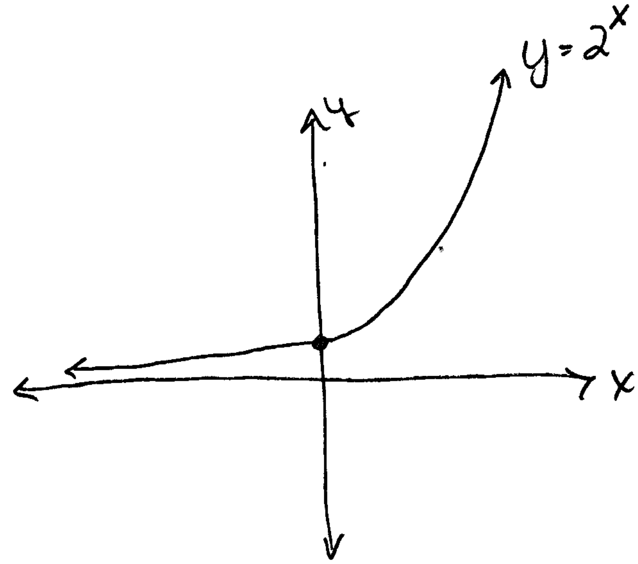
Lecture #15
MAT 123

Exponential Functions:

$y = B^x$
$y = 2^x$

"doubling"

x	y
0	1
1	2
2	4
-1	1/2
-2	1/4
-3	1/8



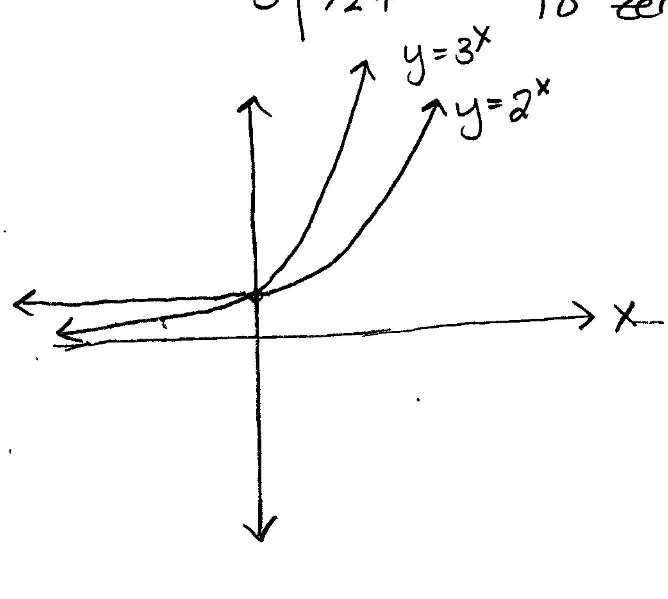
as x goes to $-\infty$ our function goes towards 0
as x goes to $+\infty$ our function go towards ∞

$y = x^2$ (10,100)
$y = 2^x$ (10, 1024) ← grows much quicker!

$y = 3^x$

x	y
0	1
1	3
2	9
3	27
-1	1/3
-2	1/9
-3	1/27

*notice the negative x does not give us a negative output, just smaller (closer to zero)



3^x grows quicker than 2^x

Problem:

There are initially 200 bacteria in a dish. Four hours later there are 400 bacteria. How many will there be after 24 hours?

time 0 = 200
time 4 = 400
time 8 = 800
time 12 = 1600
time 16 = 3200
time 20 = 6400
time 24 = 12800

$y = a \cdot b^x$

a = initial amount
b = growth/decay rate
x = time
y = amount at time x

$y = 200b^x$

$\frac{400}{200} = \frac{200b^4}{200}$

$2 = b^4$

$b = 2^{1/4}$

$y = 200(2^{1/4})^x = 200(2)^{x/4}$

at time 24 → $y = 200(2)^{24/4}$

$y = 200(2)^6 = 12,800$

Problem: Initially there are 1000 bacteria in a pond. After 3 hours there are 5,000. How many will there be after 24 hours?

$a = 1,000$
$y = 5,000$
$x = 3$

$y = ab^x$

$\frac{5000}{1000} = \frac{\cancel{1000}b^3}{\cancel{1000}}$

$5 = b^3$

$b = (5)^{1/3}$

$y = 1,000(5^{1/3})^{24}$

$y = 1,000(5)^{\frac{24}{3}}$

$y = 1,000(5)^8 = 390,625,000$

Problem: Initially there are 20 bacteria in a dish. 12 hours later there are 40 bacteria. How many will there be after 48 hours?

