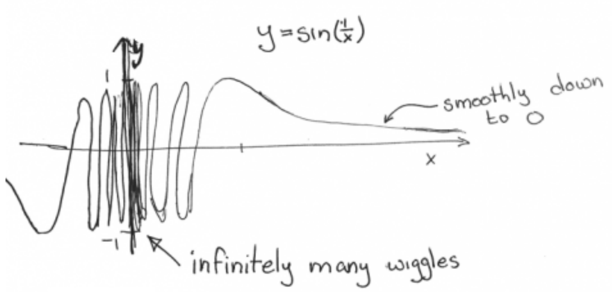
Lecture #26
MAT 123

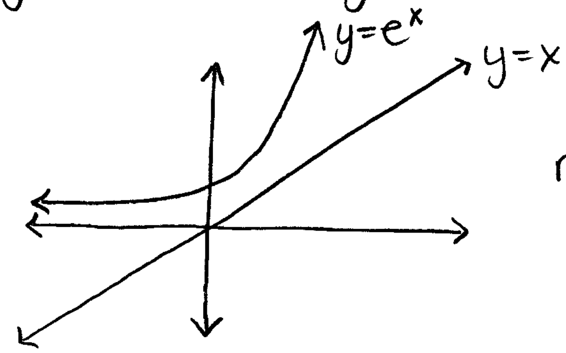
Graphs & Solving Equations

Clarification from last lecture



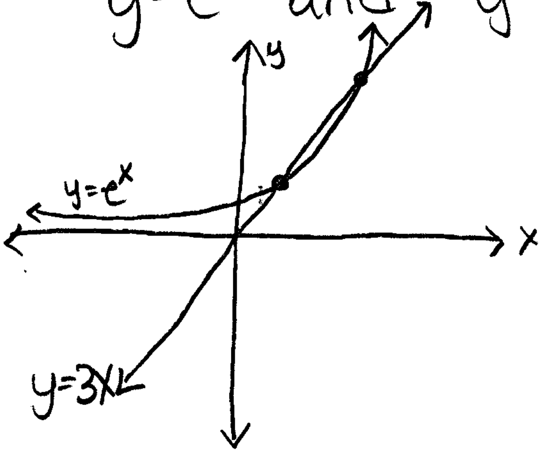
Graphs + Solving Equations

$y=e^x$ and $y=x$



no intersection point(s) $\Longrightarrow$ no solution to the set
$y=e^x$
$y=x$

$y=e^x$ and $y=3x$



2 points of intersection $\Longrightarrow$ 2 solutions to the set: $y=e^x$
$y=3x$

we can use graphs to give us insight into how many solutions an equation or systems of equations may have.

Solving Trig Equations: $\boxed{2\pi < x \leq 0}$ for all the following problems

Solve: $\sin 2x - \sin x = 0$

$2\sin x \cos x - \sin x = 0$ ← double angle formula for $\sin 2x$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$	$2\cos x - \cancel{1} = 0$
$x = 0$	$+1 \quad +1$
$x = \pi$	$\frac{\cancel{2}\cos x}{\cancel{2}} = \frac{1}{2}$
	$\cos x = 1/2$
	$x = \pi/3,\ 5\pi/3$

Solutions: $x = 0, \pi, \pi/3, 5\pi/3$

Solve: $2\sin^2 x - 5\sin x + 3 = 0$

let $\sin x = b$

$2b^2 - 5b + 3 = 0$ ← Quadratic!

$(2b + 1)(b - 3) = 0$

$2b + 1 = 0$	$b - 3 = 0$
$b = -1/2$	$b = 3$

Substitute back!

$\sin x = -1/2$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin x = 3$

No solutions

$\sin x$ is between $[-1, 1]$

Solve: $\cos^2 x - 3\cos x + 2 = 0$

let $u = \cos x$

$u^2 - 3u + 2 = 0$ → Quadratic!

$(u - 2)(u - 1) = 0$

$u - 2 = 0$	$u - 1 = 0$
$u = 2$	$u = 1$

Substitute back:

$\cos x = 2$	$\cos x = 1$
No solutions	$x = 0$

Solve: $3(1-\cos\theta) = \sin^2\theta \qquad \theta = [0, 2\pi)$

$3(1-\cos\theta) = 1-\cos^2\theta \rightarrow$ trig identity: $\sin^2\theta + \cos^2\theta = 1$

$3 - 3\cos\theta = 1 - \cancel{\cos^2\theta}$

$+1+\cos^2\theta \qquad -1 \cancel{+\cos^2\theta}$

$\cos^2\theta - 3\cos\theta + 2 = 0$

let $\cos\theta = y$

$y^2 - 3y + 2 = 0$

$(y-2)(y-1) = 0$

$y-2=0 \quad | \quad y-1=0$

$y=2 \quad | \quad y=1$

substitute back

$\cos x = 2 \quad | \quad \cos x = 1$

No solutions $\quad | \quad x = 0$

Solve: $1+\sin x = 2\cos^2 x \qquad x = [0, 2\pi)$

$1+\sin x = 2(1-\sin^2 x)$

$1+\sin x = 2 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

let $w = \sin x$

$2w^2 + w - 1 = 0$

$(2w-1)(w+1) = 0$

$2w-1=0 \quad | \quad w+1=0$

$w = 1/2 \quad | \quad w = -1$

substitute back:

$\sin x = 1/2 \quad | \quad \sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad | \quad x = \frac{3\pi}{2}$

solutions: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Solving other types of Equations:

① $2^{2x}+2^x-12=0$

let $u=2^x$

$u^2+u-12=0$

$(u+4)(u-3)=0$

$u+4=0$	$u-3=0$
$u=-4$	$u=3$

substitute back

$2^x=-4$	$2^x=3$
No solution 2^x cannot be (-)	$x=\log_2 3$

② $3\cdot 4^x+4\cdot 2^x+8=0$

$3\cdot(2^2)^x+4\cdot 2^x+8=0$

$3\cdot 2^{2x}+4\cdot 2^x+8=0$

let $w=2^x$

$3x^2+4x+8=0$

cannot factor $\Rightarrow$ No real solutions!

③ $e^{2x}-6e^x+5=0$

let $y=e^x$

$y^2-6y+5=0$

$(y-1)(y-5)=0$

$y-1=0$	$y-5=0$
$y=1$	$y=5$

substitute back:

$e^x=1$	$e^x=5$
$x=0$	$x=\ln 5$

④ $\left(e^x+\frac{3}{e^x}=4\right)\cdot e^x$

$e^{2x}+3=4e^x$

$e^{2x}-4e^x+3=0$

let $z=e^x$

$z^2-4z+3=0$

$(z-3)(z-1)=0$

$z-3=0$	$z-1=0$
$z=3$	$z=1$

substitute back:

$e^x=3$	$e^x=1$
$x=\ln 3$	$x=0$

⑤ $16^x+4^{x+1}-12=0$

$(4^2)^x+4^{x+1}-12=0$

$4^{2x}+4^{x+1}-12=0$

$4^{2x}+4\cdot 4^x-12=0$

let $b=4^x$

$b^2+4b-12=0$

$(b+6)(b-2)=0$

$b+6=0$	$b-2=0$
$b=-6$	$b=2$

substitute back:

$4^x=-6$	$4^x=2$
No solution	$x=1/2$