Index Theorems for Elliptic Operators

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Hours: T-Th 1:00-2:30

One of the major developments of 20th century mathematics was the series of results beginning with the classical Riemann-Roch Theorem and culminating in various index theorems of Atiyah and Singer. This course will track these developments, beginning with Riemann-Roch and its generalizations due to Hirzebruch. We will then present the basic Atiyah-Singer Index Theorem with a number of its consequences. Finally we will examine the Clifford Index Theorem with values in KO-theory.

The course will include a discussion of the basic theory of elliptic operators as well as the theory of characteristic classes and multiplicative sequences.

Course Outline

1. **History.**
   The classical Riemann-Roch Theorem and
   the Riemann-Roch-Hirzebruch Theorem.

2. **Vector Bundles.**
   Basic Properties
   Classifying Spaces

3. **Characteristic Classes.**
   Multiplicative Sequences
   The Chern Character

4. **K-Theory.**
   General Theory
   Bott Periodicity
   The Atiyah-Bott-Shapiro Construction
   Thom Isomorphisms
   The Chern Character and the Commutativity Defect

5. **Differential Operators.**
   Differential Operators Between Bundles on a Manifold
   The Principal Symbol
   Ellipticity
   The Symbol Class in $K_{\text{cpt}}(T^*X)$

6. **Basic Results from Analysis.**
   Sobolev Spaces and Sobolev Embedding Theorems
   The Rellich Lemma
   Schwartz Spaces
   Results for Differential Operators

7. **Pseudo-Differential Operators.**

8. **Elliptic Operators and Parametrices.**

9. **Fundamental Results for Elliptic Operators.**

10. **Fredholm Operators and the Topological Invariance of the Index.**

11. **Various Statmements of the Atiyah-Singer Index Theorem.**

12. **Various Generaliztions of the Basic Atiyah-Singer Theorem.**
    The Index Theorem for Families
    The G-Index Theorem
    The Cl_k-Index Theorem