$a = 20$
$y = 40$
$x = 12$

$y = ab^x$

$\frac{40}{20} = \frac{\cancel{20}b^{12}}{\cancel{20}}$

$2 = b^{12}$

$b = 2^{1/12}$

$y = ab^x$

$y = 20(2^{1/12})^{48}$

$y = 20(2)^{\frac{48}{12}}$

$y = 20(2)^4$

$y = (20)(16)$

$y = 320$

ex: $5^{x+3} = 25^{x-1}$

How can we solve this? We need to make them the same base!

$25 = 5^2$

$5^{x+3} = 5^{2(x-1)}$

$5^{x+3} = 5^{2x-2}$

$$\begin{array}{l} \cancel{x}+3 = 2x - \cancel{2} \\ -\cancel{x}+2 \quad -x + \cancel{2} \end{array}$$

$5 = x$

ex: $4^{3x-1} = 8^{x+1}$

$4 = 2^2$
$8 = 2^3$

$2^{2(3x-1)} = 2^{3(x+1)}$

$2^{6x-2} = 2^{3x+3}$

$$\begin{array}{l} 6x - \cancel{2} = 3\cancel{x} + 3 + 2 \\ -3x + \cancel{2} \quad -\cancel{3x} \end{array}$$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{5}{3}$

$x = 5/3$

ex: $9^{x-2} = 27^{3-x}$

$3^2 = 9$
$3^3 = 27$

$3^{2(x-2)} = 3^{3(3-x)}$

$2(x-2) = 3(3-x)$

$$\begin{array}{l} 2x - \cancel{4} = 9 - 3\cancel{x} + 4 \\ +3x + \cancel{4} \quad +\cancel{3x} \end{array}$$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{13}{5}$

$x = 13/5$

Logarithims

$10^1 = 10$
$10^2 = 100$ > $10^x = 50$

x must be between 1 and 2

$x = \log_{10} 50$ "log base 10 of 50"

$2^5 = 32$
$2^6 = 64$ > $2^x = 50$

$x = \log_2 50$

x must be between 5 and 6.

"log base 2 of 50"

$4^2 = 16$
$4^3 = 64$ > $4^x = 50$

$x = \log_4 50$ "log base 4 of 50"

General Case: $x = \log_B A$

$B^x = A$

ex: $\log_5 25 = x$

$5^x = 25$

$\boxed{x = 2}$

ex: $\log_5 125 = x$

$5^x = 125$

$\boxed{x = 3}$

ex: $\log_2 16 = x$

$2^x = 16$

$\boxed{x = 4}$

ex: $\log_3 x = 5$

$3^5 = x$

$\boxed{x = 243}$

ex: $5^x = 20$

we need logs to solve this!

$\log_5 20 = x$

ex: $7^x = 100$

$x = \log_7 100$

$\log_B 1 = 0$ "anything to the zero power is 1"

↳ no matter what B is!!!

$\log_B B = 1 \Rightarrow B^1 = b$

$\log_B B^x = x \Longrightarrow B^x = B^x$

Log Laws

$x^A \cdot x^B = x^{A+B} \Longrightarrow \log(AB) = \log A + \log B$

$\log\left(\frac{A}{B}\right) = \log A - \log B$

$\log A^B = B \log A$

example: $3^x = 40$

$\log(3^x) = \log(40)$

$\frac{x \cdot \cancel{\log(3)}}{\cancel{\log(3)}} = \frac{\log(40)}{\log(3)}$

$\boxed{x = \frac{\log(40)}{\log(3)}}$

example: $7^x = 20$

$\log 7^x = \log 20$

$\frac{x \cdot \cancel{\log 7}}{\cancel{\log 7}} = \frac{\log 20}{\log 7}$

$\boxed{x = \frac{\log 20}{\log 7}}$

example: $6^{x+3} = 35$

$\log 6^{x+3} = \log 35$

$(x+3) \cdot \log 6 = \log 35$

$x + \cancel{3} \; \cancel{-3} = \frac{\log 35}{\log 6} - 3$

$\boxed{x = \frac{\log 35}{\log 6} - 3}$