MAT 200: Logic,Language and Proof Fall 2013

Lectures:

Lecture	Time	Location	Instructor	Office hours
L01	MW 4:00pm-5:20pm,	Frey Hall 112	Tanya Firsova `tanya AT math.sunysb.edu`	MTh 2:30-3:30pm, 3-121, Math Tower

Grader: Jorge Contreras

General Course Information: Syllabus

Schedule: Schedule

Homeworks:

Homework 1 is due Monday, September 9 on the lecture. Homework 1

Homework 2 is due Monday, September 16 on the lecture. Homework 2

Homework 3 is due Wednesday, September 25 on the lecture. Homework 3

Homework 4 is due Wednesday, October 9 on the lecture. Homework 4

Homework 5 is due Wednesday, October 16 on the lecture. Homework 5

Homework 6 is due Wednesday, October 23 on the lecture. Homework 6

Homework 7 is due Wednesday, October 30 on the lecture. Homework 7

Homework 8 is due Wednesday, November 6 on the lecture. Homework 8

Homework 9 is due Monday, December 2 on the lecture. Homework 9

Midterms: There will be two midterms:

Midterm I: Monday, 9/30, 4-5:20pm

Midterm I will be in Harriman Hall room 116. The first Midterm covers Chapters 1-7. Practice problems for midterm I

Midterm II: Wednesday, 11/13, 4-5:20pm

Room TBA. Midterm II will cover Chapters 8-14 Practice problems for midterm II

Final: The final exam will be December 10, 8:30pm-11:00pm in Frey Hall, 105 (PLEASE NOTE THE CHANGE OF ROOM)

Practice problems for the final

Textbook: An introduction to Mathematical Reasoning, by Peter E. Eccles, 13th edition.

Special Needs

If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact Disability Support Services at (631) 632-6748 or online. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their instructors and Disability Support Services. For procedures and information go to the following website.

Academic Integrity

Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instances of academic dishonesty to the Academic Judiciary. Faculty in the Health Sciences Center (School of Health Technology & Management, Nursing, Social Welfare, Dental Medicine) and School of Medicine are required to follow their school-specific procedures. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website .

MAT 200: Logic, Language and Proof

1 About the course

The goal of the course is to introduce the student to mathematical reasoning and proofs. This course serves as an introduction to rigorous mathematics used in upper-division mathematics courses, but the rigorous reasoning skills are also generally useful in life, whatever your pursuits.

2 Course web page

http://www.math.sunysb.edu/ tanya/mat200-fall13/index.html

3 Textbook

Peter J.Eccles, "An Introduction to Mathematical Reasoning". Reading the chapters of the book before the lecture will greatly help you.

4 Homework

There will be homework assigned each week and collected during the Monday lecture. The list of problems will be posted on the course webpage.

5 Grading Scheme

There will be two midterms and a final.

What	Where	% of the Final Grade
Midterm 1	TBA	20%
Midterm 2	TBA	20%
Final	TBA	40%
Homeworks		20%

6 Office hours

Monday, Thursday 2:30-3:30pm, and by appointment

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MAT 200 Schedule

Date	Material Covered	Chapters
Monday, 8/26	The Language of Mathematics, Implications	Chapters 1, 2
Wednesday 8/28	Proofs, Proofs by Contradiction	Chapters 3, 4
Monday 9/2	**NO CLASSES, Labour day**	
Wednesday 9/4	Contrapositive Statements, The Induction Principle	Chapters 4,5
Monday 9/9	The Induction Principle continued	Chapters 5
Wednesday 9/11	finishing The Induction Principle	Chapter 5
Monday 9/16	The Language of Set Theory, Venn Diagrams	Chapter 6
Wednesday 9/18	Venn Diagrams, Quantifies	Chapters 6,7
Monday 9/23	Functions, Injections, Surjections, Bijections	Chapters 8,9
Wednesday 9/25	Review	
Monday 9/30	**Midterm I**	
Wednesday 10/2	Counting	Chapter 10
Monday 10/7	Properties of Finite Sets, Counting Functions and Subsets	Chapter 11,12
Wednesday 10/9	Number Systems	Chapter 13
Monday 10/14	Counting Infinite Sets	Chapter 14
Wednesday 10/16	Counting Infinite Sets	Chapter 14
Monday 10/21	Arithmetic, the Division Theorem	Chapter 15
Wednesday 10/23	The Euclidean Algorithm	Chapter 16
Monday 10/28	The Euclidean Algorithm	Chapter 16
Wednesday 10/30	Review	
Monday 11/4	Midterm II	

