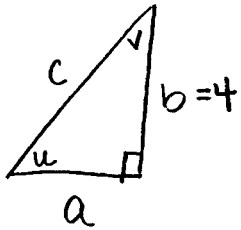


Simple Trig Identities



$\sin v = \frac{3}{11}$ find a :

$$\frac{3}{11} = \frac{a}{c}$$

$$a^2 + 4^2 = c^2$$

this is what we know so far.

$$\frac{3}{11} = \frac{a}{c}$$

$$3c = 11a$$

$$c = \frac{11a}{3}$$

$$\Rightarrow a^2 + 16 = \left(\frac{11a}{3}\right)^2$$

$$\left(a^2 + 16 = \frac{121a^2}{9}\right) \times 9$$

$$9a^2 + 144 = 121a^2$$

$$-9a^2$$

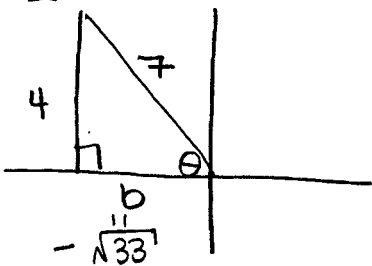
$$\frac{144}{112} = \frac{112a^2}{112}$$

$$a^2 = \frac{144}{112}$$

Answer.

$$a = \sqrt{\frac{144}{112}}$$

$\frac{\pi}{2} \leq \theta \leq \pi$ $\sin \theta = \frac{4}{7}$ find $\cos \theta$



$$a^2 + b^2 = c^2$$

$$4^2 + b^2 = 7^2$$

$$16 + b^2 = 49$$

$$b^2 = 33$$

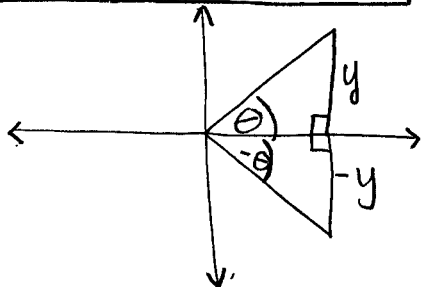
$$b = \pm \sqrt{33}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

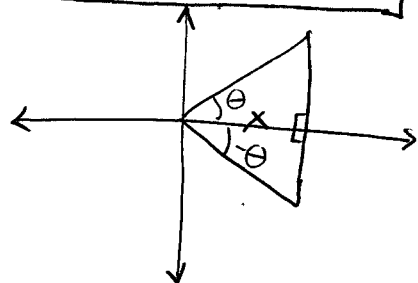
$$\cos \theta = \frac{-\sqrt{33}}{7}$$

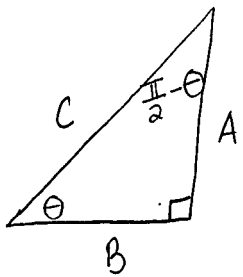
Answer

* $\sin(-\theta) = -\sin \theta$ *



* $\cos(-\theta) = \cos \theta$ *





$$\sin \theta = \frac{A}{c} \quad \cos\left(\frac{\pi}{2} - \theta\right) = \frac{A}{c}$$

$$* \boxed{\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)} *$$

$$* \boxed{\cos \theta = \sin\left(\frac{\pi}{2} - \theta\right)} *$$

example:

$$\sin(2x-5) = 0$$

find the smallest positive x that makes that true

$$2x-5 = 0$$

$$2x = 5$$

$$\boxed{x = \frac{5}{2}} \text{ Answer}$$

example:

$$\cos(2x-5) = 0$$

find the smallest positive x that makes this true

where is $\cos = 0$? $90^\circ = \frac{\pi}{2}$

$$2x-5 = \frac{\pi}{2}$$

$$2x = \frac{\pi}{2} + 5$$

$$\boxed{x = \frac{\frac{\pi}{2} + 5}{2}} \text{ Answer}$$

example:

$$\sin(2x-1) = \frac{1}{2}$$

find the smallest possible x

where is $\sin = \frac{1}{2}$? $\frac{\pi}{6}$

$$2x-1 = \frac{\pi}{6}$$

$$2x = \frac{\pi}{6} + 1$$

$$\boxed{x = \frac{\frac{\pi}{6} + 1}{2}} \text{ Answer}$$

example:

$$\tan(4x-3)=1$$

find the smallest possible x value.

where is $\tan = 1$? $\frac{\pi}{4}$

$$4x-3 = \frac{\pi}{4}$$

$$4x = \frac{\pi}{4} + 3$$

Answer

$$x = \frac{\frac{\pi}{4} + 3}{4}$$

example:

$$\sin(2x-1) = \cos(x)$$

find the smallest x -value

$$\therefore \sin(2x-1) = \sin\left(\frac{\pi}{2} - x\right) \leftarrow \text{identity from before} \quad * \cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$2x-1 = \frac{\pi}{2} - x$$

$$3x-1 = \frac{\pi}{2}$$

$$3x = \frac{\pi}{2} + 1$$

Answer

$$x = \frac{\frac{\pi}{2} + 1}{3}$$

example:

$$\cos(4x-5) = \sin(x-2)$$

$$\cos(4x-5) = \cos\left(\frac{\pi}{2} - (x-2)\right) \leftarrow \text{identity} \quad * \sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$4x-5 = \frac{\pi}{2} - x + 2$$

$$5x-5 = \frac{\pi}{2} + 2$$

$$5x = \frac{\pi}{2} + 7$$

Answer

$$x = \frac{\frac{\pi}{2} + 7}{5}$$

example:

