MAT 539
Algebraic Topology

Instructor   Sorin Popescu   (office: Math 4-119, tel. 632-8358, e-mail sorin@math.sunysb.edu)

Time and Place   TuTh 09:50am-11:10am, Old Chem 135

Prerequisites

A basic introduction to geometry/topology, such as **MAT 530** and **MAT 531**. Thus prior exposure to basic point set topology, homotopy, fundamental group, covering spaces is assumed, as well as a reasonable acquaintance with differentiable manifolds and maps, differential forms, the Poincaré Lemma, integration and volume on manifolds, Stokes' Theorem. We will briefly review some of this material in the first week of classes.

Textbook

Differential forms in algebraic topology, by Raoul Bott and Loring W. Tu, GTM **82**, Springer Verlag 1982.

The guiding principle of the book is to use differential forms and in fact the de Rham theory of differential forms as a prototype of all cohomology thus enabling an easier access to the machineries of algebraic topology in the realm of smooth manifolds. The material is structured around four core sections: de Rham theory, the Cech-de Rham complex, spectral sequences, and characteristic classes, and includes also some applications to homotopy theory.

Other recommended texts:

- *Algebraic Topology: A first Course*, W. Fulton, GTM **153**, Springer Verlag 1995
- *Topology from the Differentiable Viewpoint*, J. Milnor, U. of Virginia Press 1965
- *Algebraic Topology*, A. Hatcher, Cambridge University Press 2002 (also available online at the author's web page)
- *Elements of Homotopy Theory*, G.W. Whitehead, GTM **61**, Springer Verlag 1995
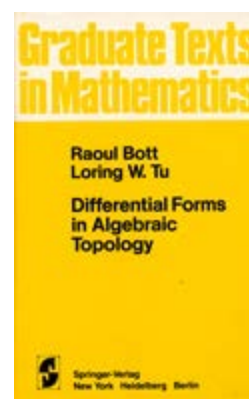
Course description

The book contains more material than can be resonably covered in a one-semester course. We will hopefully cover the following sections:

- **De Rham theory:** the de Rham complex, orientation and integration, Poincaré lemmas, the Mayer-Vietoris argument, Poincaré duality on an orientable manifold, Thom class and the Thom isomorphism (orientable vector bundle case)
- **The Cech-de Rham complex:** the generalized Mayer-Vietoris argument, sheaves and Cech cohomology, the de Rham theorem, sphere bundles, Euler class, the Hopf index theorem, the Thom isomorphism in general, monodromy
- **Spectral sequences**: basics, spectral sequence of a double complex, products, applications and some explicit computations
- **Homotopy theory:** homotopy groups, long homotopy sequence of a fibration, loop spaces, Eilenberg-MacLane spaces, the Hurewicz isomorphism, a few low dimensional homotopy groups of spheres (Hopf invariant, etc) (all of these, perhaps more only if time permits)

Homework & Exams

I will assign problems in each lecture, ranging in difficulty from routine to more challenging. Course grades will be based on these problems, class participation, and final exam.

Software

Here are some pointers to software that may be used to visualize topological objects:

- KnotPlot. Download binaries from the following site.
- Java View: a 3d geometry viewer written in Java. Among the demos you may find a Klein Bottle
- LiveGraphics3D: a Java applet to display and rotate three-dimensional graphics.
  Here used to display a version of the *Borromean Rings* (use your mouse to give them a "spin"):
- Geomview: another interactive 3D viewing program.
- JZB: a Java 3D viewer with MATHEMATICA export
- Xj3D: an open source VRML/X3D Toolkit

Links & 3D-models

History of topology:

- "Topology enters mathematics": a brief overview of the early developments (MacTutor History of Mathematics archive).
- A Brief History of Topology by E.C. Zeeman
- Stable algebraic topology 1945-1966, by J.P. May

Topological zoo:

- "The Topological Zoo" at the Geometry Center: a visual dictionary of surfaces and other mathematical objects.
- Images of "classical" topological "objects" from the Geometry Center.
- "A Knot Zoo". Here is another one. Or Hyperbolic knots. All these sites are part of an exciting collection of knots and links available at "The KnotPlot Site". Very instructive are also the VRML knot models.
- Raytraced images: Sphere, Torus, projective plane: a Crosscap, a Steiner surface, a Boy surface, and a genus 3 orientable surface.
- VRML models: a Möbius band, a Klein bottle and a Trefoil Knot. Download here a vrml viewer for Linux.
- David Eppstein's "Geometry Junkyard": a collection of pointers, clippings, research blurbs, and other stuffs related to discrete, computational geometry, and topology.
- Paul Bourke's collection of raytraced surfaces. Here is for instance the animation of a transition from a Steiner surface into a Boy surface.
- A picture of the Hopf fibration created by Ken Shoemake. Click here for a better quality TIFF version of the picture. The picture visualizes well the remarkable geometric fact that any two fibres (=circles) of the Hopf fibration are *linked*. Here is another page and an mpeg animation of the Hopf fibration (created with Knotplot).
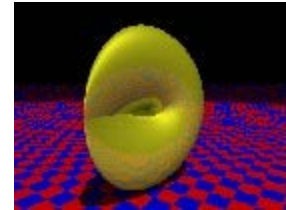
Art & Topology:

- "Symbolic Sculpture and Mathematics"
- "Mathematics & Knots Exhibition"
- Benno Artmann's Topological Models
- The Scherk-Collins Sculpture Generator: a program to generate Scherk-Collins towers and toroids (by Carlo H. Séquin)
- Helaman Ferguson's sculptures. For instance here is "Klein's modular quartic" which is on the patio of MSRI Berkeley. Or Alexander's horned sphere...
- More art links on Carlo H. Séquin's web site.
- Knots from the Alhambra de Grenada

Archives:

- The "Topology Atlas"
- The Hopf Topology Archive

- Rob Kirby's Problems in Low-Dimensional Topology (380 pages)
- An Algebraic Topology Discussion List

Fun:

- Torus and Klein Bottle Games: a collection of Java applets/games played on the surface of a torus or a Klein bottle (chess, tic-tac-toe, crossword puzzles, and more).
- Glass Klein Bottles!
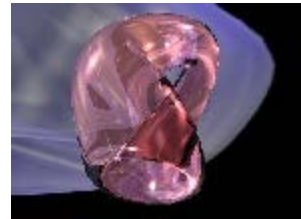
*Sorin Popescu*

2002-12-29

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This is one of the prime links pictured in Appendix C of Dale Rolfsen's book *Knots and Links*, Publish or Perish, Inc. Houston, Texas 1976, reprinted in 1990 ("a classic"). In the notation of that appendix it is

$$9^3_4$$

which means that it is the fourth link on that list, with 3 components and 9 crossings. In Conway's (1970) notation this link is

$$21, 2, 2, 2$$
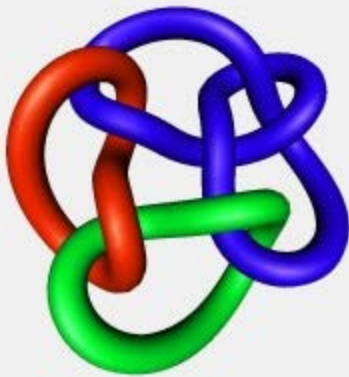
The picture was created with KnotPlot, a fairly elaborate program to visualize knots and links. The software is already installed on some of the linux machines in the department. Binaries can be downloaded from the following site.
Click here to read/download a PDF version of the KnotPlot manual.

An exciting collection of knots and links is available at The KnotPlot Site

Sorin Popescu

Department of Mathematics
Stony Brook University
Stony Brook, NY 11794-3651

email: sorin@math.sunysb.edu
Office: Math 3-109
Phone: (631)-632-8255
Fax: (631)-632-7631

Research Interests: Algebraic Geometry, Commutative Algebra, Combinatorics and Computational methods

Teaching:

Spring 2006   MAT 311 Number Theory   MAT 614 Topics in Algebraic Geometry

Previous years   Teaching Archive

Algebra, Geometry and Physics seminar: Spring 2006

Publications & E-Prints: Unless otherwise indicated, the files below are DVI files (🗎), PostScript files (🗎), PDF files (🗎), or tar gziped DVI and PostScript files (📕). Files marked as (📕) or (🔥) are hyperlinked PDF or Macromedia Flash files formated for screen viewing. Other formats (source, PS using Type I fonts) can be obtained via the UC Davis Front to the Mathematics ArXiv. Click on (🟢) or (⛰) for related *Macaulay2*, or *Macaulay* code.