Homework 1:

1. Problem 1.1
2. Problem 1.2
3. Which of the statements below is a contrapositive to the statement "If you do not work hard, you fail the course":
   (a) If you work hard, you do not fail the course.
   (b) If you failed the course, you did not work hard.
   (c) If you did not fail the course, you worked hard.
4. Find a contrapositive to the statement "If an animal is a crocodile, it is green"
5. Problem 1.5
6. Problem 1.11

Homework 2:

1. A triangle is *obtuse* if one of its angles is bigger than $90°$. Which of the following conditions are sufficient for the triangle to be obtuse?

   (a) One of the angles is $120°$.

   (b) One angle is bigger than sum of the other two angles.

   (c) There are exactly two acute angles in the triangle.

   (d) The cosine of one angle in the triangle is negative.

   (e) Two angles in the triangle are less than $30°$.

   Which of the conditions are necessary? Give explanations.

2. Problem I.7 (page 54)

3. Prove by induction

$$1 + \frac{1}{2} + \cdots + \frac{1}{2^n} = 2 - \frac{1}{2^n}.$$

4. Problem I.12 (page 54)

5. Problem I.14 (page 55)

6. Problem I.15 (page 55)

7. Problem I.16 (page 55)

8. Draw n lines on a plane. We say that a point is a point of intersection if there are at least two lines that pass through the point. Assume that all points of intersection are points of double intersection (there are exactly two lines that pass through a point). Prove by induction that the number of points of intersection is $\frac{(n-1)n}{2}$.

   Hint: $1 + 2 + \cdots + (n-1) = \frac{(n-1)n}{2}$.

Homework 3.

1. Recall that the power set $P(A)$ is the set of all subsets of the set A. Let $A = \{1, 4, 9, 16\}$. List all elements in the set $P(A)$.

2. Prove that $\{x \in \mathbb{Z} | -2.1 < x < 0\} = \{x \in \mathbb{R} | \ x^2 + 3x + 2 = 0\}$

3. Let A, B, C be sets. Prove that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

   Draw the Venn diagram to illustrate the proof.

4. Let A, B be sets. Prove that

$$A \cap (A \cup B) = A;$$

$$(A - B) \cup B = A \cup B$$

5. Let $(a, \infty) = \{x \in \mathbb{R} | \ x > a\}$. Prove that

$$(a, \infty) \subset (b, \infty) \Leftrightarrow a > b.$$

6. Prove that $(A \cap B)^c = A^c \cup B^c$

7. Take the crocodiles to be the universal set. Let $A = \{\text{amible crocodiles}\}$, $B = \{\text{hungry crocodiles}\}$. Each of the statements below is equivalent to some of the sets $A \cap B$, $A \cap B^c$, $A^c \cap B$, $A^c \cap B^c$ being empty or not empty. Read the statements are draw the conclusions:

   (a) No crocodiles are amiable when hungry.

   (b) Some crocodiles, when not hungry, are amiable; but some are not.

   (c) No crocodiles are amiable, and some are hungry.

   (d) All crocodiles, when not hungry, are amiable; and all unamiable crocodiles are hungry.

   (e) Some hungry crocodiles are amiable, and some that are not hungry are unamiable.

8. There are 19 cats and dogs in the room. 5 of them are hungry, the rest are not. There are 4 cats that are not hungry and 10 dogs. Show that all dogs are not hungry.

9. Give a proof or counterexample to the statement:

   (a) $\forall x \in \mathbb{R}\ \exists y,\ \ x + y > 0;$

   (b) $\forall x \in \mathbb{R}\ \exists y,\ \ xy > 0$

Homework 4.

