

SYLLABUS – MAT 568  
RIEMANNIAN GEOMETRY 2011-12

**Fall Term**

1. Review: Vector fields and flows, Lie derivatives, Stokes' Theorem, The de Rham Theorem, Lie groups and Lie algebras.
2. The theory of connections on vector bundles: connections, curvature, holonomy, Bianchi identities, direct sum and tensor product connections.
3. Connections on the tangent bundle: geodesics and the exponential map, The Fundamental Theorem of Riemannian Geometry, the existence of convex neighborhoods.
4. Completeness and the Hopf-Rinow Theorem.
5. Manifolds of negative curvature: The Cartan-Hadamard Theorem, existence of closed geodesics, Preissmann's Theorem, The Maximal Torus Theorem.
6. The First and Second Variations of Energy.
7. The Bonnet-Myers Theorem (for complete manifolds of positive Ricci curvature).
8. Manifolds of positive curvature: Theorems of Synge, Weinstein, Frankel and others.
9. Conjugate points and The Index Lemma.

**Spring Term**

1. The Rauch Comparison Theorem.
2. The Cartan-Ambrose-Hicks Theorem.
3. Bishop-Gromov Comparison.
4. The Toponogov Triangle Theorem.
5. Complete manifolds with  $K \geq 0$  – The Soul Theorem.
6. Hodge Theory and Bochner's Method.
7. Spinors and the Dirac operator  $\not{D}$ .
8. The Singer-Lichnerowicz Formula – manifolds of positive scalar curvature.
9. Statement of the Atiyah-Singer Index Theorem for  $\not{D}$ .
10. Submanifold Theory – minimal submanifolds.
11. Chern Gauss Bonnet and the Hirzebruch Signature Theorem.