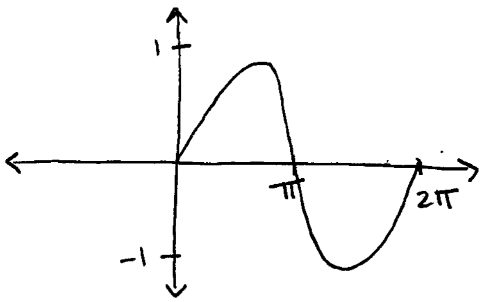
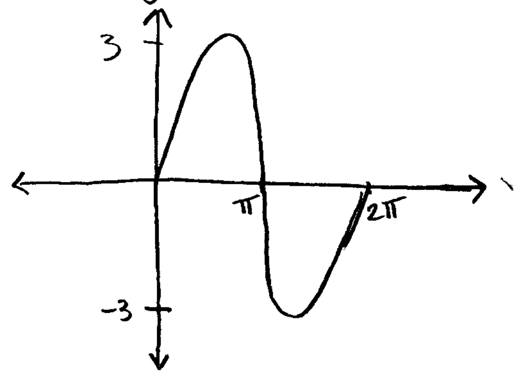
Lecture #9
MAT 123

More Graph Transformations and Inverse Functions

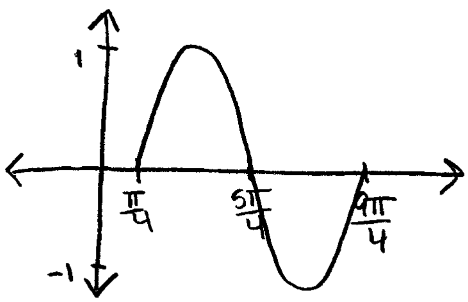
Review: $y = \sin x$



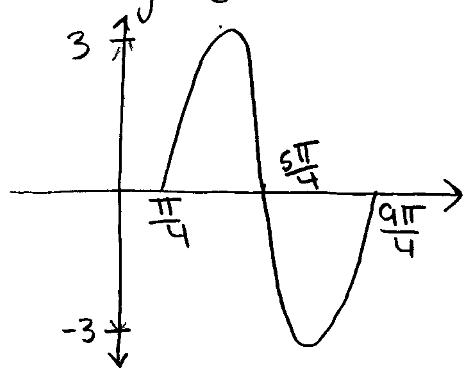
$y = 3\sin x$ vertically stretch



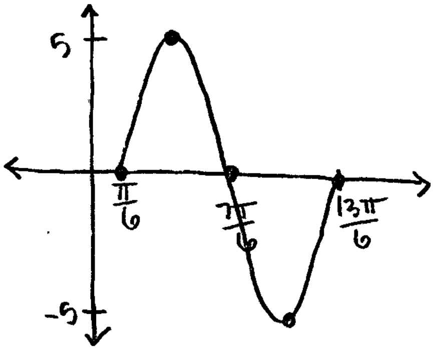
$y = \sin(x - \frac{\pi}{4})$ shift horizontally.



$y = 3\sin(x - \frac{\pi}{4})$ vert. stretch AND horizontal shift

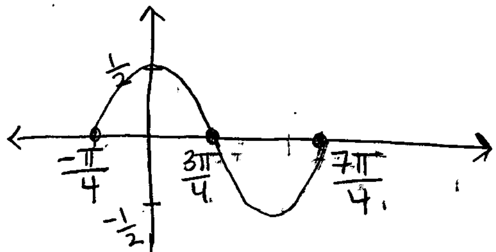


Given this graph state the transformations made:
Original function => $f(x) = \sin x$



New function => $f(x) = 5\sin(x - \frac{\pi}{6})$
horizontal shift right by $\pi/6$ and vertical stretch by 5

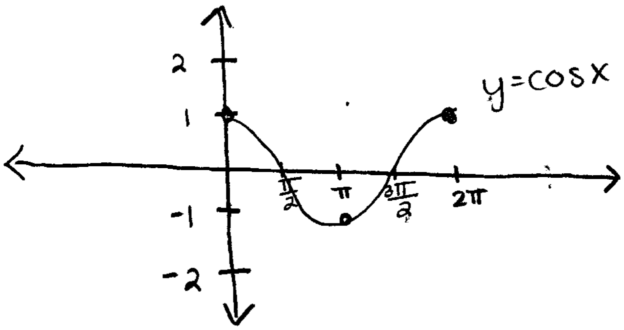
Graph: $y = \frac{1}{2}\sin(x + \frac{\pi}{4})$