1. Let $X = \{a, b, c\}$, $Y = \{1, 2\}$. Show that the following subset $\{(a, 2), (b, 1), (c, 1)\} \subset X \times Y$ defines a function $f : X \to Y$. Show that f is surjective, but not injective. Does there exist a injective function $g : X \to Y$? Justify your answer.

2. Let $A = \{(n, m) \in \mathbb{Z} \times \mathbb{Z} |\ mn = 12\}$. List all elements in A. Find a subset $B \subseteq \mathbb{Z}$, so that $A \subset B \times \mathbb{Z}$, and A defines a function from B to $\mathbb{Z}$.

3. Determine which of the following functions $f_i : \mathbb{R} \to \mathbb{R}$ are injections, bijection or surjections. Find an inverse function of each of the bijections:

   (a) $f_1 = x^3$

   (b) $f_2 = x^3 + x$

   (c) $f_3 = e^x$

   (d) $f_4 = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

4. Let $\mathbb{Z}^+$ be a subset of positive integer numbers. Which of the following subsets of $\mathbb{Z}^+ \times \mathbb{Z}^+$ define a function? For subsets that define a function, check where the function is an injection, surjection, bijection or neither. Justify your answer.

   (a) $\{(n, m) \in \mathbb{Z}^+ \times \mathbb{Z}^+ |\ m = n - 1\}$

   (b) $\{(n, m) \in \mathbb{Z}^+ \times \mathbb{Z}^+ |\ m = n + 1\}$

   (c) $\{(n, m) \in \mathbb{Z}^+ \times \mathbb{Z}^+ |\ m = 2n\}$

   (d) $\{(n, m) \in \mathbb{Z}^+ \times \mathbb{Z}^+ |\ m = n^3\}$.

Homework 5.

1. Construct a function $f : \mathbb{Z} \to \mathbb{Z}$, such that
   (a) f is an injection, but not a surjection;
   (b) f is a surjection, but not an injection.
2. Construct a bijection between the sets $\mathbb{Z}^+$ and $\mathbb{Z}$.
3. Prove that the sequence $f(n) = n$ does not have a limit.
4. Guess what is the limit of the sequence $\frac{1}{n^2}+1$. Prove your answer using the definition of the limit.
5. Let $f : X \to Y$ be a function. We say that $g : Y \to X$ is a right inverse of f if $f \circ g = Id_Y$ and $g : Y \to X$ is a left inverse of f if $g \circ f = Id_X$.
   (a) Prove that f has a right inverse if and only if f is a surjection.
   (b) Prove that f has a left inverse if and only if f is an injection.
   (c) Construct an example of a function $f : \mathbb{R} \to \mathbb{R}$ that has a right inverse, but it is not unique.
   (d) Construct an example of a function $f : \mathbb{R} \to \mathbb{R}$ that has a left inverse, but it is not unique.
6. (a) Take a set $V = \{v_1, \dots, v_n\}$, and a subset E of a set of distinct pairs of elements in V. The elements in E are referred to as edges. The data (V, E) defines **a graph**. You can think of a graph as a set of points in the plane, where some pairs of points are connected by paths. Let $n(v)$ be a number of edges that have v as a vertex. Prove that for two vertices v_1 and v_2, $n(v_1) = n(v_2)$.
   (b) Prove that for any group of people in the room, there are two people who have the same number of friends.

   Hint: Use pigeonhole principle.

Homework 6.

1. Let $A = \{a, b\}$, $B = \{1, 2, 3\}$. Count the number of functions $f : A \to B$. List them all.

2. Let A, B be finite sets. Assume that A contains n elements, B contains m elements. Count the number of functions $f : A \to B$.

3. Let $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = n^3$. Construct a function $g : \mathbb{Z} \to \mathbb{Z}$, such that $g(f(n)) = n$.

4. Let $f(x) : \mathbb{R} \to \mathbb{R}$, $f(x) = \arctan(x)$. Draw the graph of f. What is the range of f? Find a function $g : \mathbb{R} \to \mathbb{R}$, such that $g(f(x)) = x$. Give two examples of such functions.

