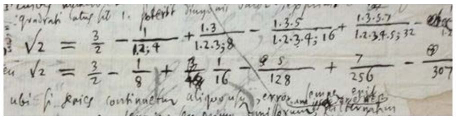
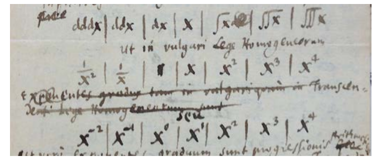


MAT 131
Calculus I

Summer Session I 2014



MAT 131 is the first course in the 2-semester single variable calculus sequence. It covers limits, continuous functions, derivatives and their applications, antiderivatives and the fundamental theorem of calculus.

The course moves rather quickly. Students who would like to learn the same material at a somewhat slower pace should take MAT 125. The three-semester sequence MAT 125-126-127 covers the same material as the two-semester sequence MAT 131-132.

It is the student's responsability to check this page frequently for changes and updates. Changes will be announced in class and, if appropriate, on the web page. Students are responsible for announcements made in class and/or on the web-page. Academic Calendar.

Tentative WEEK-BY-WEEK SYLLABUS. (Subject to Change)

Week	Sections	Homework
Week 1	May 28: 1.1, 1.2, APPENDIX C. May 29: section 1.5. May 30: sections 1.6, 2.1.	**1.1:** 12, 26, 30, 46, 56 **1.2:** 13 **App. C:** 8, 12, 34, 40 **1.5:** 16, 18, 22 **1.6:** 11, 12, 26, 52, 56 **2.1:** 2, 8
Week 2	June 2: sections 2.2, 2.3. June 4: sections 2.4, 2.5. June 5: Midterm I (solutions here)	**2.2:** 5, 7, 8, 14, 16 **2.3:** 1, 4, 10, 18, 38 **2.4:** 12, 13, 18, 32, 36 **2.5:** 12, 20, 26, 36, 40
Week 3	June 9: sections 2.6, 2.7, 2.8. June 11: sections 3.1, 3.2, 3.3. June 12: sections 3.4, 3.5, 3.6	**2.6:** 11, 16, 24, 46 **2.7:** 4, 10, 26, 50 **2.8:** 2, 12, 18, 28 **3.1:** 10, 16, 30, 42 **3.2:** 2, 10, 12, 32 **3.3:** 10, 14, 22, 34
Week 4	June 16: sections 3.7, 3.9. June 18: sections 4.1, 4.2. June 19: Midterm II (solutions here)	**3.4:** 24, 34, 36, 44 **3.5:** 12, 18, 26 **3.6:** 24, 28, 32 **3.7:** 12, 16, 42 **3.9:** 16, 21, 28, 30 **4.1:** 8, 12, 32 **4.2:** 16, 22, 52, 54
Week 5	June 23: sections 4.3, 4.5. June 25: 4.6, 4.8. June 26: 5.1, 5.2.	**4.3:** 6, 8, 14, 30, 38 **4.5:** 8, 12, 22, 54 **4.6:** 18, 22, 23, 26, 27 **4.8:** 10, 16, 26, 30
Week 6	June 30: 5.3. July 2: 5.4, 5.5. July 3: Final Exam.	**5.1:** 2, 4, 18 **5.2:** 4, 10, 18, 24 **5.3:** 6, 10, 12, 16, 20, 28 **5.4:** 8, 14, 16, 20 **5.5:** 18, 22, 34, 44, 54

Office Hours
M 1-2pm in 2-115

WTh 1-2pm in S-240 (MLC)

Bring your Stony Brook ID to all exams.

No: books, notes, calculators, cell phones, etc.

You can practice for each exam with the exams posted on the webpage for MAT 131 Fall 2011. . This will give you an **approximate** idea of the exam content, length and difficulty. However, 1) the sections covered by the 2011 exams may differ a bit from the sections covered by our exams and 2) the actual problems may be quite different.