Syzygies:

- *Gale Duality and Free Resolutions of Ideals of Points* [🗎], [🗎] [🗎] [🟢] [⛰], *Invent math* **136** (1999) 2, 419-449
  David Eisenbud and Sorin Popescu

- *The Projective Geometry of the Gale Transform* [🗎], [🗎] [🗎] [🟢], *J. Algebra* **230** (2000), no. 1, 127-173

  David Eisenbud and Sorin Popescu
  (in the D. Buchsbaum anniversary volume of *J. Algebra*)

- *Syzygy Ideals for Determinantal Ideals and the Syzygetic Castelnuovo Lemma* [🗎] [🗎], [**MathSci**],

Springer 1999
David Eisenbud and Sorin Popescu

- *Extremal Betti Numbers and Applications to Monomial Ideals* [▣] [▣] [▣] [▣], *J. Algebra* **221** (1999), no. 2, 497-512
  Dave Bayer, Hara Charalambous and Sorin Popescu

- *Lagrangian Subbundles and Codimension 3 Subcanonical Subschemes* [▣], [▣] [▣] [▣], *Duke Math. J.* **107** (2001), no. 3, 427-467
  David Eisenbud, Sorin Popescu and Charles Walter

- *Enriques Surfaces and other Nonpfaffian Codimension 3 Subcanonical Subschemes* [▣] [▣] [▣] [ƒ], *Comm. Algebra* **28** (2000), 5629-5653
  David Eisenbud, Sorin Popescu and Charles Walter
  (in the Hartshorne anniversary volume of *Comm. Algebra*)

- *Syzygies of Unimodular Lawrence Ideals* [▣] [▣] [▣] [▣], *J. Reine Angew. Math* **534** (2001), 169-186
  Dave Bayer, Sorin Popescu and Bernd Sturmfels

- *Hyperplane Arrangement Cohomology and Monomials in the Exterior Algebra* [▣] [▣] [▣] [▣] [▣], *Trans. AMS.* **355** (2003), 4365-4383
  David Eisenbud, Sorin Popescu and Sergey Yuzvinsky

- *Exterior algebra methods for the Minimal Resolution Conjecture* [▣] [▣] [▣] [▣], *Duke Math. J.* **112** (2002), no. 2, 379-395
  David Eisenbud, Frank-Olaf Schreyer, Sorin Popescu and Charles Walter

- *Symmetric resolutions of coherent sheaves* [▣] [▣] [▣]
  David Eisenbud, Sorin Popescu and Charles Walter

- *A note on the Intersection of Veronese Surfaces* [▣] [▣] [▣] [▣] [ƒ]
  David Eisenbud, Klaus Hulek and Sorin Popescu

- *Restricting linear syzygies: algebra and geometry* [▣] [▣] [▣] [▣] [ƒ], *Compositio Math.* **141** (2005), no.6, 1460-1478
  David Eisenbud, Mark Green, Klaus Hulek and Sorin Popescu

- *Small schemes and varieties of minimal degree* [▣] [▣] [▣] [▣] [ƒ], *Amer. J of Math* (2005), to appear
  David Eisenbud, Mark Green, Klaus Hulek and Sorin Popescu

Abelian varieties, modular varieties and equations:

- *Equations of (1,d)-polarized abelian surfaces* [▣] [▣] [▣], *Math. Ann.* **310** (1998), no. 2, 333-377
  Mark Gross and Sorin Popescu

- *The moduli space of (1,11)-polarized abelian surfaces is unirational* [▣] [▣] [▣], *Compositio Math.* **126** (2001), no. 1, 1-24
  Mark Gross and Sorin Popescu

- *Calabi-Yau threefolds and moduli of abelian surfaces I* [▣] [▣] [▣], *Compositio Math.* **127**, no. 2, (2001), 169-228
  Mark Gross and Sorin Popescu

[▣] [▣] [▣]

Calabi-Yau threefolds and moduli of abelian surfaces II [   ] [   ] [   ]
Mark Gross and Sorin Popescu

- *Elliptic functions and equations of modular curves* [📄] [📄] [📄] [⚡], *Math. Ann.* **321** (2001), no. 3, 553-568
  Lev A. Borisov, Paul Gunnells, and Sorin Popescu

Surfaces in P^4 and threefolds in P^5:

- *The Geometry of Bielliptic Surfaces in* P^4 [📄], [📄] [📄], *Internat. J. Math.* **4** (1993), no. 6, 873-902
  A. Aure, W. Decker, K. Hulek, S. Popescu and K. Ranestad
- *On Surfaces in* P^4 *and Threefolds in* P^5 [📄] [📄] [📄], [**MathSci**], LMSLN **208**, 69--100
  W. Decker and S. Popescu
- *Surfaces of degree* 10 *in* P^4 *via linear systems and linkage* [📄] [📄] [📄] [📄] [📄], *J. Algebraic Geom.* **5** (1996), no. 1, 13-76
  S. Popescu and K. Ranestad
- *Syzygies of Abelian and Bielliptic Surfaces in* P^4 [📄] [📄] [📄], *Internat. J. Math.* **8** (1997), no. 7, 849-919
  A. Aure, W. Decker, K. Hulek, S. Popescu and K. Ranestad
- *Examples of smooth non general type surfaces in* P^4 [📄] [📄] [📄] [📄] [📄], *Proc. London Math. Soc.* (3) **76** (1998), no. 2, 257-275
  S. Popescu
- *Surfaces of degree* >= 11 *in the Projective Fourspace* [📄] [📄] [📄]+ *Appendix* [📄] [📄] [📄]
  S. Popescu