5. Let $G = (V, E)$ be a graph, such that V is the set of vertices, E is the set of edges. Let $(v_1 v_2)$ be an edge that connects vertices v_1 and v_2. A path is a sequence of edges, so that any adjacent edges have a common vertex. A cycle is a path that starts and ends at the same vertex. Draw an example of a connected graph with 5 vertices with a cycle. Draw an example of a connected graph with 5 vertices without a cycle. Count the number of edges in each graph.

6. A coach chooses a team of 6 out of 12 possible players. Find the number of different teams she can form. Give an explanation for the answer.

Homework 7.

1. Let X be a set, fix a subset $R \subset X \times X$. It defines a relation on X, $x \sim y$ if $(x, y) \in R$. The relation is an equivalence relation if the following three conditions are satisfied:

   (a) $x \sim x$;

   (b) If $x \sim y$, then $y \sim x$;

   (c) If $x \sim y$, $y \sim z$, then $x \sim z$.

   Assume that we have an equivalence relation on X. Let $S_x = \{y \in X : y \sim x\}$ be the equivalence class of the element x. Prove that for any pair $x, y \in X$, one of the following holds:

   (a) $S_x \cap S_y = \emptyset$;

   (b) $S_x = S_y$.

   So, the set X splits into the set of equivalence classes.

2. (a) Consider a relation on $\mathbb{Z}$, $n \sim m$, if $n - m$ is even. Show that this is the equivalence relation. Give a description of the equivalence classes. How many equivalence classes are there?

   (b) Consider a relation on $\mathbb{Z}$, $n \sim m$, if $n - m$ is odd. Show that the relation is not an equivalence relation.

3. Consider a relation on the set of triangles: triangles T_1 and T_2 are equivalent if they have the same angles. Show that the relation is an equivalence relation.

4. Represent the number $1.23\overline{45}$ as a fraction in the lowest terms.

5. Find the decimal expression for $\frac{7}{13}$.

Homework 9.

1. Let r be the remainder after the division of a by b. Prove that

$$gcd(a, b) = gcd(b, r).$$

2. Use Euclidean algorithm to find
   (a) $gcd(165, 252)$;
   (b) $gcd(4284, 3480)$.

3. Use Euclidean algorithm to find a solution of the equation $13x + 35y = 1$.

4. Use Euclidean algorithm to find a solution of the equation $16x + 27y = 1$.

5. Use Euclidean algorithm to solve congruence equations
   (a) $11x = 1 (\text{mod } 27)$;
   (b) $11x = 5 (\text{mod } 27)$.

6. (**BONUS:**) Prove that if ab is divisible by m, and $gcd(a, m) = 1$, then b is divisible by m.

   Hint: Use Euclidean algorithm.

Practice problems.

1. Prove using the truth table that

   (P and Q) implies (P or Q).

2. Let A be a set of triangles. Are the following conditions equivalent?

   (a) If a triangle in A is not isosceles, then it has a right angle.

   (b) Each triangle in A is either isosceles, or right, or both.

3. Formulate a contrapositive to the statements:

   (a) If berries are not red, they are not yummy.

   (b) All bears that had berries for breakfast are happy.

4. Let A, B, C be sets. Prove that $(A - B) \cap C = A \cap C - B \cap C$. Draw Venn diagram to illustrate the proof.

5. Find all positive integers n, such that $2^n > n(n+1)$. Justify your answer using mathematical induction.

6. Are the following statements true or false? Give a proof.

   (a) $\forall n \in \mathbb{Z}$, $\exists m \in \mathbb{Z}$, nm is even;

   (b) $\exists n \in \mathbb{Z}$, $\forall m \in \mathbb{Z}$, nm is even;

   (c) $\exists n \in \mathbb{Z}$, $\forall m \in \mathbb{Z}$, nm is odd.