Important: there will be **no make-ups for missed exams**, unless an acceptable and documented reason is given. In such situation the corresponding grade will be dropped in computing your course grade.

We will be following ***Single Variable Calculus (Stony Brook Edition 4)***, by **James Stewart**. This edition is essentially the same as *Single Variable Calculus: Concepts and Contexts, 4th ed.* There are many purchasing options.

A course numerical grade will be calculated according to the following rule: Midterm I = 25%, Midterm II =25%, Final = 30%, Homework = 20%. The numerical grade will be converted to a final letter grade only **after** the final test has been graded.

In order to do well in this class you are strongly encouraged to: read the section to be covered before class, do the homework, and start preparing for tests well in advance. You may find it useful to visit the Math Learning Center. You are specially encouraged to attend my office hours.

Should you have any questions, please do not hesitate to contact me at **claudio.meneses@stonybrook.edu**

- **Disability support services (DSS) statement**:
  If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact Disability Support Services (631) 632-6748 or http://studentaffairs.stonybrook.edu/dss/. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential. Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to the following website: http://www.stonybrook.edu/ehs/fire/disabilities/asp.
- **Academic integrity statement:**
  Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at http://www.stonybrook.edu/uaa/academicjudiciary/ .
- **Critical incident management:**
  Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, and/or inhibits students' ability to learn.

1.- Find the domain of the following functions. Show all your work and justify your answers:

(a) (*5 points*): $f(x) = \dfrac{2x}{x^4 - 1}$.

This is a rational function, undefined only if $x^4 - 1 = 0$

$x^4 - 1 = (x^2 - 1)(x^2 + 1)$; $x^2 + 1 \geq 1$ always.

$x^2 - 1 = 0 \iff x = 1$ or -1.

Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(b) (*10 points*): $g(t) = \sqrt{4 - t^2} - \sqrt{t^2 - 1}$.

$\sqrt{4 - t^2}$ is defined if $4 - t^2 \geq 0 \iff 4 \geq t^2 \iff -2 \leq t \leq 2$

$\sqrt{t^2 - 1}$ is defined if $t^2 - 1 \geq 0 \iff t^2 \geq 1 \iff t \geq 1$ or $t \leq -1$

Domain: $[-2, -1] \cup [1, 2]$

(c) (*10 points*): $h(x) = \ln(\sin(x))$, (Hint: consider first the interval $[0, 2\pi]$).

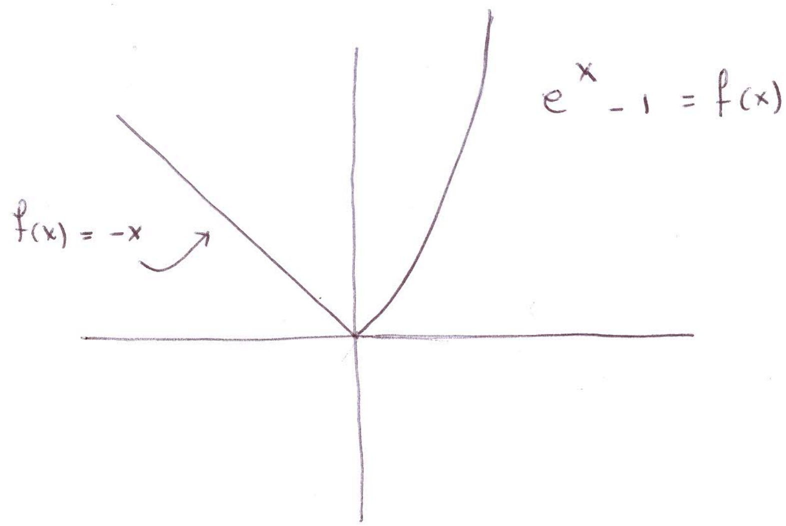
$\ln(\sin(x))$ is defined as long as $\sin(x) > 0$

This happen in $[0, 2\pi]$ when $0 < x < \pi$.

