Chebyshev Polynomials	Dr. Sergei Tabachnikov, PSU
Tuesday, December 20	Polynomials and Symmetry	Dr. Nigel Higson, PSU
Friday, January 6	---	cancelled
Tuesday, January 17	Curvature of Space (Einsteinian Gravity)	Dr. John Roe, PSU
Friday, February 3	---	cancelled
Tuesday, February 21	The Mathematical Mechanic	Dr. Mark Levi, PSU
Friday, March 9	Linear Equations and "Lights Out"	Dr. Misha Guysinski, PSU
Tuesday, March 20	From Complex Numbers to Quaternions, and Beyond	Dr. Sergei Tabachnikov, PSU
Tuesday, April 17	The Mathematics of Rock Climbing	Dr. John Roe, PSU
Monday, April 23	Discontinuous calculus, indiscrete calculus	Dr. Tadashi Tokieda, University of Cambridge
Tuesday, May 15	Counting infinite sets	Dr. Misha Guysinski, PSU
Thursday, May 24	Picnic Party	

My end of the 2011 season GEM talk:

Polynomials and Pyramids.



How many golf balls are there? (I brought a box of about 500).

Answer: we proved that the 'pyramidal numbers' are

$$\binom{n+2}{3} = \frac{n(n+1)(n+2)}{6},$$

the closest to 500 being 455, corresponding to $n = 13$.

As to polynomials, we proved that the polynomials, having integer values for all integer arguments, are

$$f(x) = \sum_{k=0}^{n} c_k \binom{x}{k},$$

where $c_k \in \mathbf{Z}$.

Math Circle at Radio Park Elementary School

Some favorite problems:

- Cut a hole in a sheet of paper, large enough for you to pass through.

- How many (topologically) different 7-pronged stars are there? 9-pronged ones? 10-pronged ones? 12-pronged ones?

- There are 4 Mondays and 4 Fridays in January. Which day is January 1-st?

- Solve: $ABC + ACB = BCA$.

- Make the earth equator 1 yard longer. Will a dog be able to crawl underneath it?

- Two people run down a moving escalator. Who will count more steps: the one who runs faster or slower?

- The population of the United States is about 300 million. Explain why there is not enough room for 300 people on the $1 : 1,000,000$ scale map of the USA.

- Throw a tennis ball vertically up. Does it take more time going up or down?

Certificate of Merit

Is awarded to

Anna Graef

For the successful completion of

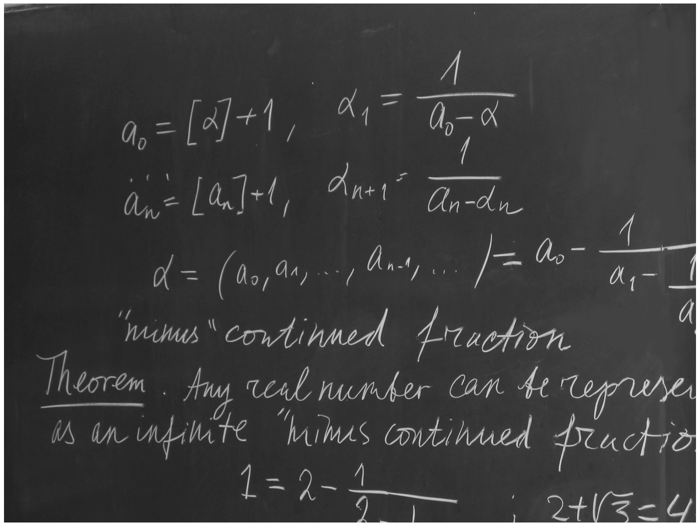
Math Club Program

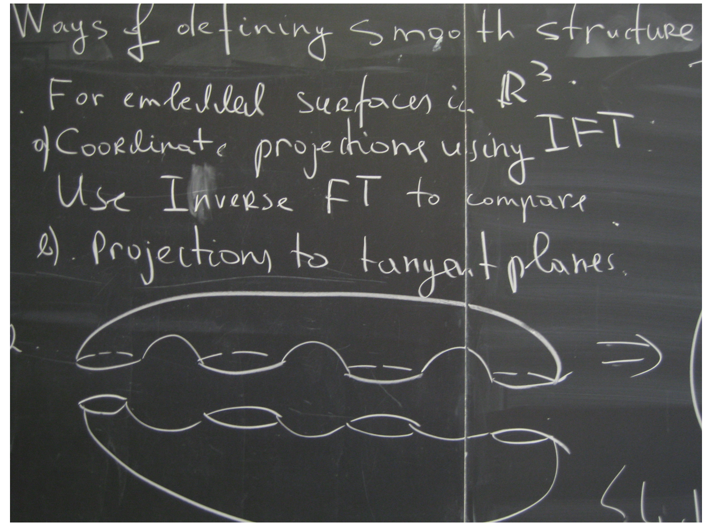
During the 2004-2005 academic year
Radio Park Elementary School
Third grade, class of Mrs. J. Jobe

Math Club Advisors

Sergei Tabachnikov, "Dr. T."
Professor of Mathematics, PSU

Howard Weiss
Professor of Mathematics, PSU


$a_0 = [\alpha]+1, \quad \alpha_1 = \frac{1}{a_0-\alpha}$
$a_n = [\alpha_n]+1, \quad \alpha_{n+1} = \frac{1}{a_n-\alpha_n}$
"minus" continued fraction
Theorem. Any real number can be
as an infinite "minus continued


Ways of defining smooth structure
For embedded surfaces in $\mathbb{R}^3$.
a) Coordinate projections using IFT.
Use Inverse FT to compare
b). Projections to tangent planes.

Thank you!