PRAGMATIC 1997: A summer school in Catania, Sicily

- *Research Problems for the summer school* [📄], [📄] [📄], [**MathSci**], *Matematiche* (Catania) **53** (1998), 1-14
  David Eisenbud and Sorin Popescu

Algorithmic Algebra and Geometry: Summer Graduate Program (1998) at MSRI:

- Poster [📄] [📄], lecture slides and streaming video , CD ROM,
  Dave Bayer and Sorin Popescu

Linear algebra notes

- *On circulant matrices* [📄], [📄] [📄] [📄] [⚡],
  Daryl Geller, Irwin Kra, Sorin Popescu and Santiago Simanca

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Upcoming conferences:

- DARPA FunBio Mathematics-Biology Kick-off meeting, Princeton, September 21-23, 2005
- MAGIC 05: Midwest Algebra, Geometry and their Interactions Conference, University of Notre Dame, Notre Dame, October 7-11, 2005
- AMS Special Session on Resolutions, Eugene, OR, November 12-13, 2005
- Clay Workshop on Algebraic Statistics and Computational Biology, Clay Mathematics Institute, November 12-14, 2005
- CIMPA School on Commutative Algebra, December 26, 2005 - January 6, 2006, Hanoi, Vietnam
- AMS Special Session on Syzygies in Commutative Algebra and Geometry, San Antonio, TX, January 12-15, 2006
- KAIST Workshop on Projective Algebraic Geometry, January 23-25, 2006, Korean Advanced Institute of Science and Technology, Daejeon
- AMS Special Session on the Geometry of Groebner bases, San Francisco, CA, April 29-30, 2006
- Castenuovo-Mumford regularity and related topics, Workshop at CIRM, Luminy, France, May 9-13, 2006
- Commutative Algebra and its Interaction with Algebraic Geometry, Workshop at CIRM, Luminy, France, May 22-26, 2006
- Syzygies and Hilbert Functions, Banff International Research Meeting, Canada, October 14-19, 2006

Past conferences:

- A conference on alegbraic geometry to celebrate Robin Hartshorne's 60th birthday, Berkeley, August 28-30, 1998
- Western Algebraic Geometry Seminar, MSRI, Berkeley, December 5-6, 1998
- Conference on Groebner Bases, Guanajato, Mexico, February 8-12, 1999
- The Pacific Northwest Geometry Seminar
- Computational Commutative Algebra and Combinatorics, Osaka, July 21-30, 1999.
- Kommutative Algebra und Algebraische Geometrie, Oberwolfach, August 8-14, 1999.
- AMS Western Section Meeting Salt Lake City, UT, September 25-26, 1999.
- Algebra and Geometry of Points in Projective Space, Napoli, February 9-12, 2000.
- AMS Spring Eastern Sectional Meeting Lowell, MA, April 1-2, 2000.
- Algèbre commutative et ses interactions avec la géométrie algébrique, Centre International de Rencontres Mathématiques, June 5-9, 2000.
- Topics in Classical Algebraic Geometry, Oberwolfach, June 18-24, 2000
- AMS Fall Central Section Meeting Toronto, Ontario Canada, September 22-24, 2000
- AMS Fall Eastern Section Meeting, New York, Columbia U. in New York, November 4-5, 2000
- Exterior algebra methods and other new directions in Algebraic Geometry, Commutative Algebra and Combinatorics, 8-15 September 2001, Ettore Majorana Centre, Erice, Sicily, Italy. Photos from the conference.
- Classical Algebraic Geometry, Oberwolfach, May 26 - June 1, 2002
- Current trends in Commutative Algebra, Levico, Trento, June 17-21, 2002
- Birational and Projective Geometry of Algebraic Varieties, Ferrara, September 2-8, 2002
- Commutative Algebra, Singularities and Computer Algebra, Sinaia, September 17-22, 2002. Photos from the conference.
- James H. Simons Conference on Quantum and Reversible Computation , Stony Brook, May 25-31, 2003