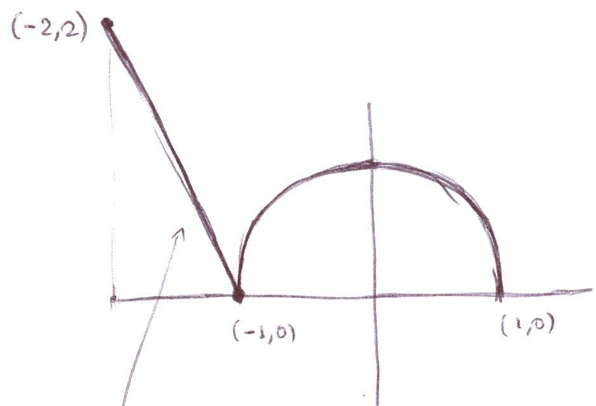
In general, by the periodicity of $\sin(x)$, the domain will be the union of al intervals of the form

$(2\pi k, 2\pi k + \pi)$, k integer.

2.- (a) (*10 points*): Sketch the graph of the function $f(x) = \begin{cases} -x & \text{if } x < 0 \\ e^x - 1 & \text{if } x \geq 0 \end{cases}$



(b) (*20 points*): Find an expression for the function whose graph consists of the line segment from the point $(-2, 2)$ to the point $(-1, 0)$ together with the top half of the circle with center the origin and radius 1.



$$f(x) = \begin{cases} -2x - 2 & -2 \leq x < -1 \\ \sqrt{1 - x^2} & -1 \leq x \leq 1 \end{cases}$$

slope $= \dfrac{0-2}{-1-(-2)} = \dfrac{-2}{1} = -2$

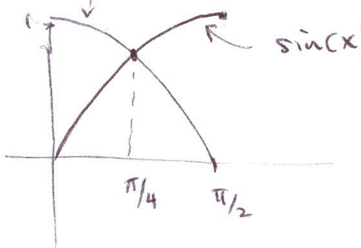
y-intercept: $y = -2$

$L(x) = -2x - 2$

3.- (a) (*10 points*): Determine for which values of x in the interval $[0, \pi/2]$ the inequality

$$\cos(x) < \sin(x)$$

holds.



From the graphs of sin(x) and cos(x), we can conclude that the inequality is satisfied if

$$\boxed{\frac{\pi}{4} < x \leq \frac{\pi}{2}}$$

(b) (*10 points*): If $\sin(x) = 1/3$ and $\sec(y) = 5/4$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$. Do not approximate.

$$\sec(y) = 5/4 \Leftrightarrow \cos(y) = \frac{4}{5}$$

Since $0 \leq x, y \leq \pi/2$, both $\sin(x)$, $\cos(x)$, $\sin(y)$, $\cos(y)$ are positive

so $$\cos(x) = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\sin(y) = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$$

$$= \left(\frac{1}{3}\right)\left(\frac{4}{5}\right) + \left(\frac{3}{5}\right)\left(\frac{2\sqrt{2}}{3}\right) = \boxed{\frac{4 + 6\sqrt{2}}{15}}$$

4.- Let

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } \ x < 0 \\ 3 - x & \text{if } \ 0 \leq x < 3 \\ (x-3)^2 & \text{if } \ x > 3 \end{cases}$$

(a) (*10 points*) Evaluate each limit, if it exists: (i) $\lim_{x\to 0^+} f(x)$, (ii) $\lim_{x\to 0} f(x)$, (iii) $\lim_{x\to 3^-} f(x)$.

(i) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (3-x) = 3$

(ii) Since $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sqrt{-x} = \sqrt{0} \neq 3$, $\lim_{x\to 0} f(x)$ does not exist.

