Math 307 - Fall 2015 - Course Information

Week of:	Monday	Wednesday	Homework (Due the following Wednesday unless stated otherwise).
8/24	1.1,1.2	1.3,1.4	p.7; 2,10,12 // p.16; 2,3,6,8,32 // p.23; 2,6,8,18,22 // p.31; 2,4,6,8,14,16,18
8/31	2.1,2.2	2.3-2.4	p.69; 4,6,8,11,15 // p.73; 2,4,6 // p.80; 7,8,9,14,15,16 // p.87; 8,9,16
9/7	Labor Day	2.5,1.6	p.98; 1,8,13,14,22 // p.42 3,4,8,21,22
9/14	1.6,4.1	4.1,4.2	p.182; 1,2,11,14,32 // p.187; 2,3,9,14 // p.192; 4,16 // p.198; 8
9/21	4.3	4.4	p.210; 1,2,4,11,22 // p215; 24 Due Monday October 5, in class.
9/28	ReviewProblems (click here)	Midterm 1	none
10/5	5.1	5.2-5.3	p.224; 4,6,8,20,21,35,36 // p.232; 1,2,4,14,18,19,20 (Due 10/14)
10/12	5.4	6.1	p.236; 1,2,4,15 // p.243 1,2,18 // p.258; 7,8,17,23,24,33 (Due 10/21)
10/19	6.2,6.3	6.4	p.270; 1,2,6 // p.281; 3,6,16 // p.292; 9,10,11 // p.298; 3,4,5,6 (Due 10/28)
10/26	6.4,7.1	7.2,7.3	p.321; 3,4,12 // p.332; 13,14,15 // p.337; 2,6,9,10
11/2	6.5,7.4	7.6,8.1	p.309; 11 // p.346; 12,13,21,22,24 // p.358; 3,4,8
11/9	8.2,8.3	8.4	p.376; 1,2,4,5 // p.382; 11,12 // p.385; 4,8 // p.394; 1,2,3,4 (Due 11/23, but is covered on exam 2).
11/16	Click For Review Problems	Midterm 2	
11/23	9.1,9.3	Thanksgiving	
11/30	Mathematica in Harriman 320	Last Class	p.408; 1,2 // p. 418; 3,4 //p. 429; 8a,9 // p.437; 6,9,10 // p.447 5,6. This HW is optional but the material will be on the final. If you hand it in I will replace your lowest HW score with your score on this assignment. If so, due at the final exam.
12/7			Final Exam: Tuesday Dec 8, 8:30-11 PM

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Office Hours: M 1:30-3:00; W: 11:00-12:30 or by appt.
Lecture: MW 4:00-5:20 in Library E4310
Recitation: M 5:30-6:23 in Library E4310
Recitation Instructor: Jean-Francois Arbour
Textbook: Multivariable Mathematics by Williamson and Trotter 4th ed.
We will cover chapters 1-9, except chapter 3.
Final Exam: Tuesday Dec 8, 8:30-11 PM
More course information is available on the syllabus.

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Comments

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1. Let $\vec{v} = (3,1,1), \vec{x} = (1,2,-3), \vec{w} = (2,-1,4)$ be vectors in $\mathbb{R}^3$.

(a) Calculate the following:

i. $e_1 \bullet \vec{v}$

ii. $\vec{v} \bullet \vec{x}$

iii. $(\vec{v} \times \vec{x}) \times \vec{w}$

iv. $\vec{v} \times (\vec{x} \times \vec{w})$

v. All angles between the three vectors.

(b) Are these 3 vectors linearly independent?

(c) Find a parametrization for the k-plane of solutions to $A\vec{x} = 0$, where A is the matrix whose rows are $\vec{v}$, $\vec{x}$ and $\vec{w}$.

2. What would you say if I asked the question 'is the dot product associative?' ?

3. Let $\vec{v}$ and $\vec{w}$ be vectors on a plane Γ in $\mathbb{R}^3$. Let $\vec{x}$ be a vector orthogonal to Γ. Prove that $\vec{v} \bullet \vec{x} = \vec{w} \bullet \vec{x}$.

4. Let A be the following matrix:

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

(a) Find A^{-1}.

(b) Calculate A^4.