- Conference on Commutative Algebra, Lisbon, June 23-27 2003. Photos from the conference. Also photos from Belém.
- Commutative Algebra and Interactions with Algebraic Geometry and Combinatorics, ICTP, Trieste, June 6-11
- III Iberoamerican Congress on Geometry, Salamanca, June 7-12
- Projective Varieties: A Conference in honour of the 150th anniversary of the birth of G. Veronese, Siena, June 8-12 , 2004. Photos from the conference.
- Algebraic Geometry: conference in honour of Joseph Le Potier & Christian Peskine, Paris, June 15-18, 2004
- Classical Algebraic Geometry, Oberwolfach, June 27-July 3, 2004
- Combinatorial Commutative Algebra, Oberwolfach, July 4-10th, 2004
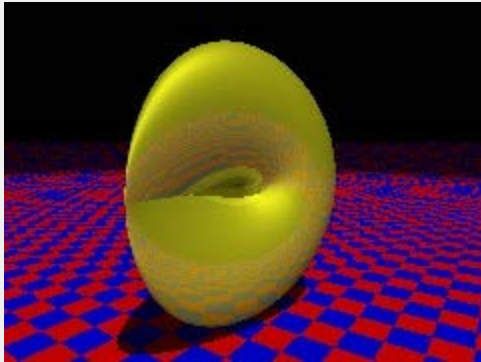

Last updated on 10 Dec 2003

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The Crosscap is an image of the real projective plane. It has a segment of double points, which terminates in two "pinch points", or "Whitney singularities". Click here to view a VRML 1.0 model. Use the mouse to navigate around, turn on/off the light sources, and so on...
Click here to see a table of immersions and embeddings of real projective spaces.
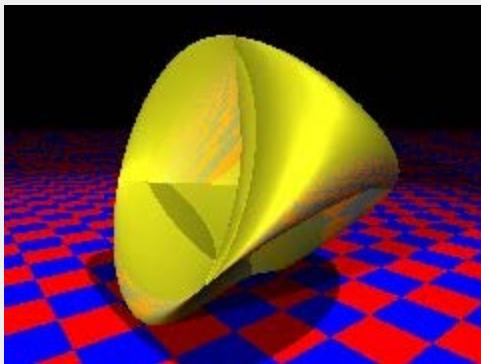
The Steiner surface is also an image of the real projective plane. It contains three segments of double points each of which terminates in two "pinch points", or "Whitney singularities". A triple point is created where the three double point segments intersect. Click here to view a VRML 1.0 model.
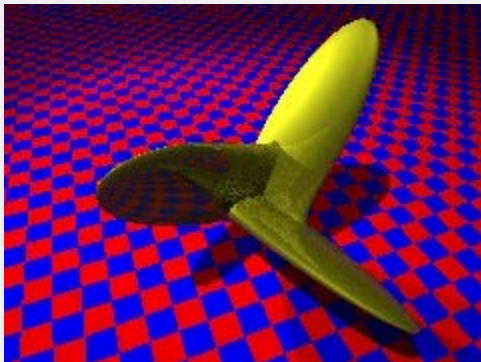
Boy's surface is also image of a map from the real projective plane to R^3. It contains a continuous double point curve, which meets itself in a triple point. Click here to view a VRML 1.0 model.

Here is another page with other pictures (some in the Geomview ".oogl" format) of the Boy surface. Here is an OOGL model of the Boy surface; open it in Geomview.

There is also a nice steel model in front of the library of the Mathematisches Forschungsinstitut Oberwolfach.

To view the above VRML files you will need to download first a VRML plug-in for your browser. Here are several choices:

1. You may try the older and unsupported Cosmo player, which works reasonably well with Internet Explorer and Netscape on older windows platforms, or
2. The newer WorldView, or
3. Better, you may install the Cortona plug-in.

All these plug-ins are freely available, but the last plug-in provides better graphics support on Windows XP/2000, or Mac OSX platforms.