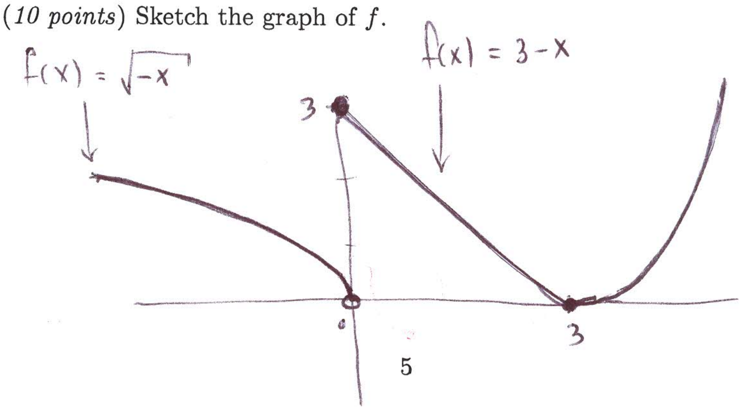
(iii) $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (3-x) = 0.$

(b) (*10 points*) Where is f discontinuous? Justify your answer.

Since $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x-3)^2 = 0$,

we see that $f(x)$ is discontinuous only at $x=0$.

(c) (*10 points*) Sketch the graph of f.



5.- Evaluate the limit, if it exists.

(a) (*10 points*): $\lim_{x \to 16} \frac{4-\sqrt{x}}{16x - x^2}$ (undefined at $x=16$)

$$\lim_{x \to 16} \frac{4-\sqrt{x}}{16x-x^2} = \lim_{x \to 16} \frac{4-\sqrt{x}}{x(4-\sqrt{x})(4+\sqrt{x})}$$

$$= \lim_{x \to 16} \frac{1}{x(4+\sqrt{x})} = \frac{1}{16(4+\sqrt{16})}$$

$$= \boxed{\frac{1}{128}}$$

(b) (*10 points*): $\lim_{x \to \pi^-} \ln(\sin(x))$

since $\lim_{x \to \pi^-} \sin(x) = \sin(\pi) = 0$, $\boxed{\text{but } \sin(x) > 0 \text{ when } 0 < x < \pi,}$

we conclude that

$$\lim_{x \to \pi^-} \ln(\sin(x)) = \boxed{-\infty}$$

(c) (*10 points*): $\lim_{x \to 1} \left(\frac{1}{x-1} + \frac{1}{x^2 - 3x + 2} \right)$

$$\lim_{x \to 1} \left(\frac{1}{x-1} + \frac{1}{(x^2-3x+2)} \right) = \lim_{x \to 1} \left(\frac{1}{(x-1)} + \frac{1}{(x-1)(x-2)} \right)$$

$$= \lim_{x \to 1} \left(\frac{1}{x-1} \right) \left(1 + \frac{1}{(x-2)} \right)$$

since $\lim_{x \to 1} \left(1 + \frac{1}{x-2}\right)$ 6. $(1-1) = 0$, we need a trick:

$$\lim_{x \to 1} \left(\frac{1}{x-1} + \frac{1}{(x-1)(x-2)} \right) = \lim_{x \to 1} \left(\frac{(x-1)(x-2) + (x-1)}{(x-1)^2(x-2)} \right) = \lim_{x \to 1} \left(\frac{(x-1)^2}{(x-1)^2(x-2)} \right) = \lim_{x \to 1} \left(\frac{1}{x-2} \right)$$

$$= \boxed{-1}$$

1.- Determine the following limits. Justify all your work.

(a) (*10 points*): $\lim_{h\to 0} \dfrac{(h-1)^3+1}{h}$.

This is the derivative of $f(x) = x^3$ at $x_0 = -1$. Therefore, $f'(x) = 3x^2$ and $f'(-1) = 3(-1)^2 \boxed{= 3.}$

We can also calculate

$$\lim_{h\to 0} \frac{(h-1)^3+1}{h} = \lim_{h\to 0} \frac{(h^3-3h^2+3h-1)+1}{h} = \lim_{h\to 0} \frac{h^3-3h^2+3h}{h} = \lim_{h\to 0} (h^2-3h+3)$$

$$\boxed{= 3}$$

(b) (*10 points*): $\lim_{x\to\infty} \left(\sqrt{x^2+4x+1} - x\right)$.

