



# MAT 552

## Lie groups, Lie algebras and their representations

**Place and time** MF 09:25 - 10:45, Mon Math 4-130, Fri SBS N-102

**Instructor** [Sorin Popescu](mailto:sorin@math.sunysb.edu) (office: Math 4-119, tel. 632-8358, e-mail [sorin@math.sunysb.edu](mailto:sorin@math.sunysb.edu))

**Course starts on September 9!**

### Course description

This course will cover basic theory of Lie groups and Lie algebras. One of its aims is to provide a brief introduction to modern representation theory by working with the classical examples: the general linear, orthogonal and symplectic Lie groups. The framework will be mainly algebraic, and/or sometimes analytic. We will assume material covered in **MAT 530**, **MAT 531** (Geometry/Topology I-II), **MAT 534**, **MAT 535** (Algebra I-II), as well as certain basic facts from **MAT 544** (Analysis). It should however be possible to fill in some of the gaps during the semester.

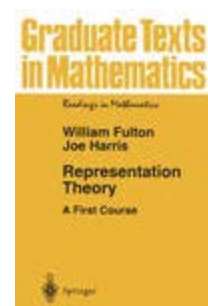
### Textbook(s)

The basic textbooks are:

- *Lie Groups*, Hans Duistermaat, Johan Kolk, Springer Universitext
- *Representation Theory: A First Course*, William Fulton, Joe Harris, Springer, Graduate Texts in Mathematics, Vol. **129**

Both books will be on reserve in Math/Phys library. In addition, you may find the following books useful:

- J. F. Adams, *Lectures on Lie Groups* - a "classic"
- J.-P. Serre, *Lie algebras and Lie groups* - an excellent exposition of the theory of simple Lie algebras
- Broucker and tom Dieck, *Representation of compact Lie groups*.
- A. Knapp, *Lie groups beyond an introduction*, Birkhauser, 1996
- Chevalley, *Theory of Lie Groups I*.



## Topics

The following is a tentative list of what we will try to cover in class. Actual material will depend also on the interests of the course participants.

### Part I. Lie groups

- Basic definitions; examples.
- Linear groups; exponential mapping. Classical groups:  $SL$ ,  $SO$ ,  $Sp$ ,  $SU$ .
- Group representations; adjoint representation. Simple and semi-simple representations. Schur lemma
- Compact groups. Haar measure; complete reducibility of representations. Representations of  $S^1$ .

### Part II. Lie algebras: basic definitions

- Lie algebra of a Lie group: different definitions.
- Lie theory; exponential mapping, third Lie theorem
- Solvable, simple and semisimple Lie algebras
- Killing form; criterion of semisimplicity. Real forms and complexification.

### Part III. Structure theory of simple Lie algebras

- Example:  $sl_2$  and its representations
- Cartan subalgebra, roots
- Abstract root systems, Cartan matrices, Weyl group. Dynkin diagrams
- Correspondence between root systems and simple Lie algebras

### Part IV. Representations of simple Lie algebras.

- Representations of  $sl_2$ ,  $SO(3)$  and spectrum of the hydrogen atom
- Highest weight representations. Verma modules
- Weyl character formula
- Example: representations of  $sl_n$

## Homework & Exams

I will assign problems in each lecture, ranging in difficulty from routine to more challenging. There will be also a take-home midterm and a final exam. Course grades will be based on these exams and homework problems (and any other participation).

*Sorin Popescu*

2002-08-25



## Sorin Popescu

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**Research Interests:** Algebraic Geometry, Commutative Algebra, Combinatorics and Computational methods

**Teaching:**

Spring 2006









[MAT 311 Number Theory](#)

[MAT 614 Topics in Algebraic Geometry](#)












Previous years

[Teaching Archive](#)

**Algebra, Geometry and Physics seminar:** [Spring 2006](#)



**Publications & E-Prints:** Unless otherwise indicated, the files below are DVI files () , PostScript files () , PDF files () , or tar gzipped DVI and PostScript files () . Files marked as () or () are hyperlinked PDF or Macromedia Flash files formatted for screen viewing. Other formats (source, PS using Type I fonts) can be obtained via the UC Davis Front to the [Mathematics ArXiv](#). Click on () or () for related [Macaulay2](#), or [Macaulay](#) code.

