

MAT 682: INDEX THEOREMS FOR ELLIPTIC OPERATORS

Instructor: Blaine Lawson

Tuesday and Thursday, 10:00 - 11:20 in P-123

Office Hours: Tuesday and Thursday, 11:30 - 1:00 in room 5-109

I am also willing to meet at other times. See me at class or e-mail me at blaine@math.sunysb.edu to make arrangements.

References for the course: The best reference is

Spin Geometry by Professor Michelsohn and myself. QC793.3.S6 L39 1989

Further books:

[1] N. Berline, E. Getzler and M. Vergne, *Heat Kernels and Dirac Operators*, This is a great reference for an analytic approach to the result. QA377.B49 2004

[2] P. Gilkey, *Invariance Theory, The Heat Equation and the Atiyah-Singer Index Theorem*, Publish or Perish press, 1984. (Good reference for analytic background). QA614.92 G56 1995

[3] F. Hirzebruch, *Topological Methods in Algebraic Geometry*, Springer-Verlag, 1966. (A great reference for the topological side, and of course for the Hirzebruch theorems which were pre-generators of the AS Theorem.) QA564.H513

[4] M. Taylor, *Pseudodifferential Operators*, Princeton Univ. Press, 1981. (A great book on the title-subject.) QA3.L28 no. 416

[5] P. Shanahan. *The Atiyah-Singer Index Theorem*, Springer Lecture Notes 638.

[6] M. Atiyah, *K-Theory*, Benjamin Press.

[7] R. Bott, *Lectures on $K(X)$* , Benjamin Press.

The following is an outline of the syllabus for the course. The plan is to do these subjects from the beginning — that is, you will not be required to already know them.

On the other hand, the main part of the course will be topics 6, 7, 8 and 9.

SYLLABUS

1. **Riemann-Roch Theorem.** (3 Versions)
2. **Vector Bundles, Characteristic Classes, etc.**
 - (1) Vector Bundles and Classifying Spaces.
 - (2) The Splitting Principle.
 - (3) Chern Classes.
 - (4) Pontryagin Classes.
 - (5) K-Theory
 - (6) The Chern Character.
 - (7) Multiplicative Sequences.
 - (8) Examples.
3. **Differential Operators, Principal Symbol, Elliptic Operators.**
4. **The Functor K_{cpt} and the class of $\sigma(D)$.**
5. **Clifford Algebras, Clifford Bundles, $Cl(X)$ and Dirac-type Operators.**
(with some basic examples).
6. **Pseudodifferential Operators.**
 - (1) Definitions and elementary properties.
 - (2) Elliptic PSD's and parametrices.
 - (3) Fundamental results for elliptic operators.
7. **The Topological Invariance of the Index.**
8. **The Thom Isomorphism and the Chern Character Defect.**
9. **Proof of the Atiyah-Singer Theorem.**
10. **Generalizations.**
11. **Applications.**

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<http://www.stonybrook.edu/uaa/academicjudiciary>

Critical Incident Management: Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, and/or inhibits students' ability to learn.