Finally if on a Linux/Unix platform, download either vrmlview or the Java based browser vrwave. Alternatively here is a list of VRML viewers available for Linux platforms.
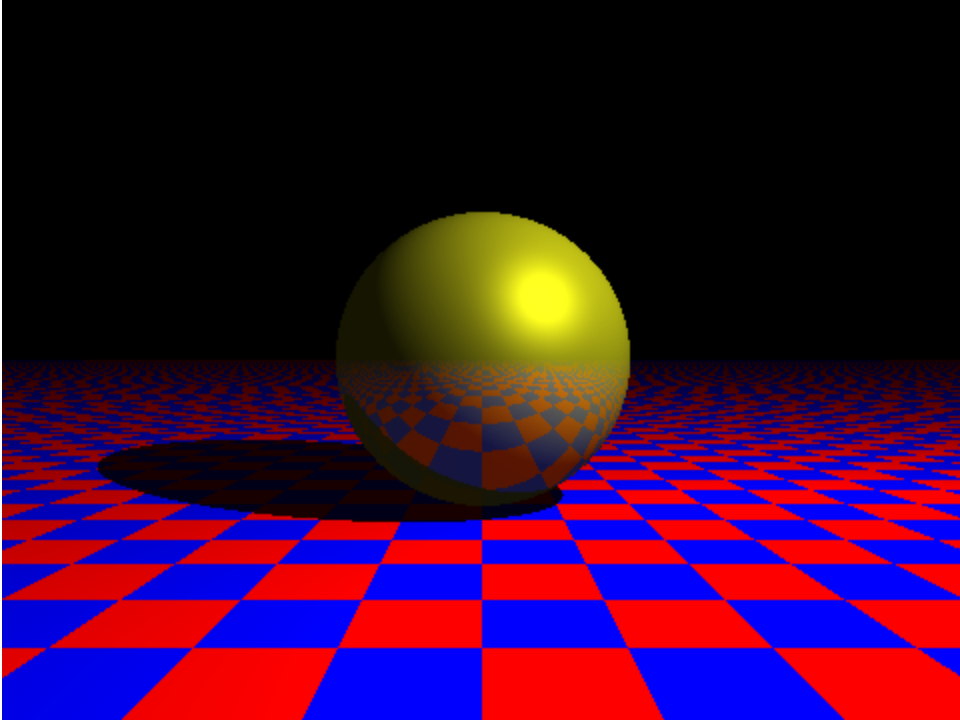
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This is a snapshot of a trefoil knot. Click here to view a VRML 1.0 version of the knot. Use