**Syzygies:**










- [Gale Duality and Free Resolutions of Ideals of Points](#) [] , [] [] [] [] , *Invent math* **136** (1999) 2, 419-449  
David Eisenbud and Sorin Popescu
- [The Projective Geometry of the Gale Transform](#) [] , [] [] [] , *J. Algebra* **230** (2000), no. 1, 127-173  
David Eisenbud and Sorin Popescu  
(in the D. Buchsbaum anniversary volume of *J. Algebra*)
- [Syzygy Ideals for Determinantal Ideals and the Syzygetic Castelnuovo Lemma](#) [] [] , [[MathSci](#)] ,

Springer 1999

David Eisenbud and Sorin Popescu

- *Extremal Betti Numbers and Applications to Monomial Ideals* [] [] [] [], *J. Algebra* **221** (1999), no. 2, 497-512  
Dave Bayer, Hara Charalambous and Sorin Popescu
- *Lagrangian Subbundles and Codimension 3 Subcanonical Subschemes* [], [] [] [], *Duke Math. J.* **107** (2001), no. 3, 427-467  
David Eisenbud, Sorin Popescu and Charles Walter
- *Enriques Surfaces and other Nonpfaffian Codimension 3 Subcanonical Subschemes* [] [] [] [] [], *Comm. Algebra* **28** (2000), 5629-5653  
David Eisenbud, Sorin Popescu and Charles Walter  
(in the Hartshorne anniversary volume of *Comm. Algebra*)
- *Syzygies of Unimodular Lawrence Ideals* [] [] [] [], *J. Reine Angew. Math* **534** (2001), 169-186  
Dave Bayer, Sorin Popescu and Bernd Sturmfels
- *Hyperplane Arrangement Cohomology and Monomials in the Exterior Algebra* [] [] [] [] [], *Trans. AMS.* **355** (2003), 4365-4383  
David Eisenbud, Sorin Popescu and Sergey Yuzvinsky
- *Exterior algebra methods for the Minimal Resolution Conjecture* [] [] [] [], *Duke Math. J.* **112** (2002), no. 2, 379-395  
David Eisenbud, Frank-Olaf Schreyer, Sorin Popescu and Charles Walter
- *Symmetric resolutions of coherent sheaves* [] [] []  
David Eisenbud, Sorin Popescu and Charles Walter
- *A note on the Intersection of Veronese Surfaces* [] [] [] [] [],  
David Eisenbud, Klaus Hulek and Sorin Popescu
- *Restricting linear syzygies: algebra and geometry* [] [] [] [] [], *Compositio Math.* **141** (2005), no.6, 1460-1478  
David Eisenbud, Mark Green, Klaus Hulek and Sorin Popescu
- *Small schemes and varieties of minimal degree* [] [] [] [] [], *Amer. J of Math* (2005), to appear  
David Eisenbud, Mark Green, Klaus Hulek and Sorin Popescu





#### Abelian varieties, modular varieties and equations:

- *Equations of (1,d)-polarized abelian surfaces* [] [] [], *Math. Ann.* **310** (1998), no. 2, 333-377  
Mark Gross and Sorin Popescu
- *The moduli space of (1,11)-polarized abelian surfaces is unirational* [] [] [], *Compositio Math.* **126** (2001), no. 1, 1-24  
Mark Gross and Sorin Popescu
- *Calabi-Yau threefolds and moduli of abelian surfaces I* [] [] [], *Compositio Math.* **127**, no. 2, (2001), 169-228  
Mark Gross and Sorin Popescu




























[Calabi-Yau threefolds and moduli of abelian surfaces II](#) [  ] [  ] [  ]




Mark Gross and Sorin Popescu

- [Elliptic functions and equations of modular curves](#) [  ] [  ] [  ] [  ], *Math. Ann.* **321** (2001), no. 3, 553-568  
Lev A. Borisov, Paul Gunnells, and Sorin Popescu

#### Surfaces in $P^4$ and threefolds in $P^5$ :

- [The Geometry of Bielliptic Surfaces in  \$P^4\$](#)  [  ], [  ] [  ], *Internat. J. Math.* **4** (1993), no. 6, 873-902  
A. Aure, W. Decker, K. Hulek, S. Popescu and K. Ranestad
- [On Surfaces in  \$P^4\$  and Threefolds in  \$P^5\$](#)  [  ] [  ] [  ], [ **MathSci** ], LMSLN **208**, 69--100  
W. Decker and S. Popescu
- [Surfaces of degree 10 in  \$P^4\$  via linear systems and linkage](#) [  ] [  ] [  ] [  ] [  ], *J. Algebraic Geom.* **5** (1996), no. 1, 13-76  
S. Popescu and K. Ranestad
- [Syzygies of Abelian and Bielliptic Surfaces in  \$P^4\$](#)  [  ] [  ] [  ], *Internat. J. Math.* **8** (1997), no. 7, 849-919  
A. Aure, W. Decker, K. Hulek, S. Popescu and K. Ranestad
- [Examples of smooth non general type surfaces in  \$P^4\$](#)  [  ] [  ] [  ] [  ] [  ], *Proc. London Math. Soc.* (3) **76** (1998), no. 2, 257-275  
S. Popescu
- [Surfaces of degree  \$\geq 11\$  in the Projective Fourspace](#) [  ] [  ] [  ] + [Appendix](#) [  ] [  ] [  ]  
S. Popescu

#### PRAGMATIC 1997: A summer school in Catania, Sicily

- [Research Problems for the summer school](#) [  ], [  ] [  ], [ **MathSci** ], *Matematiche* (Catania) **53** (1998), 1-14  
David Eisenbud and Sorin Popescu

