

Urheberrechtlich geschütztes Material

Science Networks

Historical Studies

45

Arie Hinkis

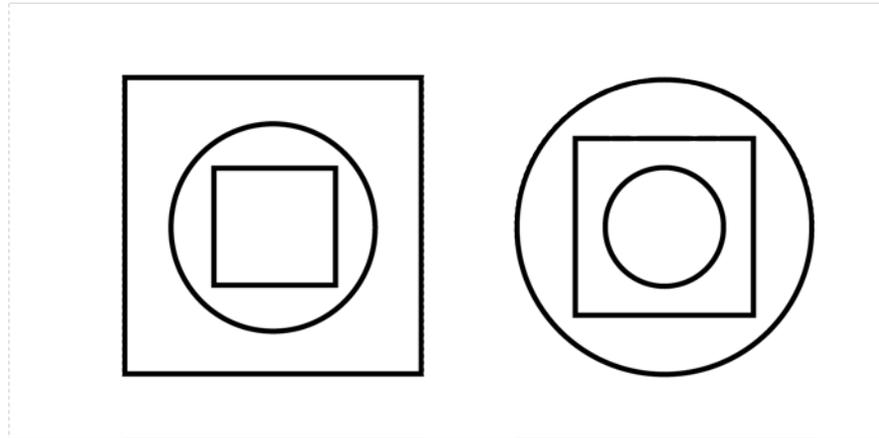
Proofs of the Cantor-Bernstein Theorem

A Mathematical Excursion

 Birkhäuser

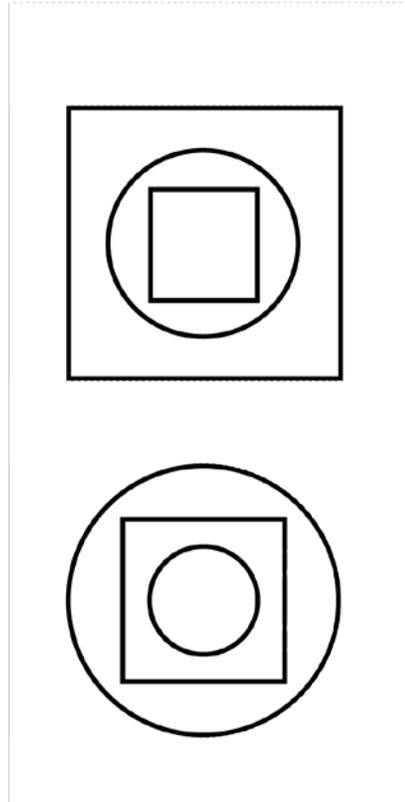
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The Cantor Bernstein Theorem