the mouse to navigate around, turn on/off the light sources, and so on...
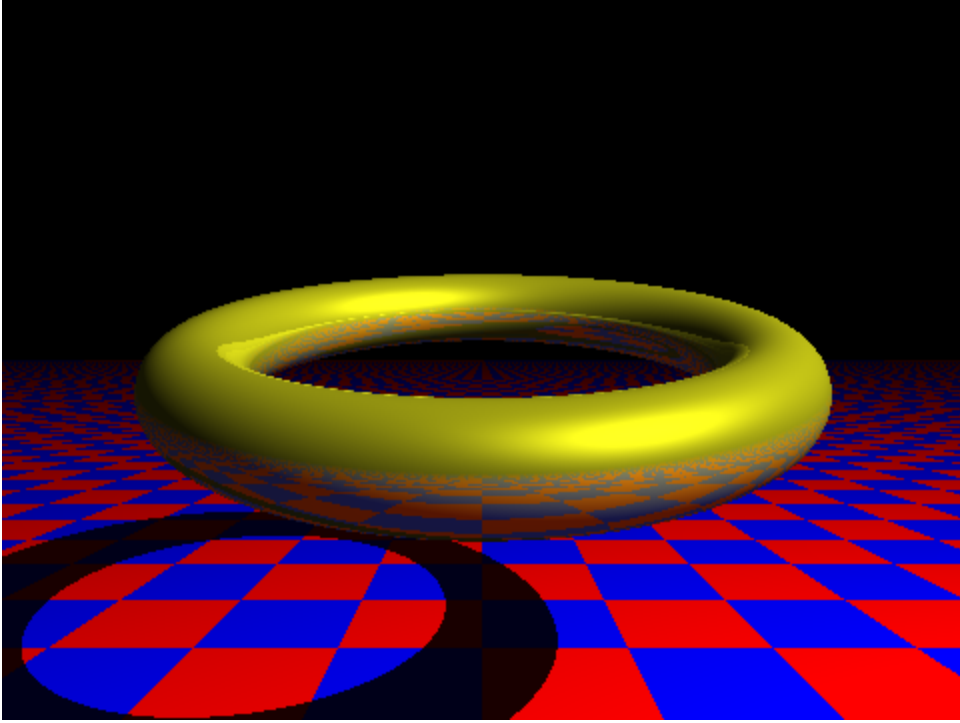
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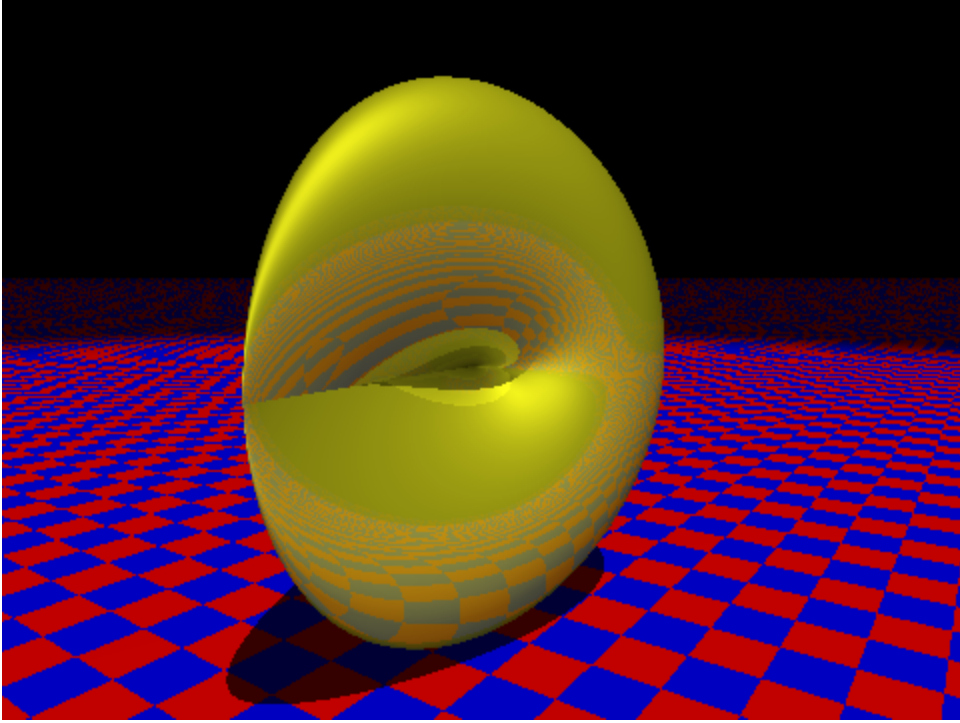
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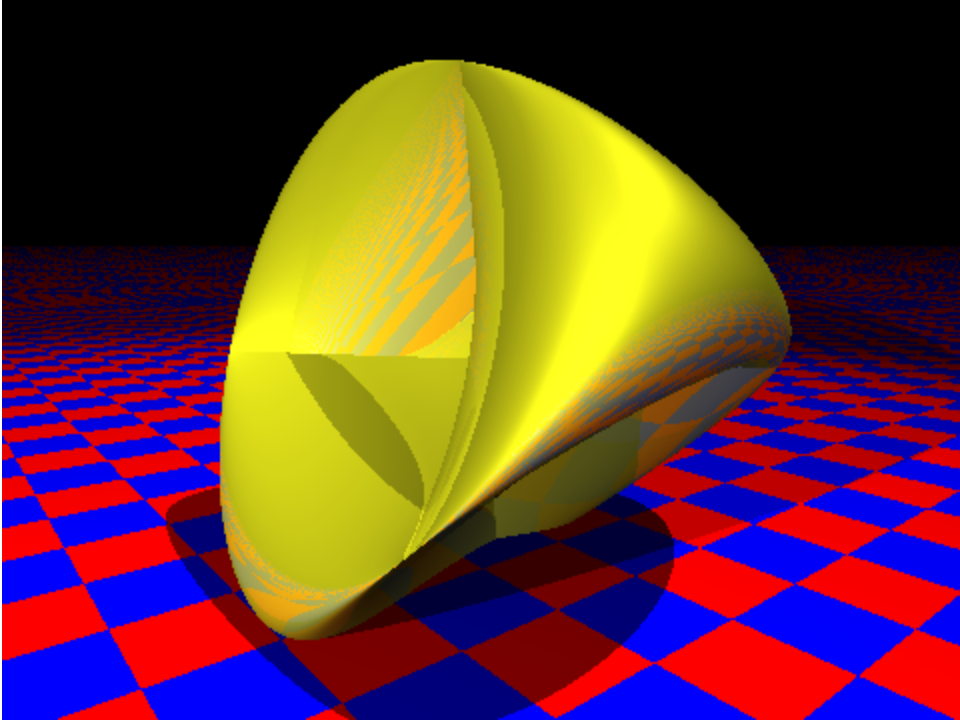3. Better, you may install the Cortona plug-in.