#### Algorithmic Algebra and Geometry: Summer Graduate Program (1998) at MSRI:

- Poster [  ] [  ], [lecture slides and streaming video](#) , CD ROM,  
Dave Bayer and Sorin Popescu

#### Linear algebra notes

- [On circulant matrices](#) [  ], [  ] [  ] [  ] [  ],  
Daryl Geller, Irwin Kra, Sorin Popescu and Santiago Simanca






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#### Upcoming conferences:

- DARPA FunBio Mathematics-Biology Kick-off meeting, Princeton, September 21-23, 2005
- [MAGIC 05: Midwest Algebra, Geometry and their Interactions Conference](#), University of Notre Dame, Notre Dame, October 7-11, 2005
- [AMS Special Session on Resolutions](#), Eugene, OR, November 12-13, 2005
- [Clay Workshop on Algebraic Statistics and Computational Biology](#), Clay Mathematics Institute, November 12-14, 2005
- [CIMPA School on Commutative Algebra](#), December 26, 2005 - January 6, 2006, Hanoi, Vietnam
- [AMS Special Session on Syzygies in Commutative Algebra and Geometry](#), San Antonio, TX, January 12-15, 2006
- [KAIST Workshop on Projective Algebraic Geometry](#), January 23-25, 2006, Korean Advanced Institute of Science and Technology, Daejeon
- [AMS Special Session on the Geometry of Groebner bases](#), San Francisco, CA, April 29-30, 2006
- [Castnuovo-Mumford regularity and related topics](#), Workshop at CIRM, Luminy, France, May 9-13, 2006
- [Commutative Algebra and its Interaction with Algebraic Geometry](#), Workshop at CIRM, Luminy, France, May 22-26, 2006
- [Syzygies and Hilbert Functions](#), Banff International Research Meeting, Canada, October 14-19, 2006

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#### Past conferences:

- A [conference](#) on algebraic geometry to celebrate Robin Hartshorne's 60th birthday, Berkeley, August 28-30, 1998
- [Western Algebraic Geometry Seminar](#), MSRI, Berkeley, December 5-6, 1998
- [Conference on Groebner Bases, Guanajato](#), Mexico, February 8-12, 1999
- [The Pacific Northwest Geometry Seminar](#) 
- [Computational Commutative Algebra and Combinatorics](#), Osaka, July 21-30, 1999. 
- [Kommutative Algebra und Algebraische Geometrie](#), Oberwolfach, August 8-14, 1999. 
- [AMS Western Section Meeting](#) Salt Lake City, UT, September 25-26, 1999.
- [Algebra and Geometry of Points in Projective Space](#), Napoli, February 9-12, 2000.
- [AMS Spring Eastern Sectional Meeting](#) Lowell, MA, April 1-2, 2000.
- [Algèbre commutative et ses interactions avec la géométrie algébrique](#), Centre International de Rencontres Mathématiques, June 5-9, 2000.
- [Topics in Classical Algebraic Geometry](#), Oberwolfach, June 18-24, 2000 
- [AMS Fall Central Section Meeting](#) Toronto, Ontario Canada, September 22-24, 2000
- [AMS Fall Eastern Section Meeting](#), New York, Columbia U. in New York, November 4-5, 2000
- [Exterior algebra methods and other new directions in Algebraic Geometry, Commutative Algebra and Combinatorics](#), 8-15 September 2001, Ettore Majorana Centre, Erice, Sicily, Italy. [Photos](#) from the conference.
- [Classical Algebraic Geometry](#), Oberwolfach, May 26 - June 1, 2002 
- [Current trends in Commutative Algebra](#), Levico, Trento, June 17-21, 2002
- [Birational and Projective Geometry of Algebraic Varieties](#), Ferrara, September 2-8, 2002
- [Commutative Algebra, Singularities and Computer Algebra](#), Sinaia, September 17-22, 2002. [Photos](#) from the conference.
- [James H. Simons Conference on Quantum and Reversible Computation](#), Stony Brook, May 25-31, 2003

- [Conference on Commutative Algebra](#), Lisbon, June 23-27 2003. [Photos](#) from the conference. Also [photos](#) from Belém.
- [Commutative Algebra and Interactions with Algebraic Geometry and Combinatorics](#), ICTP, Trieste, June 6-11
- [III Iberoamerican Congress on Geometry](#), Salamanca, June 7-12
- [Projective Varieties: A Conference in honour of the 150<sup>th</sup> anniversary of the birth of G. Veronese](#), Siena, June 8-12 , 2004. [Photos](#) from the conference.
- [Algebraic Geometry: conference in honour of Joseph Le Potier & Christian Peskine](#), Paris, June 15-18, 2004
- [Classical Algebraic Geometry](#), Oberwolfach, June 27-July 3, 2004
- [Combinatorial Commutative Algebra](#), Oberwolfach, July 4-10th, 2004

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