Lecture #18
MAT 123

Review: logs & exponentials

problem: $\log_5 \sqrt{125} = ?$

$5^x = \sqrt{125}$

$5^x = \sqrt{5^3}$ same bases!

$5^x = (5^3)^{\frac{1}{2}}$

$5^x = 5^{\frac{3}{2}}$

$\boxed{x = \frac{3}{2}}$

problem: $\log_8 16 = ?$

$8^x = 16$

$(2^3)^x = 2^4$ same base!

$2^{3x} = 2^4$

$\frac{3x}{3} = \frac{4}{3}$

$\boxed{x = \frac{4}{3}}$

problem: $\log_2(4x+1) - \log_2(x-2) = 4$ Solve for x

$\log_2\left(\frac{4x+1}{x-2}\right) = 4$

$2^4 = \frac{4x+1}{x-2}$

$(x-2) \cdot 16 = \frac{4x+1}{x-2} \cdot (x-2)$

$16(x-2) = 4x+1$

$16x - 32 = 4x + 1 + 32$

$-4x \quad\quad -4x$

$\frac{12x}{12} = \frac{33}{12}$

$\boxed{x = \frac{33}{12}}$

make sure you check your x value

$4\left(\frac{33}{12}\right)+1 > 0$ ✓ must be (+)

$\frac{33}{12} - 2 > 0$ ✓ since we cannot take the log of a negative

problem: Solve: $\log_3(2x+1) - \log_3(x+5) = 2$

$$\log_3\left(\frac{2x+1}{x+5}\right) = 2$$

$$3^2 = \frac{2x+1}{x+5}$$

$$\frac{9}{1} = \frac{2x+1}{x+5}$$

$$2x+1 = 9(x+5)$$

$$2x+1 = 9x+45$$

$$-2x-45 \quad -2x \quad -45$$

$$\frac{-44}{7} = \frac{7x}{7}$$

$$x = \frac{-44}{7}$$

← this does not work because we cannot take the log of a negative #

check:

$2\left(-\frac{44}{7}\right)+1 = -12.6 < 0$ X

$\frac{-44}{7} + 5 = -1.29 < 0$ X

No Solution!

problem: $\log_5(16x-2) - \log_5(x-3) = 2$ solve.

$$\log_5\left(\frac{16x-2}{x-3}\right) = 2$$

$$5^2 = \frac{16x-2}{x-3}$$

$$\frac{25}{1} = \frac{16x-2}{x-3}$$

$$16x-2 = 25(x-3)$$

$$16x-2 = 25x-75$$

$$-16x+75 \quad -16x+75$$

$$\frac{73}{9} = \frac{9x}{9}$$

$$x = \frac{73}{9}$$

check:

$16\left(\frac{73}{9}\right) - 2 > 0$ ✓

$\frac{73}{9} - 3 > 0$ ✓

Problem: Solve: $\log_4(x-10) + \log_4(x+10) = 3$

$\log_4((x-10)(x+10)) = 3$

$4^3 = (x-10)(x+10)$

$64 = x^2 - 10x + 10x - 100$

$64 = x^2 - 100$

$+100 \qquad +100$

$164 = x^2$

$x = \pm\sqrt{164}$

Check:

$\sqrt{164} - 10 > 0$ ✓

$\sqrt{164} + 10 > 0$ ✓

$-\sqrt{164} - 10 < 0$ X

only one solution works!

$x = \sqrt{164}$

Problem: $\log_5(8-x) + \log_5(8+x) = 3$

$\log_5((8-x)(8+x)) = 3$

$5^3 = (8-x)(8+x)$

$125 = 64 - x^2$

$+x^2 \qquad +x^2$

$125 + x^2 = 64$

$-125 \qquad -125$

$x^2 = -61$ → this cannot happen! the square of any # can never be negative

NO Solution

problem: $\log_3(9x^2-x+1) - \log_3(x^2-2) = 2$

$$\log_3\left(\frac{9x^2-x+1}{x^2-2}\right) = 2$$

$$3^2 = \frac{9x^2-x+1}{x^2-2}$$

$$\frac{9}{1} = \frac{9x^2-x+1}{x^2-2}$$

$$9x^2-x+1 = 9(x^2-2)$$

$$9x^2-x+1 = 9x^2-18$$

$$-x+1 = -18$$

$$-1 \quad -1$$

$$-x = -19$$

$$-1 \quad -1$$

$$\Rightarrow \boxed{x=19}$$

check:

$9(19)^2-19+1 > 0$ ✓

$9^2-2 > 0$ ✓

problem: Suppose a pond has 200 in^2 of algae initially and 4 days later has 600 in^2 of algae. How much will there be after 10 days?