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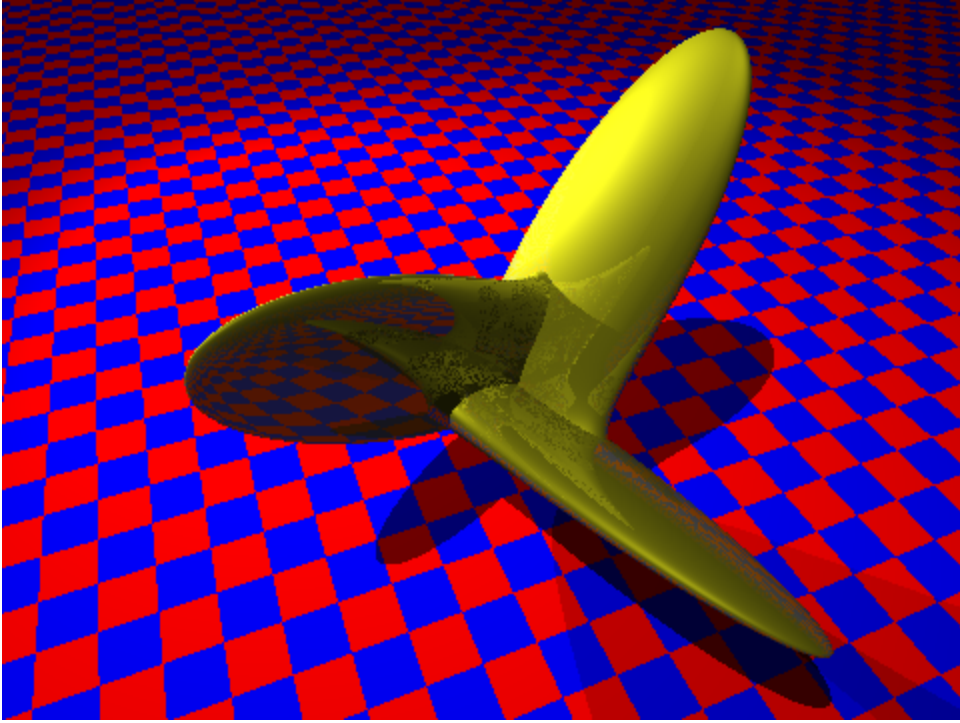
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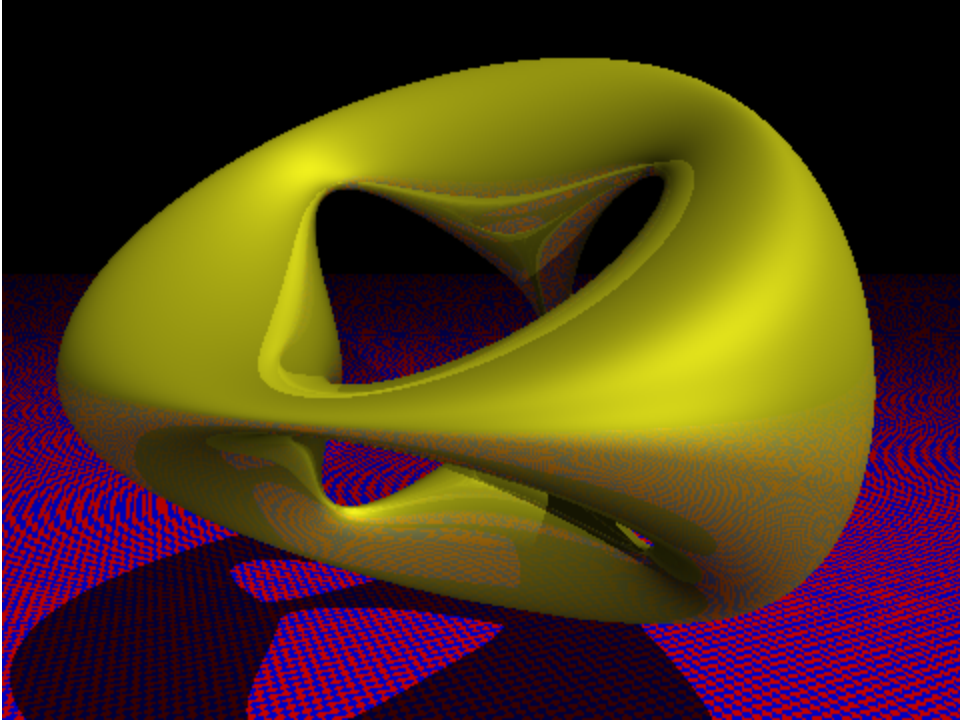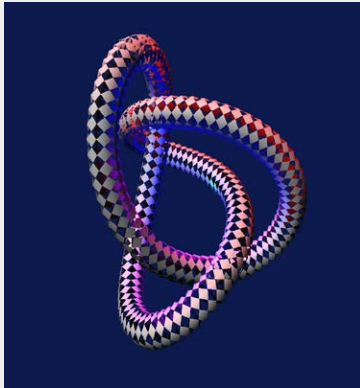
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This is a snapshot of a figure eight knot. Click here to view a VRML 1.0 version of the knot. Use the mouse to navigate around, turn on/off the light sources, and so on...
Here is another (real) picture of the figure eight knot borrowed from Tony Phillips' web page on "Knots and their polynomials".

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