Lecture #19
MAT 123

Midterm Review

Practice Problems

① Represent $\log_8 128$ as a fraction, integer or radical

$\log_8 128 = x$

$8^x = 128$ ← Need to find a common base!

$(2^3)^x = 2^7$

$2^{3x} = 2^7$

$\frac{3x}{3} = \frac{7}{3}$

$\boxed{x = 7/3}$

② What is the largest domain on which $\log_5(5-4x)$ is defined?

*we can only take the log of a positive #

$5 - 4x > 0$

$-5 \quad -5$

$-4x > -5$

$\frac{-4x}{-4} > \frac{-5}{-4}$

$\boxed{x < 5/4}$

③ Solve for x: $16^{3x-5} = 32^{x+1}$

$16 = 2^4$

$32 = 2^5$

$(2^4)^{3x-5} = (2^5)^{x+1}$

$2^{4(3x-5)} = 2^{5(x+1)}$

$4(3x-5) = 5(x+1)$

$12x - 20 = 5x + 5 + 20$

$-5x \quad +20 \quad -5x$

$\frac{7x}{7} = \frac{25}{7}$

$\boxed{x = \frac{25}{7}}$

④ Find the equation of the line through $(8, \pi)$ and $(4, 1)$

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

point-slope formula: $y - y_1 = m(x - x_1)$

m = slope

(x_1, y_1) = a point on the line

$$m = \frac{\pi - 1}{8 - 4} = \frac{\pi - 1}{4}$$

$$\boxed{y - 1 = \frac{\pi - 1}{4}(x + 4)}$$

⑤ If $\log A = 20$ and $\log B = 30$

Simplify $\log \frac{\sqrt{A}}{B^3}$ ← Must expand before plugging in!

$$\log \frac{\sqrt{A}}{B^3} = \log \sqrt{A} - \log B^3$$

$$= \frac{1}{2} \log A - 3 \cdot \log B \quad \leftarrow \text{Now we can plug in!}$$

$$= \frac{1}{2}(20) - 3(30)$$

$$= 10 - 90 = \boxed{-80}$$

* $\sqrt{A} = A^{\frac{1}{2}}$

⑥ Write the equation of a circle with center at $(8, -4)$ and radius 10.

r = radius

(h, k) = center

$$(x - h)^2 + (y - k)^2 = r^2$$

general formula for a circle!

$$\boxed{(x - 8)^2 + (y + 4)^2 = 10^2}$$

⑦ Represent $\log_{10} 1000$ as an integer, radical, or fraction

$$\log_{10} 1000 = x$$

$$10^x = 1000$$

$$\boxed{x = 3}$$

⑧ For the function $f(x) = \frac{3x^2-12x+9}{x^2+6x+5}$

(a) what is the largest domain?

(b) what value, if any, does y approach as x approaches infinity?

(c) what, if any, are the zeroes of $f(x)$?

(d) At what value, if any, does $f(x)$ cross the y-axis?

$\frac{3x^2-12x+9}{x^2+6x+5} = \frac{3(x^2-4x+3)}{x^2+6x+5} = \frac{3(x-3)(x-1)}{(x+5)(x+1)}$ factor first!

(a) $(x+5)(x+1) \neq 0$

$x+5 \neq 0 \quad | \quad x+1 \neq 0$

$x \neq -5 \quad | \quad x \neq -1$

Domain → denominator cannot be ∅!

$x \neq -5, x \neq -1$

(b) essentially asking about asymptote (horizontal) end behavior

$\frac{3x^2}{x^2} = \frac{3}{1}$

look at the term of heighest degree in numerator and denominator

$y = 3$

(c) $3(x-3)(x-1) = 0$

$3 \neq 0 \quad | \quad x-3=0 \quad | \quad x-1=0$

$x=3 \quad | \quad x=1$

$x=3$ and $x=1$

set numerator = 0 then solve for x

(d) $y = 3(0-3)(0-1)$

$y = 3(-3)(-1)$

$y = 9$

$y = (0+5)(0+1)$

$y = (5)(1)$

$y = 5$

$y = 9/5$

plug in 0 for x then solve for y.

Side note: To figure out end behavior you look at the terms of heighest degree

let $f(x) = \frac{ax^m + \dots}{bx^n + \dots}$

① $m > n$ infinity or none

② $n > m$ zero $(y=0)$

③ $m = n$ $\frac{a}{b}$ $(y = \frac{a}{b})$

↑ horizontal asymptote

(9)

$$2x+3 \overline{)\,2x^4+7x^3+16x^2+29x+21}$$

x^3+2x^2+5x+7

$-(2x^4+3x^3)$

$4x^3+16x^2$

$-(4x^3+6x^2)$

$10x^2+29x$

$-(10x^2+15x)$

$14x+21$

$-(14x+21)$

0

polynomial long division.

No Remainder!

$$x^3+2x^2+5x+7 = \frac{2x^4+7x^3+16x^2+29x+21}{2x+3}$$

(10) Solve for x: $6^{3x+2} = 9^x$

$\log(6^{3x+2}) = \log(9^x)$ → take the log of both sides

$(3x+2)\log 6 = x \cdot \log 9$ → Distribute!

$3x\cdot\log 6 + 2\cdot\log 6 = x\cdot\log 9$ → group all terms with x and all terms w/o x.

$2\log 6 = x\cdot\log 9 - 3x\log 6$

$\frac{2\log 6}{\log 9 - 3\log 6} = \frac{x(\log 9 - 3\log 6)}{\log 9 - 3\log 6}$ → factor out the x

→ Divide.

$$x = \frac{2\log 6}{\log 9 - 3\log 6}$$

(11) A container of yeast initially contains 20 gms, 5 hrs later it contains 30 gms. Find an equation for $A(t)$, the amount at time t. How much do you have after 24 hrs?

$y = a\cdot b^x$ $\quad a=20$ $\quad x=5$ $\quad y=30$ $\quad b=?$

$\frac{30}{20} = \frac{20\cdot b^5}{20}$

$\left(\frac{3}{2}\right)^{1/5} = (b^5)^{1/5}$

$b = \left(\frac{3}{2}\right)^{\frac{1}{5}}$

$y = (20)\left(\left(\frac{3}{2}\right)^{\frac{1}{5}}\right)^x$

$$A(t) = (20)\left(\frac{3}{2}\right)^{\frac{t}{5}}$$

$t=24$

$$A(24) = (20)\left(\frac{3}{2}\right)^{\frac{24}{5}}$$

(12) If you deposit $2000 at 8% compounded continuously How much will you have after 10 yrs?

$F = Pe^{RT}$

$F = ?$

$P = 2000$

$R = .08$

$T = 10$

$F = 2000e^{(.08)(10)}$

$\boxed{F = 2000e^{.8}}$