7. Let $A \subset \mathbb{R}$. Find the negation of the statement: $\forall x \in A$, $x > 2$.

8. Which of the following conditions are necessary for n to be an integer? Which are sufficient? Justify your answer.

   (a) $n + 1 \in \mathbb{Z}$;

   (b) $n = 3k$, where $k \in \mathbb{Z}$;

   (c) n^2 is an integer;

   (d) $n = 5$.

Practice problems.

1. Let A and B be sets, check whether the subset $S \subset A \times B$ defines a function $f : A \to B$ if:

   (a) $A = \mathbb{Z}^+$, $B = \mathbb{Z}^+$, $S = \{(n, m) \in \mathbb{Z}^+ \times \mathbb{Z}^+ : n = m + 1\}$;

   (b) $A = \{a, b, c\}$, $B = \{1, 2, 3\}$, $S = \{(a, 1), (b, 3), (c, 3), (a, 2)\}$;

   (c) $A = \mathbb{R}$, $B = \mathbb{R}$, $S = \{(x, y) : x^2 = y + 1\}$

   Justify your answer.

2. Let $A = \{a, b, c\}$, $f : A \to A$, such that

   $$f(a) = b, f(b) = c, f(c) = c.$$

   Find

   (a) $f^{\circ 2}$;

   (b) $f^{\circ 3}$;

   (c) $f^{\circ n}$.

3. (a) Let $A = \{a, b, c\}$, $B = \{1, 2\}$. Draw diagrams for all maps $f : A \to B$, that are surjections.

4. Let S be a set of all positive even integer numbers. What is the cardinality of S? Justify your answer.

5. Let S be a set of positive integers less than 1000.

   (a) Find the cardinality of a subset $A \subset S$ that consists of numbers divisible by 3.

   (b) Find the cardinality of a subset $B \subset S$ that consists of numbers divisible by 15.

   (c) Find the cardinality of a subset $C \subset S$ that consists of numbers divisible by 3, but not divisible by 5.

6. Among 18 students in a room, 7 study mathematics, 10 study science, and 10 study computer programming. Also, 3 study mathematics and science, 4 study mathematics and computer programming, and 5 study science and computer programming. We know that 1 student studies all three subjects. How many of these students study none of the three subjects?

7. A teacher needs to split the class of 20 students into two teams of 10 students each. How many ways he can do it?

8. Consider a set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. We say that two numbers $n, m \in S$ are equivalent $n \sim m$, if $n - m$ is divisible by 3. Check that we defined an equivalence relation. Find the equivalence class of 4. Find all the equivalence classes.

9. (a) Let X, Y be sets. Give a definition of an injective map $f : X \to Y$.

   (b) Assume that X, Y are finite sets of the same cardinality. Prove that f is an injection if and only if f is a bijection.

10. Let $f : \mathbb{R} \to \mathbb{R}_{\geq 0}$ be a function such that $f(x) = x^2$. Find a function $g : \mathbb{R}_{\geq 0} \to \mathbb{R}$, $f \circ g = \mathrm{Id}$.

11. Write the number $1.11\overline{22}$ as a fraction in lowest terms.

12. Write the number $\frac{11}{12}$ in the decimal form.

Practice problems.

1. Consider a universe of crocodiles. For each crocodile in the universe we can say if it is hungry or not. Each crocodile is either amiable or not amiable. And all the crocodiles are unicolorous, they are either green or red.

   Assume that the following statements are true:

   (a) All amiable crocodile are hungry.

   (b) When a crocodile is not hungry and not amiable, it is green.

   (c) There are no crocodile that are hungry and red.

   Prove that all crocodiles are green.

2. Find two statements that are contrapositive to each other.

   (a) If the weather is not good, we do not go out.

   (b) If the weather is good, we go out.

   (c) If we go out, the weather is good.

3. Formulate a negation to the statement:

   (a) If the square of an integer number is divisible by 6, then the number is divisible by 18.

   (b) $\forall m \in \mathbb{Z}$, $\exists n \in \mathbb{Z}$, $mn \in \mathbb{Z}$.

   (c) $\exists m \in \mathbb{Z}$, $\forall n \in \mathbb{Z}$, mn is even.