Notice that $\left(\sqrt{x^2+4x+1} - x\right) = \left(\sqrt{x^2+4x+1} - x\right) \dfrac{\left(\sqrt{x^2+4x+1} + x\right)}{\left(\sqrt{x^2+4x+1} + x\right)}$

$$= \frac{(x^2+4x+1) - x^2}{\sqrt{x^2+4x+1} + x} = \frac{4x+1}{\sqrt{x^2+4x+1} + x}$$

Therefore

$$\lim_{x\to\infty} \left(\sqrt{x^2+4x+1} - x\right) = \lim_{x\to\infty} \frac{4x+1}{\sqrt{x^2+4x+1} + x} = \lim_{x\to\infty} \frac{4+\frac{1}{x}}{\sqrt{1+\frac{4}{x}+\frac{1}{x^2}} + 1}$$

$$= \frac{4}{1+1} = \frac{4}{2} = \boxed{2}$$

3.- (a) (*5 points*): Find the derivative of $g(x) = \dfrac{x}{x^2+1}$

Quotient rule:

$$g'(x) = \frac{(x^2+1)\cdot 1 - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(1+x^2)^2}$$

Note: $g'(x) = 0 \iff 1-x^2 = 0 \iff x = \pm 1$.

(b) (*15 points*) There is a function $f(x)$ satisfying $f(0) = 0$ and whose derivative is $f'(x) = \dfrac{x}{x^2+1}$. Use this information and $f''(x)$ to draw a graph of the function $f(x)$.

We know: $f'(x) = \dfrac{x}{x^2+1}$

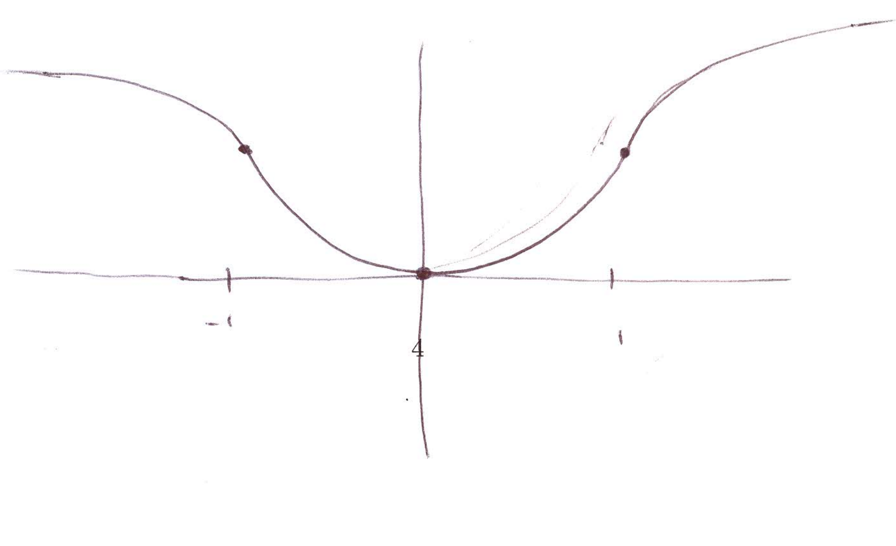
$$f''(x) = \frac{1-x^2}{(1+x^2)^2}$$

$$\left.\begin{array}{l} f'(x) > 0 \text{ if } x > 0 \\ f'(x) < 0 \text{ if } x < 0 \end{array}\right\} \left(\text{since } x^2+1 > 0\right)$$

Therefore, $f(x)$ is increasing on $(0,\infty)$, decreasing on $(-\infty, 0)$

$f''(x) > 0$ if $1-x^2 > 0 \iff -1 < x < 1$,

So f is concave up in $(-1,1)$, concave down on $(-\infty,-1)\cup(1,\infty)$



4.- Calculate the derivatives of the following functions. Simplify as much as possible.