(c) Can you give a geometric interpretation of your results above. Hint: Think of A as a function $\mathbb{R}^2 \to \mathbb{R}^2$.

5. Suppose forces acting on a particle move it in $\mathbb{R}^3$ along a trajectory parametrized by

$$f(t) = (\sqrt{t}, t^2, t)$$

for $0 < t < 4$. Suddenly at time $t = 4$ these forces cease, and the particle continues from this time maintaining a constant direction and velocity.

(a) What is the position of the particle at $t = 3$?

(b) What is the position of the particle at $t = 5$?

6. Let (a, b, c) be a point on the unit sphere in $\mathbb{R}^3$. Recall the spherical coordinate parametrization is:

$$f(u, v) = \begin{pmatrix} cos(u)sin(v) \\ sin(u)sin(v) \\ cos(v) \end{pmatrix}$$

(a) Use f to find a parametrization of the plane tangent to the sphere at (a, b, c). Hint: since (a, b, c) is a point on the sphere, there exists (r, t) such that $f(r, t) = (a, b, c)$.

(b) Show that if $\vec{v}$ is any vector on the tangent plane you found above, then $\vec{v} \bullet (a, b, c) = 1$.

(c) Give a geometric interpretation of this fact. (Draw a picture).

(d) Prove that the sphere has no singular points.

Name: ____________________

1. True or False

(a) Every continuous function is differentiable.

(b) Every differentiable function is continuous.

(c) Every integrable function is continuous.

(d) Every continuous function is integrable.

(e) If $f\colon \mathbb{R}^n \to \mathbb{R}^m$ is a differentiable function then for each vector $\vec{x}$, $f'(\vec{x})$ is an nxm matrix.

(f) If there exists a closed curve γ such that $\int_\gamma \vec{F} d\vec{x} = 0$, for some vector field $\vec{F}$ then $\vec{F}$ is the gradient of a function.

(g) If f is a smooth function, then $Curl(\nabla f) = 0$.

(h) If f is a smooth function, then $Div(\nabla f) = 0$.

2. Let $f(x, y) = x^2 + y - 4xy$.

(a) Find the direction of greatest increase of f at the point $(1, 1)$.

(b) Find the direction of greatest decrease of f at the point $(1, 1)$.

(c) Find the direction(s) of smallest change in f at the point $(1, 1)$.

3. Let $f(x, y) = (2ycos(x), 3x^2)$

(a) Find f'. Your answer will be a 2x2 matrix.

(b) Suppose f does not have a continuously differentiable inverse in a neighborhood of some point (a, b) in the interior of the rectangle $0 \leq x \leq 4, 0 \leq y \leq 4$. Find a.

4. Find and classify the critical points of $f(x, y) = sin(x)+sin(y)+sin(x+y)$ in the interior of the square region $0 \leq x \leq 4, 0 \leq y \leq 4$.

5. Find the maximum and minimum values of the function $f(x, y) = 3x^3 - xy^2 + 6y$ in the square region $0 \leq x \leq 4, 0 \leq y \leq 4$.

6. Which point(s) on the surface $xy + 3x + z^2 = 9$ are closest to the origin?

7. Compute $\int_B xyz\ dV$ where B is the solid cylinder of height 2 and radius 1 whose bottom circle is in the xy-plane centered at the origin.

8. Show that the vector field $\vec{F}(x, y) = (x+y, 1)$ is not the gradient of a function by finding two curves with the same end points but different line integrals.

I also recommend using the homework for chapter 8 to review that material.

1. Write the vector $(-1, 3, 2)$ as a linear combination of the vectors $(1, 1, 0), (1, 0, 1)$ and $(1, 0, -1)$. Draw a picture if you feel so bold.

2. Find three vectors such that the previous question would have no answer if they were substituted in.

3. Give a geometric description of vector subtraction.

4. Prove that $|r||\vec{v}| = |r\vec{v}|$.

5. Draw a picture in $\mathbb{R}^2$ that 'proves' vector addition is associative.

1. Calculate the dot product of $(2, 3, 4, 5)$ and $(-1, 2, 3, -3)$. Are these vectors orthogonal?

2. Using the dot product, find some vectors which are at a 45 degree angle to $(0, 0, 1)$ in $\mathbb{R}^3$ and whose height (z coordinate) is 1. What shape does the set of all such vectors form?

