WELCOME TO MAT 310 LinearAlgebra

Spring 2012

Time and place:

Lecture: MWF at 11:45 in the main library, room E4315

Recitations: M at 3:50PM in the main library, room N3085

Tu at 9:50AM in the main library, room N3074

Introduction: This is a mathematically rigorous course with complete proofs. Topics covered will include vector spaces, linear transformations, eigenvalues, and inner product spaces. Also the charateristic and minimal polynomial of a linear operator and the Jordan form if time permits.

Text Book: *Linear Algebra Done Right* (2nd edition) by Sheldon Axler, Springer, (c) 1997

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Office hours: Monday, Wednesday and Friday , 10:40am-11:40am or by appointment

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Office Hours: Tuesday 1:00-2:00pm

Homework: Homework will be assigned every week. Doing the homework is a *fundamental* part of the course work. Problems should be handed in in your recitation section.
1st assignment: page 19, problems 1,2,3,5,6,7,8 due the week of January 30.
2nd assignment: page 19, problems 9,11,13; page 35, problems 1,2,4,7,8,9,10 due the week of February 6.
3rd assignment: page 35, problems 11,12,13,15; page 59, problems 1,2,3,4,5,7 due the week of February 13

4th assignment: page 59, problems 9,13,15,16, 20,22,23,24,25,26 due the week of February 20
5th assignment: page 73, problems 1,2,4; page 94, problems 3,4,5,6,9,11,14 due the week of February 27
6th assignment: page 94, problems 15, 17, 19, 20, 21, 22, 23, 24; page 122, problems 2, 3 due the week of March 5
7th assignment: page 122, problems 4, 7, 8, 11, 13, 14, 21, 26, 27, 32 due the week of March 19
8th assignment: page 158, problems 1, 2, 4, 6, 7, 8, 11, 14, 16, 20 due the week of March 26
9th assignment: page 160, problems 21,22,23,24,26,27, 28,30,32,34 due the week of April 9
10th assignment: click on: 10th assignment due the week of April 23
11th assignment: page 188, problems 2, 3, 4, 5, 7, 10, 11, 13, 15, 18 due the week of April 30

Grading Policy: The overall numerical grade will be computed by the formula **20% Homework + 30 % Midterm Exam+ 50% Final Exam**

Midterm Exam: Wednesday, March 14, in class

MIDTERM EXAM REVIEW: Exam Review
Final Exam: Wednesday, May 9, 2:15PM-4:45PM in Javits 101
FINAL EXAM REVIEW: Exam Review

N. B. Use of calculators is not permitted in any of the examiniations.

Special Needs: If you have a physical, psychological, medical or learning disability that may impact on your ability to carry out assigned course work, I would urge you to contact the staff in the Disabled Student Services office (DDS), Room 113, humanities, 632-6748/TTD. DSS will review your concerns and determine, with you, what accommodations are necessary and appropriate. All information and documentation of disabilities is confidential.

10th Assignment

1. Find the (possibly complex) eigenvalues, eigenvectors and singular values of the matrix:

$$\begin{pmatrix} 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix}$$

2. Find an isometry and a positive operator whose product is the operator given by the matrix above.

3. Prove that the trace of a non-zero positive operator is positive. Give an example of a self-adjoint operator with positive trace that is not positive.

4. Prove that matrix multiplication is associative. You may use the fact that with appropriate bases, the matrix of the composition of two linear transformations is the product of the matrices of the transformations

5. Find a two by two matrix of complex numbers that does not have a square root. Can such a matrix have two distinct eigenvalues?

6. Let V be an n-dimensional vector space with basis $\{e_1, e_2, ..., e_n\}$. Define $T : V \to V$ by $T(e_i) = e_{i+1}$ for $i = 1...n-1$ and $T(e_n) = 0$. Prove that $T^n = 0$. Prove that T does not have a square root.

Review for Midterm

Description of real and complex numbers as fields. Comnplex conjugate

Definition of a vector space over the real or complex numbers.

