Lecture #24
MAT 123

Inverse Trig / Trig / Graphs

Review:

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(90°) = \sin\left(\frac{\pi}{2}\right) = 1$

ex: $\sin(90°) = \sin(60° + 30°) = \sin 60° \cos 30° + \cos 60° \sin 30°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$\sin 90° = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$ ✓

ex: $\sin(15°) = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$

(difference of two angles)

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\sin(15°) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$

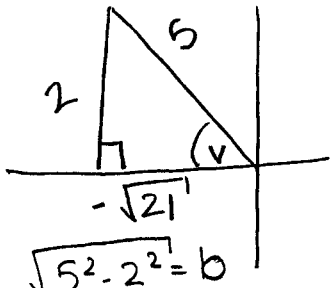
ex: $\sin\left(\frac{30}{2}\right) = \pm\sqrt{\frac{1 - \cos(30)}{2}}$ (half angle)

$= \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{2 - \sqrt{3}}{4}}$

* $\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos A}{2}}$

ex: $\sin V = \frac{2}{5}$ (opp / hyp) $\quad \frac{\pi}{2} \le V \le \pi$ (V is in Quad II). Find ① $\sin 2V$, ② $\cos 2V$, ③ $\tan 2V$

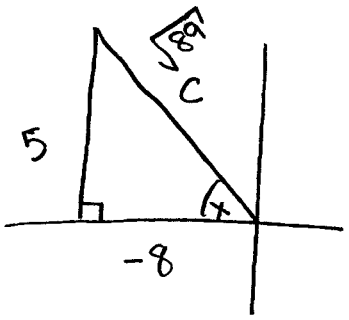


① $\sin(2V) = 2\sin V \cos V = 2\left(\frac{2}{5}\right)\left(-\frac{\sqrt{21}}{5}\right) = \frac{-4\sqrt{21}}{25}$

② $\cos(2V) = \left(-\frac{\sqrt{21}}{5}\right)^2 - \left(\frac{2}{5}\right)^2 = \frac{21}{25} - \frac{4}{25} = \frac{17}{25}$

③ $\tan(2V) = \frac{\sin(2V)}{\cos(2V)} = \frac{-4\sqrt{21}}{25} \cdot \frac{25}{17} = \frac{-4\sqrt{21}}{17}$

ex: If $\tan x = \frac{-5}{8}$, $\frac{\pi}{2} \le x \le \pi$ Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$



$c^2 = 5^2 + (-8)^2$

$c = \sqrt{89}$

$$\sin\left(\frac{x}{2}\right) = +\sqrt{\frac{1-\cos x}{2}} = +\sqrt{\frac{1-\left(\frac{-8}{\sqrt{89}}\right)}{2}}$$

$$\sin\frac{x}{2} = \sqrt{\frac{1+\frac{8}{\sqrt{89}}}{2}}$$

$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+\left(\frac{-8}{\sqrt{89}}\right)}{2}}$$

$$\cos\frac{x}{2} = \sqrt{\frac{1-\frac{8}{\sqrt{89}}}{2}}$$

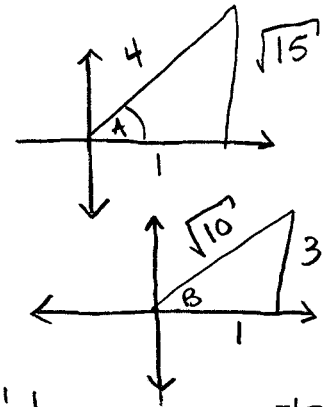
ex: $\cos(\cos^{-1}\frac{1}{4} + \tan^{-1}3) = ?$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$A = \cos^{-1}\frac{1}{4}$ $B = \tan^{-1}3$

$$\cos(\cos^{-1}\tfrac{1}{4} + \tan^{-1}3) = \left(\tfrac{1}{4}\right)\left(\tfrac{1}{\sqrt{10}}\right) - \left(\tfrac{\sqrt{15}}{4}\right)\left(\tfrac{3}{\sqrt{10}}\right)$$

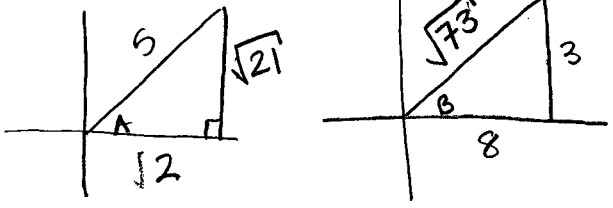
$$= \boxed{\frac{1-3\sqrt{15}}{4\sqrt{10}}}$$



* $\cos^{-1}\frac{1}{4}$ and $\tan^{-1}3$ let us draw triangles to use!

ex: $\sin(\cos^{-1}(\frac{2}{5}) + \tan^{-1}(\frac{3}{8})) = ?$

(A) (B)