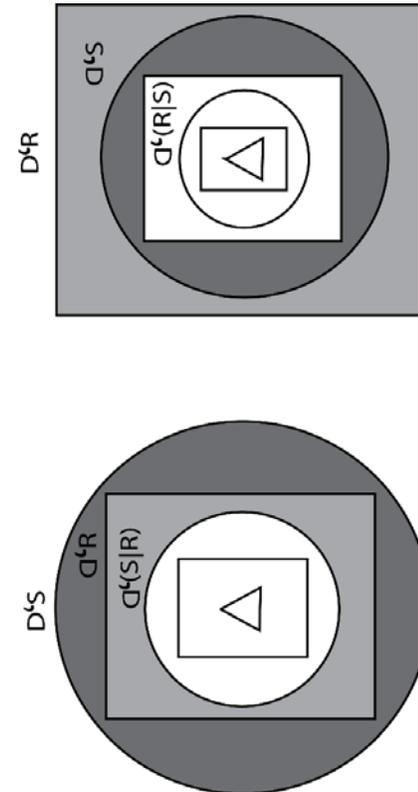


Proof of CBT

- The **GESTALT** of nesting squares and circles



- The **GESTALT SWITCH** to the frames



Metaphors for the Proof

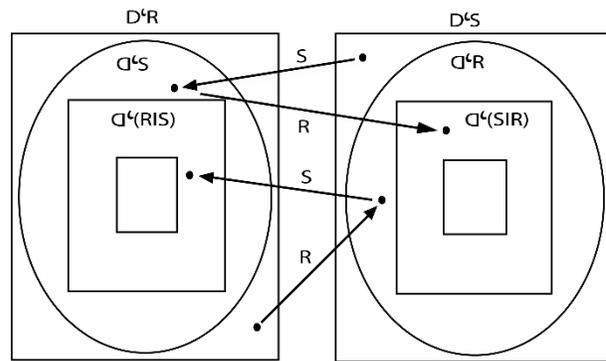
Nesting alternating sets

Similar frames

shoe lacing



The proof of J. König



The theory of gestalt and metaphor

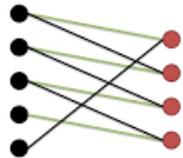
- **Every mathematical proof can be presented as a series of gestalt switches**
- **Each gestalt can be attributed with a metaphor**
- **Proof comparison can be effected through these two dimensions**



D. König theorem and infinity lemma

König's theorem

- Let G be a bipartite graph. Then $\nu(G) = \tau(G)$, where $\nu(G)$ is the maximum cardinality of a matching of G and $\tau(G)$ is the minimum cardinality of a vertex cover of G .

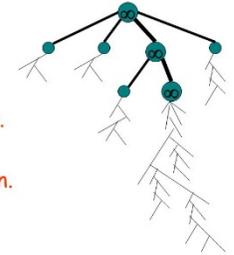


König's Infinity Lemma

Proof Idea:

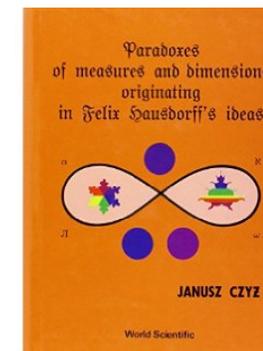
Find an infinite branch of the tree.

Go in that direction.



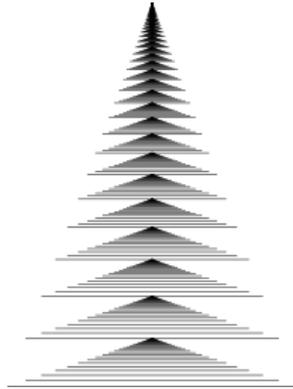
Hausdorff's paradox and BDT

BDT: $n \cdot a = n \cdot b$  $a = b$
(without AC)



The centrality of CBT

- Cantor

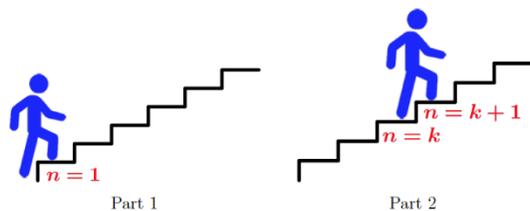


- Russell



*5443. $\vdash: a, \beta \in 1. \supset: a \cap \beta = \Lambda. \equiv. a \cup \beta \in 2$
Dem.
 $\vdash. *5426. \supset \vdash: a = t'x, \beta = t'y. \supset: a \cup \beta \in 2. \equiv. x \neq y.$
 $[*51231] \quad \equiv. t'x \cap t'y = \Lambda.$
 $[*1312] \quad \equiv. a \cap \beta = \Lambda \quad (1)$
 $\vdash. (1). *111185. \supset$
 $\vdash: (x, y). a = t'x, \beta = t'y. \supset: a \cup \beta \in 2. \equiv. a \cap \beta = \Lambda \quad (2)$
 $\vdash. (2). *1154. *521. \supset \vdash. Prop$
 From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

- Poincaré



- Brouwer



The FLAW in...
 Reductio Ad
 Absurdum

Historical surprises

- **Sets and classes**
- **Axiom system for set theory**
- **Cantor had a proof to CBT by transfinite induction**

The names of CBT

- **The Equivalence Theorem**
- **The Cantor-Schroeder-Bernstein Theorem**
- **The Cantor-Bernstein Theorem**
- **The Cantor-Dedekind Theorem**

Directions for further research

- More studies in proof comparison and the inner history of mathematics.
- Is the experience of gestalt switch the same as “understanding”? Is it the locus of the pleasure mathematicians get from their occupation?
- How can the theory of gestalt and metaphor improve the teaching and learning of mathematics?