3. Prove that two distinct parallel lines do not intersect.

4. Find a parametrization for the plane P with the following two properties: all vectors on P are perpendicular to the vector $(1, 2, 1)$ and the plane P contains the vector $(-1, 3, 2)$.

1. Find a vector which is orthogonal to the plane containing $(1, 1, 2)$ and $(0, 0, 1)$.

2. Find a pair of orthogonal vectors in the plane containing $(1, 1, 2)$ and $(0, 0, 1)$.

3. Does the inverse of the following matrix exist? How do you know?

$$\begin{bmatrix} 2 & 0 & 1 \\ 3 & -1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

4. Use the inverse matrix of A to solve the system $A\vec{x} = \vec{b}$ where A is as above and $\vec{b} = (1, 2, -2)$

5. Find the determinant of the following matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

1. Make a presentable sketch of the parametrized curve $f\colon \mathbb{R} \to \mathbb{R}^3$ given by

$$f(t) = (t, tsin(\pi t), tcos(\pi t))$$

2. Find the derivative of the above function.

3. Find the length of the curve $y = x^2$ on the interval $[0, 1]$.

4. Use level sets to graph the functions $z = 4x^2 - y^2$ and $z = 3x - 2y$.

5. Show that for the above functions (setting $z = g(x, y)$) that $g_{xy} = g_{yx}$.

Consider the function $f\colon \mathbb{R}^3 \to \mathbb{R}^3$:

$$f(r,\phi,\theta) = \begin{pmatrix} rcos(\theta)sin(\phi) \\ rsin(\theta)sin(\phi) \\ rcos(\phi) \end{pmatrix}$$

which we'll restrict to the domain: $0 \le r \le R$ $0 < \phi < \pi$ and $0 \le \theta < 2\pi$.

1. Describe the level sets of f.

2. What size will the Jacobian matrix $f'(\vec{x})$ be?

3. Find the Jacobian matrix.

4. Find the determinant of the Jacobian matrix. Simplify using trig. identities.

5. How do you know f is differentiable?

6. Integrate the determinant with respect to r, then ϕ, then θ, over the specified domain.

7. What does this number represent?

This problem asks you to find the absolute maximum and minimum of the intersection of the plane $x + y + z = 1$ and the sphere $x^2 + y^2 + z^2 = 1$.

1. First describe the plane by a function $f(x, y) = z$ of two variables and substitute this expression to describe the itersection as a level set $g(x, y) = 0$.

2. Use the method of Lagrange multipliers to find the critical points of the function. In other words find the critical points of $\mathcal{L} = f - \lambda g$.

3. Verify that the points you found live on both the plane and the sphere. Evaluate to find the maximum and minimum z values.

4. Draw a picture of the intersection of these two objects with the extrema labeled.

5. Is the intersection a circle? If so determine its radius.

1. Find the volume under the plane $x + 2y + 3z = 3$ and over the first quadrant of the xy-plane.

2. Let $g(x, y) = x^2 + sin(x) + yx$. Compute

$$\frac{d}{dy}\int_0^{\pi} g(x, y)dx$$

in two ways: directly and using the Leibniz rule.

3. Let B be the subset of $\mathbb{R}^2$ consisting of those vectors whose length is less than 10. Define a function $f : B \to \mathbb{R}$ by

$$f(x, y) = \begin{cases} 2 & \text{if } x \text{ is an integer} \\ 3 & \text{if } x \text{ is not an integer} \end{cases}$$

Is f integrable? If not, why not? If so, compute $\int_B f dA$.

Consider the vector field $\vec{F}(x, y) = (x^2 + y^2, rxy)$, where r is some unknown real number.

1. Calculate the scalar curl of $\vec{F}$.

2. Suppose $\vec{F}$ is path independent. Find r.

3. Fix r to the value you found above. Let γ be the curve consisting of the portion of the y-axis with $|y| \leq 1$ attached to the right half of the unit circle. Test Green's theorem by calculating the line integral of $\vec{F}$ around the curve γ in two ways:

   (a) Using a parametrization of γ.

   (b) Using an iterated integral in polar coordinates.

   Which way was easier?