(a) (*10 points*): $f(x) = e^{-x}\cos(x)$ Product rule

$$f'(x) = \left(e^{-x}\right)'\cos(x) + e^{-x}\left(\cos(x)\right)'$$

$$= -e^{-x}\cos(x) + e^{-x}\left(-\sin(x)\right)$$

$$= \boxed{-e^{-x}\left(\cos(x)+\sin(x)\right)}$$

(b) (*10 points*): $f(x) = \dfrac{\sin(x)}{\arcsin(x)}$ Quotient rule

$$f'(x) = \frac{\arcsin(x)\left(\sin(x)\right)' - \sin(x)\cdot\left(\arcsin(x)\right)'}{\left(\arcsin(x)\right)^2}$$

$$= \frac{\arcsin(x)\cdot\cos(x) - \sin(x)\cdot\left(\frac{1}{\sqrt{1-x^2}}\right)}{\left(\arcsin(x)\right)^2}$$

(c) (*10 points*): $f(x) = \arctan\left(e^{x^2}+1\right)$ Chain rule

$$f'(x) = \left(\frac{1}{1+\left(e^{x^2}+1\right)^2}\right)\cdot\left(e^{x^2}+1\right)'$$

$$= \frac{1}{1+\left(e^{x^2}+1\right)^2}\cdot e^{x^2}\cdot 2x = \frac{2xe^{x^2}}{1+\left(e^{x^2}+1\right)^2}$$

5.- (*20 points*): Consider the curve given by the equation $x^2\cos(y)+\sin(2y) = xy$. Find y' (this should be an expression in terms of x, y).

$$\frac{d}{dx}\left(x^2\cos(y)+\sin(2y)\right) = \frac{d}{dx}(xy) = 1\cdot y + x\frac{dy}{dx}$$

$$\parallel$$

$$2x\cos(y) + x^2\left(-\sin(y)\cdot\frac{dy}{dx}\right) + \cos(2y)\left(2\frac{dy}{dx}\right) = y + x\frac{dy}{dx}$$

Therefore

$$x\frac{dy}{dx} + x^2\sin(y)\frac{dy}{dx} - 2\cos(2y)\frac{dy}{dx} = -y + 2x\cos(y)$$

or

$$\frac{dy}{dx}\left(x + x^2\sin(y) - 2\cos(2y)\right) = y - 2x\cos(y)$$

$$\boxed{\frac{dy}{dx} = \frac{-y + 2x\cos(y)}{x + x^2\sin(y) - 2\cos(2y)}}$$

6.- (*20 points*): Find the linearization of $\sqrt[3]{x-1}$ at $x_0 = 9$. Use it to estimate $\sqrt[3]{7.99}$.

$f(x) = \sqrt[3]{x-1}$, $\quad f(9) = \sqrt[3]{9-1} = \sqrt[3]{8} = 2.$

Since $f'(x) = \frac{1}{3}(x-1)^{-2/3}$, $\quad f'(9) = \frac{1}{3}(9-1)^{-2/3} = \dfrac{1}{3(\sqrt[3]{8})^2}$

$= \dfrac{1}{3(2)^2} = \dfrac{1}{12}$

The linearization of $f(x)$ is $\boxed{L(x) = 2 + \frac{1}{12}(x-9)}$

Notice that $\sqrt[3]{7.99} = \cancel{f}\sqrt[3]{8.99-1} = f(8.99)$

Since $f(8.99) \approx L(8.99) = 2 + \frac{1}{12}(8.99-9) = 2 - \dfrac{0.01}{12}$

$\sqrt[3]{7.99} \approx \boxed{2 - \dfrac{1}{1200}}$