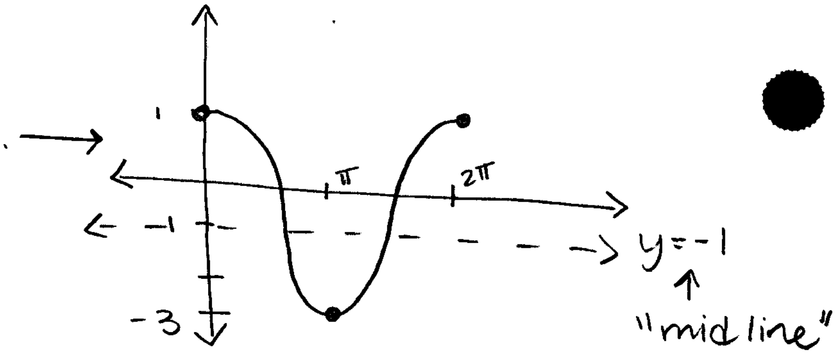


note: $\pi - \frac{\pi}{4} = \frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$

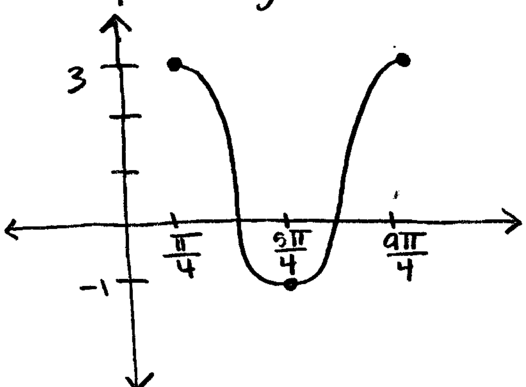
$2\pi - \frac{\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

ex: Graph: $y = 2\cos x - 1$





ex: Graph: $y = 2\cos\left(x - \frac{\pi}{4}\right) + 1$



step #1: shift right by $\frac{\pi}{4}$ units
step #2: scale by factor of 2
step #3: shift up 1 unit

Inverse Functions

"Reversing the original operation"

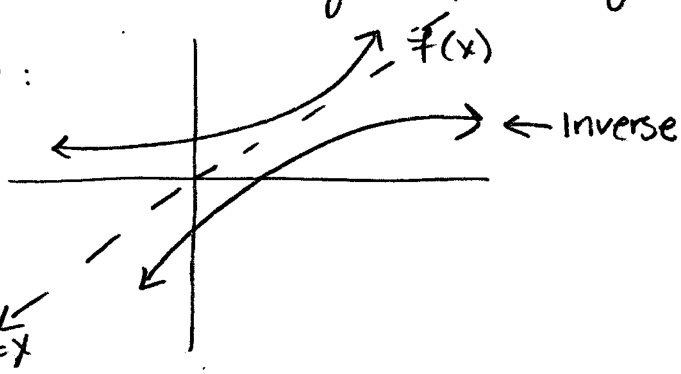
input = Range
output = Domain.

switching the Domain and range.

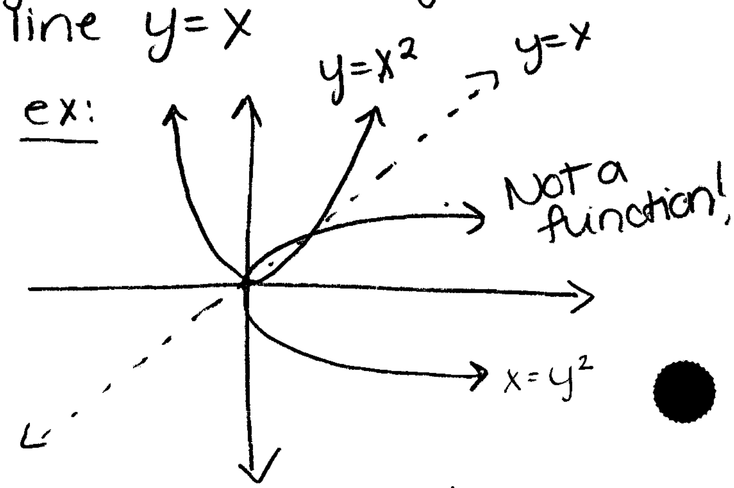
$(x, y) \rightarrow (y, x)$

What's actually happening? you are reflecting over the line $y = x$

ex:



ex:



$y = x^2$ has no inverse! since it doesn't pass the horizontal line test.

(*) if a function passes both the vertical and horizontal line test it is called a one-to-one function.

ex: $f(x) = \frac{5x-4}{7}$ find $f^{-1}(x)$

*notation for inverse function.