Subspaces, sums and direct sums

Span, linear independence, bases and dimension.

If a space has n linearly independent vectors, then a spanning set must have at least n elements. It follows that any two bases have the same number of elements.

Linear maps, their null spaces and ranges. The rank plus nullity theorem.

The matrix of a linear map. Matrix multiplication and composition of linear maps.

Invertible maps and their matrices. Injective and surjective maps.

Given a linear map $T : V \to W$, find bases of V and W that make the matrix of T as simple as possible. It should have r ones and the rest zeros, where r is the rank of the map.

Polynomials with real or complex coefficients. The degree of a polynomial. The division algorithm for polynomials. The fundamental theorem of algebra (You needn't know the proof.)

Factoring a polynomial with real coefficients into linear and quadratic factors

Eigenvalues and their eigenvectors.

Invariant subspaces of a linear map from a vector space into itself. A one-dimensional invariant space contains an eigenvector.

A set of eigenvectors with distinct eigenvalues is linearly independent. Therefore there cannot be more eigenvalues than the dimension of the space.

If $T \in L(V)$, and $p(x)$ is a polynomial, define $p(T)$.

If V is a finite dimensional complex vector space of positive dimension, and $T \in L(V)$, then T has an eigenvalue. Also there is a basis of V for which the matrix of T has upper-triangular form. Also the diagonal elements of this matrix are the eigenvalues of T.

If T is in $L(V)$ and V is given a basis, then with respect to this basis, there is a matrix for T. How does the matrix change if one changes the basis?

If T is as above, and T has n distinct eigenvalues where n is the dimension of V, then one can choose a basis of V consisting of eigenvectors of T, and the matrix of V with respect to this basis has diagonal form.

If V is a real finite positive dimensional vector space, and $T \in L(V)$, then T must have either an eigenvector or a two-dimensional invariant supspace.

Give an example of such a T which has no eigenvalue or eigenvector.

Projections onto and along subspaces.

If V is real as above and odd-dimensional, and T is as above, then T has an eigenvector.

Definition of Inner Product for real and complex vector spaces. Dot product as an example.

Norm defined by inner product. $< v, w >= 0$ implies $||u||^2 + ||v||^2 = ||u+v||^2$

Prove the Schwartz inequality and triangle inequality.

Review for Final

Everything that is on the Midterm exam review sheet.

Definition of an inner product for real and complex vector spaces.

Schwartz inequality and its proof

Triangle inequality

Parallelogram equality

Norms and how an inner product gives rise to a norm

Orthogonal and orthonormal sets, orthonormal bases

Gram-Schmidt procedure to get an orthonormal set

If an operator from V to itself has a basis for which it's matrix is upper trianglular, then it has an orthonormal basis for which its matrix is upper triangular.

Subspaces and their perpendicular subspaces.

Orthogonal projections and the closest point in a subspace

Linear functionals

Let V be a finite dimensional vector space. Then for any linear functional $\phi : V \to \mathbf{F}$, there exists a vector $w \in V$ such that $\phi(v) =< v, w >$ for all $v \in V$. Know how to prove this and show by example that it may not be true if V is not finite dimensional.

Definition of the adjoint T^* of a linear transformation T.

The matrix of T^* with respect to orthonormal bases is the conjugate of the transpose of the matrix of T

Self-adjoint and normal operators

Self-adjoint operators have only real eigenvalues and they have an orthonormal basis of eigenvectors.

T is normal iff Tv and T^*v have the same norm for all vectors v.

If T is normal and λ is an eigenvalue of T, then $\bar{\lambda}$ is an eigenvalue of T^* with the same eigenvector.

Spectral theorem for normal operators on a complex vector space and for self-adjoint operators on a real vector space

Invariant subspaces of an operator and block diagonal matrices

Positive operators and their square roots

Isometries

Polar and Singular value decompositions

Traces and their properties