$y = ab^x \quad a = 200$

$y = 200b^x \quad (4, 600)$

$$\frac{600}{200} = \frac{200b^4}{200}$$

$$3 = b^4$$

$$b = 3^{\frac{1}{4}}$$

Now we know: $y = 200(3^{\frac{1}{4}})^x = 200(3)^{\frac{x}{4}}$

$x=10$: $y = 200(3)^{\frac{10}{4}} = 200(3)^{\frac{5}{2}}$

problem: $8^{x+3} = 4^{2x-1}$ common bases!

$(2^3)^{x+3} = (2^2)^{2x-1}$

$2^{3(x+3)} = 2^{2(2x-1)}$

$3(x+3) = 2(2x-1)$

$3x+9 = 4x-2$

$-3x+2 \quad -3x+2$

$11 = x$

$x = 11$

problem: $8^{x+3} = 11$ we can't find a like base!

$\log_8 8^{x+3} = \log_8 11$

$x+3 = \log_8 11 - 3$

-3

$x = \log_8 11 - 3$ } take the log base 8

OR

$\log 8^{x+3} = \log 11$

$\frac{x+3 \cdot \log 8}{\log 8} = \frac{\log 11}{\log 8}$

$x+3 = \frac{\log 11}{\log 8} - 3$

-3

$x = \frac{\log 11}{\log 8} - 3$ } take the log of both sides (any base!)

Both of these are correct solutions

problem: $8^{x+3} = 11^x$

$\log 8^{x+3} = \log 11^x$

$(x+3)\log 8 = x \cdot \log 11$ ← Distribute

$x\log 8 + 3\log 8 = x\log 11 - x\log 8$ ← you need to put all terms with an "x" on one side of the equation, and all terms w/o an "x" on the other side of the equation.

$-x\log 8$

$3\log 8 = x\log 11 - x\log 8$

$\frac{3\log 8}{\log 11 - \log 8} = \frac{x(\log 11 - \log 8)}{\log 11 - \log 8}$ ← factor out an "x"

$x = \frac{3\log 8}{\log 11 - \log 8}$ ← divide.

problem: $3^{2x-1} = 25^x$

$\log 3^{2x-1} = \log 25^x$

$(2x-1)\log 3 = x \cdot \log 25$ ① Distribute

$2x\log 3 - \log 3 = x\log 25 - 2x\log 3$ ② Group "x" terms

$-2x\log 3$

$-\log 3 = x\log 25 - 2x\log 3$ ③ factor out "x"

$\frac{-\log 3}{\log 25 - 2\log 3} = \frac{x(\log 25 - 2\log 3)}{\log 25 - 2\log 3}$ ④ Divide

$x = \frac{-\log 3}{\log 25 - 2\log 3}$

problem: If you deposit $1000 for 6 yrs at 12% how much will you have if the interest compounds:

1. annually
2. monthly
3. continuously

(1) $F = P(1+R)^T$

$F = 1000(1.12)^6$ ← this is as far as you have to go on the test since you don't have a calculator

$F = 1973.82$

(2) $F = P\left(1+\frac{R}{N}\right)^{NT}$

$F = 1000\left(1+\frac{.12}{12}\right)^{12\cdot 6}$

$F = 1000(1.01)^{72}$

$F = 2047.01$

(3) $A(t) = A_0 e^{RT}$

$A(6) = 1000e^{(.12)(6)}$

$A(6) = 2054.43$

Now what if we want to know how long will it take to double our money? compounded continuously

$A(t) = A_0 e^{RT}$

$\frac{2000}{1000} = \frac{1000e^{(.12)T}}{1000}$

$\ln 2 = \ln e^{.12T}$

$\frac{\ln 2}{.12} = \frac{.12T}{.12}$

$$T = \frac{\ln 2}{.12}$$