① $y = \frac{5x-4}{7}$

② $x = \frac{5y-4}{7}$

③ $7x = 5y-4$

$7x+4 = 5y$

$y = \frac{7x+4}{5}$

④ $\boxed{f^{-1}(x) = \frac{7x+4}{5}}$

Steps: ① replace f(x) with y
② switch x and y
③ solve for y
④ replace y with $f^{-1}(x)$

(*) be careful if they ask for $f^{-1}(y)$ do one last step ⑤ switch all x's to y's.

so $f^{-1}(y) = \frac{7y+4}{5}$

ex: $f(x) = \frac{2x^3+7}{11}$, find $f^{-1}(x)$

① $y = \frac{2x^3+7}{11}$

② $x = \frac{2y^3+7}{11}$

③ $\frac{11x-7}{2} = y^3$

$\sqrt[3]{\frac{11x-7}{2}} = y$

④ $\boxed{f^{-1}(x) = \sqrt[3]{\frac{11x-7}{2}}}$

ex: $f(x) = \frac{6x^5+3}{4}$, find $f^{-1}(x)$

$y = \frac{6x^5+3}{4}$

$x = \frac{6y^5+3}{4}$

$\frac{4x-3}{6} = y^5$

$y = \sqrt[5]{\frac{4x-3}{6}}$

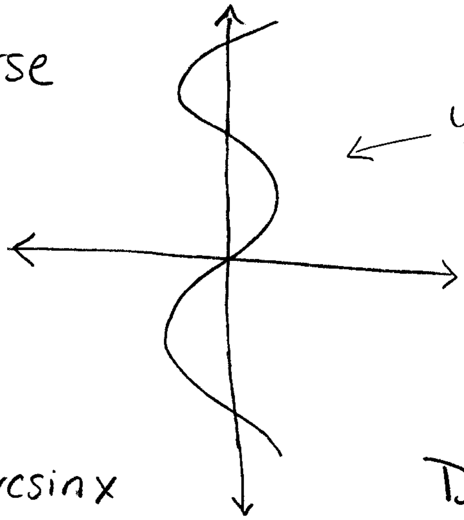
$\boxed{f^{-1}(x) = \sqrt[5]{\frac{4x-3}{6}}}$

ex: $y = \sin x$
find the inverse

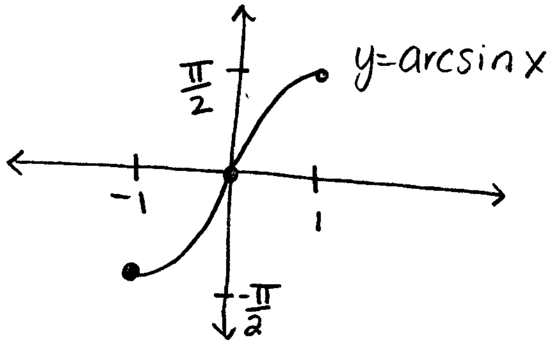
$x = \sin y$

$\arcsin x = y$

$\sin^{-1} x = y$



$\leftarrow$ $y = \arcsin x$
This in NOT a function so we restrict the domain to make it a function.

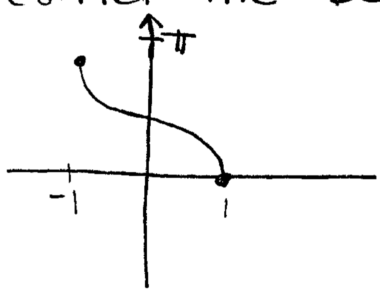


D: $\{ -\frac{\pi}{2}, \frac{\pi}{2} \}$

$\sin \frac{\pi}{6} = \frac{1}{2}$ $\leftarrow$ input = angle

$\arcsin \frac{1}{2} = \frac{\pi}{6}$ $\leftarrow$ output = angle.

ex: $y = \cos x$
find the inverse.
restrict the Domain to $[0, \pi]$



	x = positive	x = negative
Arcsin x	quadrant I	quadrant IV
Arccos x	quadrant I	quadrant II

ex:

$\text{Arcsin}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

problem: If $f(x) = 3\sin\left(x + \frac{\pi}{4}\right)$ find $f^{-1}(x)$

$y = 3\sin\left(x + \frac{\pi}{4}\right)$

$x = 3\sin\left(y + \frac{\pi}{4}\right)$

$\frac{x}{3} = \sin\left(y + \frac{\pi}{4}\right)$

$\arcsin\left(\frac{x}{3}\right) = y + \frac{\pi}{4}$

$\arcsin\left(\frac{x}{3}\right) - \frac{\pi}{4} = y$

$\boxed{f^{-1}(x) = \arcsin\left(\frac{x}{3}\right) - \frac{\pi}{4}}$