$$\sin(3x+4) = \cos(2-x)$$

$$\sin(3x+4) = \sin\left(\frac{\pi}{2} - (2-x)\right)$$

$$3x+4 = \frac{\pi}{2} - 2 + x$$

$$2x+4 = \frac{\pi}{2} - 2$$

$$2x = \frac{\pi}{2} - 6$$

Answer

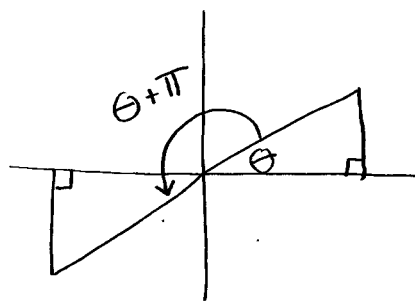
$$x = \frac{\frac{\pi}{2} - 6}{2}$$

*

$$\cos(\theta + \pi) = -\cos(\theta)$$

$$\sin(\theta + \pi) = -\sin(\theta)$$

*



*

$$\cos(\theta + 2\pi) = \cos(\theta)$$

$$\sin(\theta + 2\pi) = \sin(\theta)$$

*

→ the "period" of the circle is 2π

$$\sin(\theta + k\pi) = \begin{cases} \sin\theta & \text{if } k \text{ is even} \\ -\sin\theta & \text{if } k \text{ is odd} \end{cases}$$

note: k is an integer!

$$\cos(\theta + k\pi) = \begin{cases} \cos\theta & \text{if } k \text{ is even} \\ -\cos\theta & \text{if } k \text{ is odd} \end{cases}$$

example: $\sin\left(\frac{13\pi}{6}\right) = ?$

$$\sin\left(\frac{13\pi}{6}\right) = \sin\left(\frac{\pi}{6} + \frac{12\pi}{6}\right) = \sin\left(\frac{\pi}{6} + 2\pi\right) = \sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$$

example: $\cos\left(\frac{17\pi}{4}\right) = ?$

$$\frac{17\pi}{4} - 2\pi = \frac{17\pi}{4} - \frac{8\pi}{4} = \frac{9\pi}{4}$$

$$\frac{9\pi}{4} - 2\pi = \frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$$

$$\cos\left(\frac{17\pi}{4}\right) = \cos\left(\frac{\pi}{4} + 4\pi\right)$$

$$= \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$$

↑
Answer

example:

$$\sin\left(\frac{41\pi}{6}\right) = ?$$

$$\frac{41\pi}{6} - 2\pi = \frac{41\pi}{6} - \frac{12\pi}{6} = \frac{29\pi}{6}$$

$$\frac{29\pi}{6} - 2\pi = \frac{29\pi}{6} - \frac{12\pi}{6} = \frac{17\pi}{6}$$

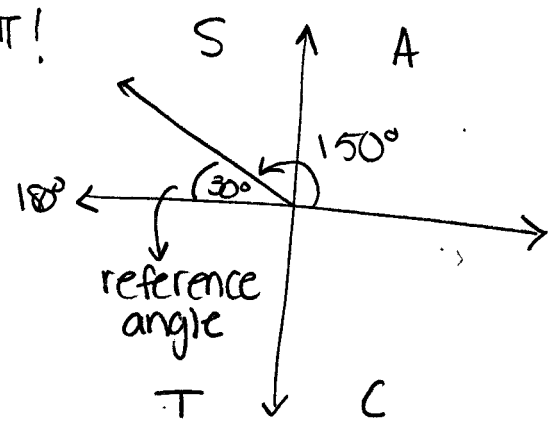
$$\frac{17\pi}{6} - 12\pi = \frac{5\pi}{6} \quad \text{STOP less than } 2\pi!$$

Keep subtracting 2π until you get a # less than 2π

$$\sin\left(\frac{41\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right) = \sin(150^\circ)$$

$$\frac{5\pi}{6} \times \frac{180}{\pi} = 150^\circ$$

$$\sin 150^\circ = \sin 30^\circ = \boxed{\frac{1}{2}} \text{ Answer}$$



example:

$$\cos\left(\frac{23\pi}{4}\right) = ?$$

$$\frac{23\pi}{4} \times \frac{180}{\pi} = 1035^\circ$$

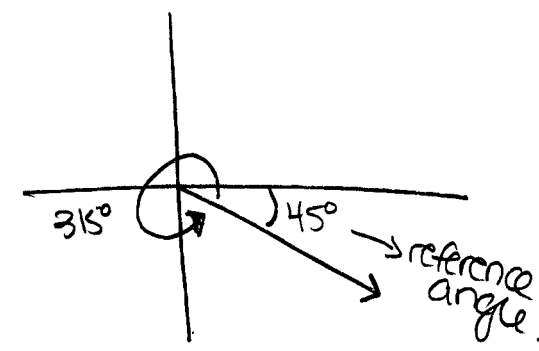
$$1035^\circ - 360^\circ = 675^\circ$$

$$675^\circ - 360^\circ = 315^\circ \leftarrow \text{STOP less than } 360^\circ!$$

$$\cos\left(\frac{23\pi}{4}\right) = \cos(315^\circ)$$

$$\cos(315^\circ) = \cos(45^\circ) = \boxed{\frac{\sqrt{2}}{2}} \text{ Answer}$$

Keep subtracting until you get a # less than 360°



(*) note you can do these problems in radians or you can convert to degrees. Either way you will get the same answer!