   (d) Any 3 lines on a plane separate the plane into 7 regions.

4. Check whether the following statements are true:

   (a) $\forall n \in \mathbb{Z}_5$, $\exists m \in \mathbb{Z}_5$, $mn \equiv 2(\text{mod}5)$

   (b) $\forall n \in \mathbb{Q}$, $\forall m \in \mathbb{Q}$, $mn \in \mathbb{Q}$

   (c) $\exists n \in \mathbb{Q}$, $\forall m \in \mathbb{Q}$, $mn \in \mathbb{Z}$

   (d) $\exists n \in \mathbb{Z}_{10}$, $\forall m \in \mathbb{Z}_{10}$, $mn \not\equiv 1(\text{mod}10)$

5. Prove using truth tables that $P \Rightarrow (Q or R)$ is equivalent to $(P \Rightarrow Q) or (P \Rightarrow R)$.

6. Let $G = (V, E)$, $\tilde{G} = (\tilde{V}, \tilde{E})$ be two graphs, where V, $\tilde{V}$ are the sets of vertices; E, $\tilde{E}$ are the sets of edges. We say that G and $\tilde{G}$ are equivalent if there exists a bijection between V and $\tilde{V}$, such that v_1 and v_2 are connected by an edge in E if and only if their images are connected by an edge in $\tilde{E}$.

   (a) Prove that the defined above realation is an equivalence relation on graphs.

   (b) Draw two non-equivalent trees with 5 vertices.

   (c) Draw two non-equivalent graphs with the following set of indices $2, 2, 2, 2, 2, 2$. (*Hint:* The graphs are not necessarily connected.)

   (d) Show that there is only one equivalence class of graphs with the set of indices $2, 2, 2$.

7. The function $f : \mathbb{Z} \to \mathbb{Z}$ induces a well-defined function $\tilde{f} : \mathbb{Z}_n \to \mathbb{Z}_m$, if for any $a \equiv b (\text{mod n})$, $f(a) \equiv f(b) (\text{mod m})$. Check whether the following functions induce well-defined functions and list all the values of the induced functions $\tilde{f}$

   (a) $f = x^2$, $m = n = 3$;

   (b) $f = 2x$, $n = 5$, $m = 10$;

   (c) $f = x$, $n = 5$, $m = 3$.

8. Construct a left inverse for the function $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = 2n$.

9. Let $A = \{a, b, c, d\}$. Construct an example of a function $f : A \to A$, such that $f \neq Id$, $f^2 = Id$. Can you construct f that is not a bijection? Justify your answer.

10. Construct an example of a function $f : \mathbb{R} \to \mathbb{Z}$, such that f is a surjection. Can you construct an example of a bijection? Justify your answer.

11. Find all integer solutions of the equation $930m + 131n = 1$.

12. Find the last digit of 389^{389}.

13. Prove that for any $n \in \mathbb{Z}$, the number $n^{101} - n$ is divisible by 3.

14. Solve the congruence equation $44x \equiv 5(\mathrm{mod} 21)$.

15. Represent a number $1.25\overline{67}$ as a fraction.

16. Find the cardinality of the set of even integer numbers not divisible by 4.

17. Draw a connected graph with 6 vertices that has two vertices of index 5.

18. (a) Prove that among any 14 integer numbers, there are 2 that belong to the same congruence class mod13.

    (b) Prove that among any 100 integer numbers, there are 8 that belong to the same congruence class mod13.

19. Prove by induction that $n! > 3n^2$ for $n \geq 5$.

20. Find all positive integer numbers such that $2^{n+3} > 3^n$.

21. Find a coefficient of ab in the expansion of $(a + b + 2)^{10}$.

22. Let A be the set of the first n integer numbers, B be the set of the first m integer numbers. Find the number of injections from A to B. List all injections if $n = 2$, $m = 4$.

23. Find the number of positive integers less than 2000 that are divisible by 7, but not divisible by 5 and not divisible by 3.

24. Let A be a set of cardinality n. Find the number of 3-element subsets of A.