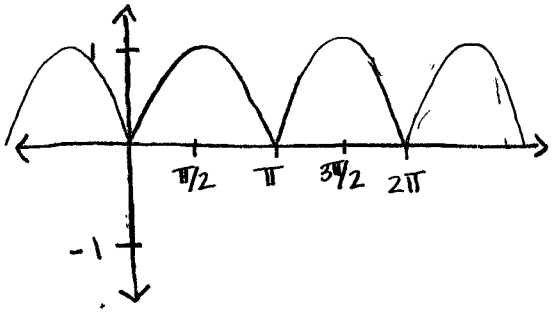


$= \sin A\cos B + \cos A\sin B$

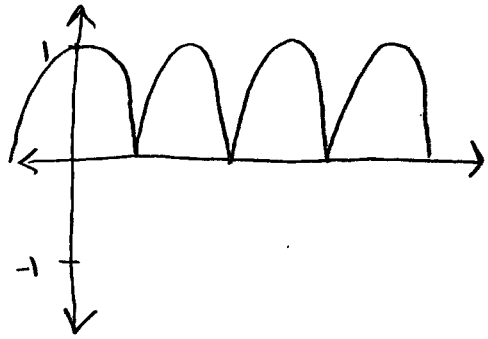
$$= \left(\frac{\sqrt{21}}{5}\right)\left(\frac{8}{\sqrt{73}}\right) + \left(\frac{2}{5}\right)\left(\frac{3}{\sqrt{73}}\right)$$

$$= \frac{8\sqrt{21}+6}{5\sqrt{73}}$$

$y = |\sin x|$



$y = |\cos x|$



the absolute value takes any negative values and reflects them over the x-axis

No negative values!

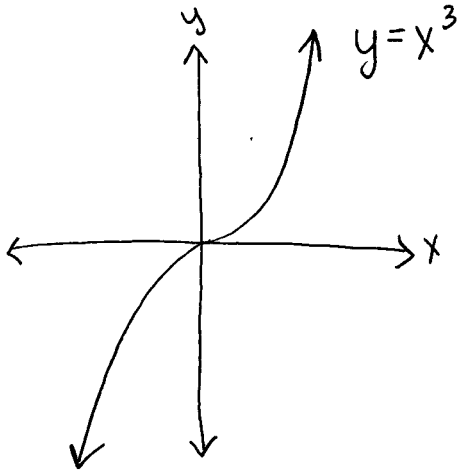
helpful graphs to know:

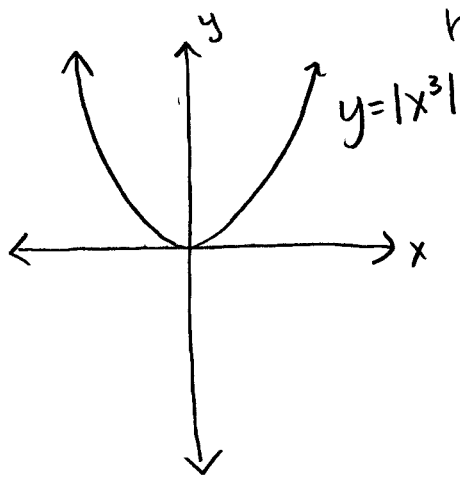
$f(x) = x^2$ $f(x) = \sin x$

$f(x) = x^3$ $f(x) = \cos x$

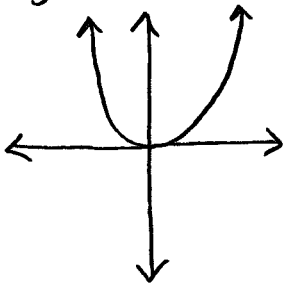
$f(x) = e^x$ $f(x) = 1/x$

$f(x) = \ln x$ $f(x) = x$

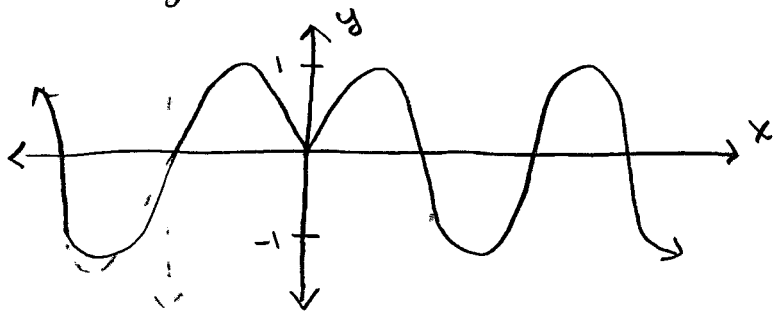




$y = x^2 = |x^2|$

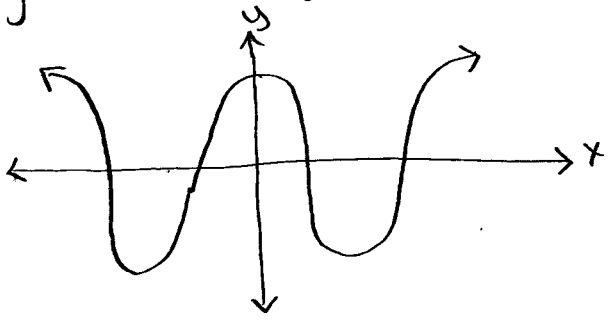


$y = \sin|x|$

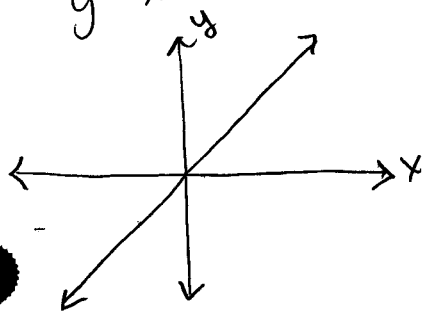


* when you take the absolute value of the input you reflect the right side of the x-axis over the y-axis (mirror).

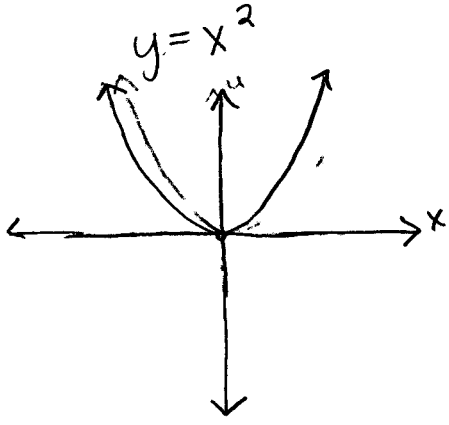
$y = \cos|x| = y = \cos x$



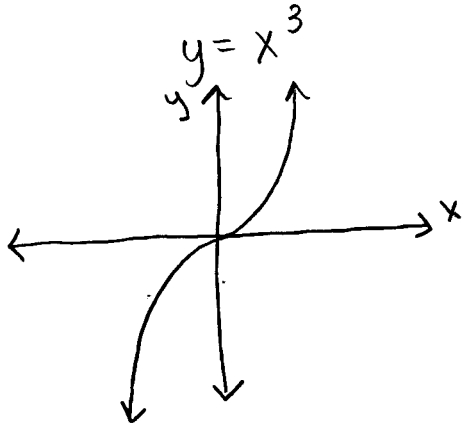
$y = x$



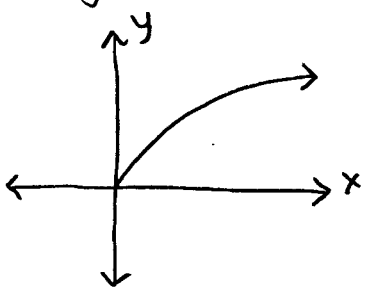
$y = x^2$



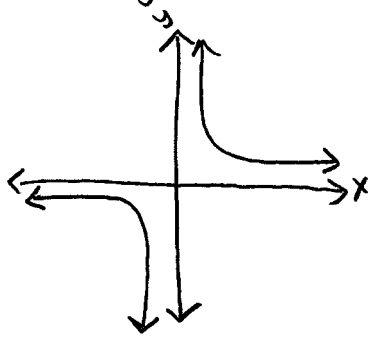
$y = x^3$



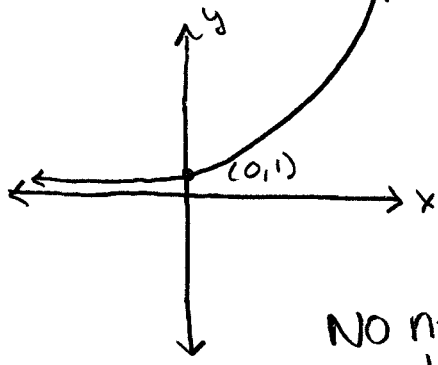
$y = \sqrt{x}$



$y = \frac{1}{x}$

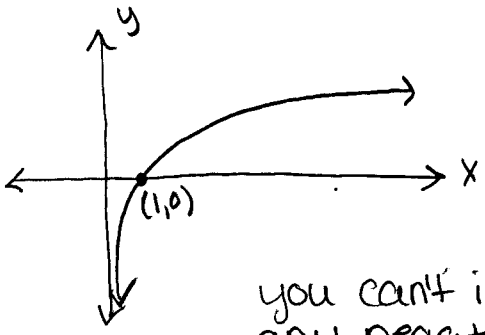


$y = e^x$



No negative outputs!

$y = \ln x$



you can't input any negative values!