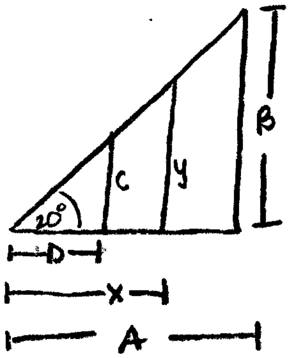
Lecture #1
MAT 123

Introduction to Trigonometry

① Right Triangle Trig

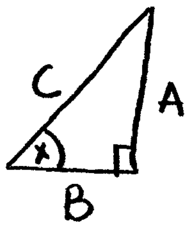


$\frac{y}{x} = \frac{B}{A} = \frac{C}{D}$ ← these ratios hold because all three of these triangles are similar

← these 3 triangles are similar because they all have the same angle measures (20°, 70°, 90°)

We use trigonometry to give meaning to these ratios!

The Basics



$\sin x = \frac{A}{C}$

$\cos x = \frac{B}{C}$

$\tan x = \frac{A}{B}$

"SOH CAH TOA"

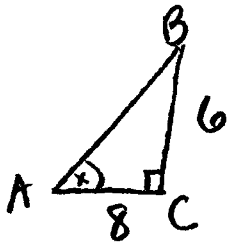
$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan x = \frac{\text{opposite}}{\text{adjacent}}$

note: adjacent = side next to the angle
hypotenuse = the longest side; the side across from the right angle

Pythagorean Theorem: A formula that relates the 3 sides of a right triangle

$a^2 + b^2 = c^2$ (c is the hypothenuse)

example:
Given $\triangle ABC$



① find the length of side c
② find $\sin x$
$\cos x$
$\tan x$

① $a^2+b^2=c^2$
$6^2+8^2=c^2$
$36+64=c^2$
$100=c^2$
$\pm 10=c$
$\boxed{c=10}$

↖ use the Pythagorean Theorem

② $\sin x=\frac{6}{10}$

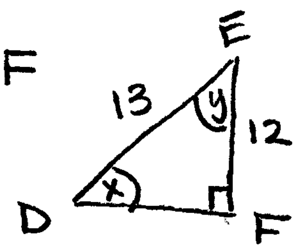
$\cos x=\frac{8}{10}$

$\tan x=\frac{6}{8}$

} use SOH CAH TOA to find these ratios.

example: Given $\triangle DEF$

① find the missing side
② find: $\sin x$
$\cos x$
$\tan x$
③ find: $\sin y$
$\cos y$
$\tan y$



① $a^2+b^2=c^2$
$12^2+b^2=13^2$
$144+b^2=169$
$b^2=25$
$b=\pm 5$
$\boxed{b=5}$

② $\sin x=\frac{12}{13}$

$\cos x=\frac{5}{13}$

$\tan x=\frac{12}{5}$

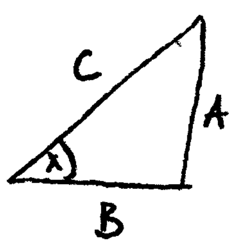
③ $\sin y=\frac{5}{13}$

$\cos y=\frac{12}{13}$

$\tan y=\frac{5}{12}$

⊛ notice $\sin x=\cos y$ and $\cos x=\sin y$
sine and cosine are "co-functions"
angle x and y are complementary angles (they add to 90°)

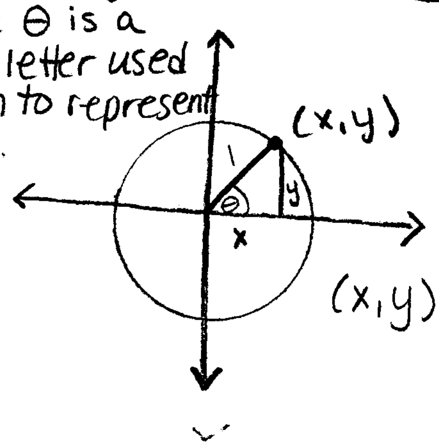
② Connecting Right triangle trig to the Unit Circle → a circle w/ radius=1 and center (0,0)



$\sin x=\frac{A}{C}$

$\cos x=\frac{B}{C}$

*note θ is a greek letter used in math to represent an angle.

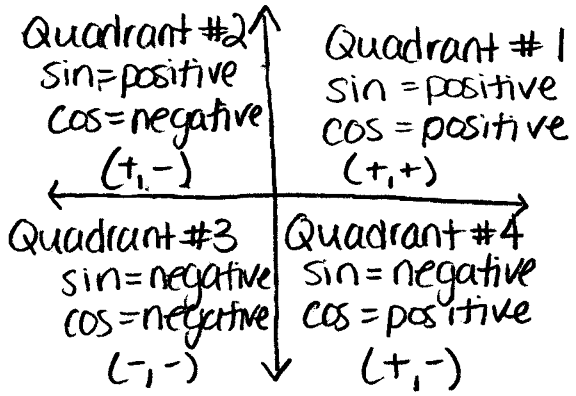


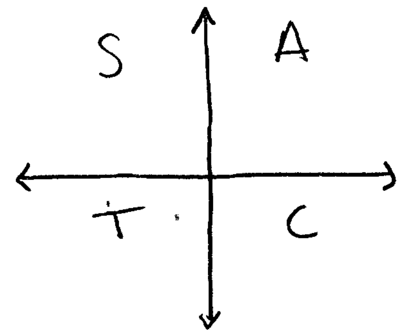
$\sin\theta=\frac{y}{1}=y$

$\cos\theta=\frac{x}{1}=x$

$(x,y)\rightarrow(\cos\theta,\sin\theta)$

this allows us to connect trig to all real numbers.

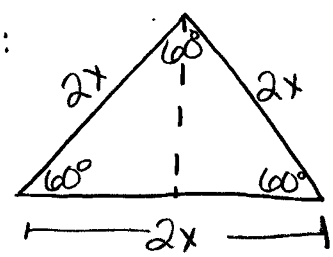




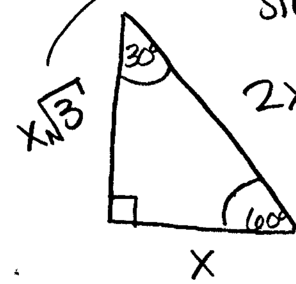
"All Students Take Calculus"

③ Special Right Triangles

equilateral Δ :



cut in half →



→ find this missing side using Pythagorean Theorem!

30°-60°-90° Triangle.

$\sin 30° = \frac{x}{2x} = \frac{1}{2}$

$\cos 30° = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$

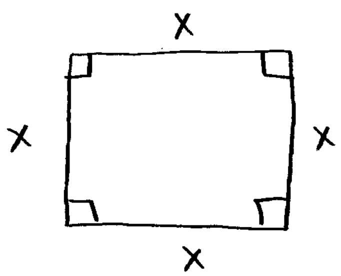
$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

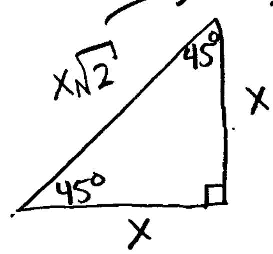
$\tan 60° = \sqrt{3}$

these exact values need to be memorized for the exam!

square:



cut in half diagonally →



→ use Pythagorean Thm to find this missing side!

rationalize the denometer.

★ memorize!

$\sin 45° = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{x}{x\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45° = \frac{x}{x} = 1$

Helpful Table

	$30°$	$45°$	$60°$
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

* note: $\tan x = \frac{\sin x}{\cos x}$