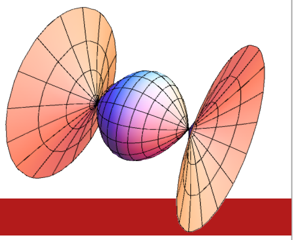
MAT 126: Calculus B
Spring 2016



Schedule & Homework

Home Course Information Schedule & Homework

Schedule

This is our tentative weekly schedule and it will be updated as we advance in the semester, please check regularly. Students are expected to attend class regularly and to keep up with the material presented in the lecture and the assigned reading. As a general rule, the written homework assignment is due in your recitation the following week (for example, HW1 is due Week 2). For the online HW assignment, please check on **WebAssign**.

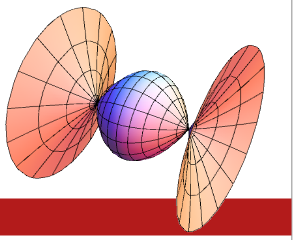
Week	Lectures (Assigned Reading)	Videos	Assignments
Week 1 (1/25 - 1/31)	**4.8:** Review of Antiderivatives **5.1:** Area under a Curve	**Lec 1** **Lec 2**	**4.8:** 10, 14, 18 **5.1:** 2, 22
Week 2 (2/1 - 2/7)	**5.2:** The Definite Integral **5.3:** Evaluating Definite & Indefinite Integrals	**Lec 3** **Lec 4**	**5.2:** 22, 31 **5.3:** 12, 25, 75
Week 3 (2/8 - 2/14)	**5.4:** The Fundamental Theorem of Calculus	**Lec 5**	**5.4:** 2, 6, 18, 22
Week 4 (2/15 - 2/21)	**5.5:** The Substitution Rule Midterm 1 Review	**Lec 6** **Lec 7**	*only WebAssign* *hw due week 5*
	Midterm 1 – Tuesday, February 23, 8:45 - 10:15pm, *covers Sections* **4.8 - 5.5** **Midterm Solutions**		
Week 5 (2/22 - 2/28)	Midterm 1 Review **5.6:** Integration by Parts	**Lec 8** **Lec 9**	*only WebAssign* *hw due week 6*
Week 6 (2/29 - 3/6)	**5.6:** Integration by Parts (continued) **5.7:** Trigonometric Integrals **5.7:** Trigonometric Substitutions	**Lec10** **Lec11**	**5.6:** 14, 36 **5.7:** 8, 10 **5.7:** 12, 13, 16
Week 7 (3/7 - 3/13)	**Appendix G:** Integration by Partial Fractions **5.7:** Partial Fractions	**Lec12**	**Appendix G:** 21, 24, 31 **5.7:** 22, 24, 25, 30
Week 8 (3/14 - 3/20)	*no classes (Spring Break!)*		
Week 9 (3/21 - 3/27)	**5.10:** Improper Integrals	**Lec14** **Lec15**	**5.10:** 18, 21, 26, 28, 30
Week 10 (3/28 - 4/3)	**6.1:** More on areas **6.2:** Volumes (Disk & Washer Method)	**Lec16** **Lec17**	*only WebAssign* *hw due week 11*
	Midterm 2 - Wednesday, April 6, 8:45 - 10:15pm *covers Sections* **5.6 - 5.7, 5.10, 6.1 - 6.2** **Midterm Solutions** **Review Session**		

Week 11 (4/4 - 4/10)	Midterm 2 Review **6.3:** Volumes by Cylindrical Shells	**Lec18** **Lec20**	*hw due week 13* **6.3:** 2, 4, 8, 10, 16
Week 12 (4/11 - 4/17)	**6.3:** Volumes by Cylindrical Shells **6.4:** Arc Length	**Lec21** **Lec22**	
Week 13 (4/18 - 4/24)	**6.5:** Average Value of a Function **6.6:** Applications to Physics and Engineering (Work, Hydrostatic Force and Pressure)	**Lec23** **Lec24**	**6.4:** 11, 12 **6.5:** 8, 11, 16
Week 14 (4/25 - 5/1)	**6.6:** Applications to Physics and Engineering **6.7:** Applications to Biology and Economics	**Lec25**	**6.6:** 17, 19, 29, 32, 34
Week 15 (5/2 - 5/8)	Catch-up and Review	**Lec26** **Lec27**	
	Final exam - Wednesday, May 11, 11:15am - 1:45pm, *covers everything!*		

Last updated May 5, 2016

MAT 126: Calculus B
Spring 2016

Course Information



Course Description

The goal of this course is to extend your knowledge from Differential Calculus to Integral Calculus. You will develop a deeper understanding of Calculus and learn how to apply these concepts in a variety of areas. MAT 126 is a continuation of MAT 125, covering: definite and indefinite integrals, the Fundamental Theorem of Calculus, symbolic and numeric methods of integration, area under a curve, volumes, complex numbers. Applications of integrals will also be discussed.

Click here to download a copy of the course syllabus. Please visit also the **course website on Blackboard**.

Lectures & Office Hours

Please check the **last page of the syllabus** for the list of Lectures and Recitations.
The Office Hours are posted on Blackboard, under Faculty Information, and also on the **Stony Brook website**. The office hours held in the Math Learning Center (MLC) can be found **here**; their schedule will be fully updated by the second week of classes.

Blackboard

Almost all course administration will take place on Blackboard: **https://blackboard.stonybrook.edu**
All course announcements will be posted on Blackboard. Your exam, homework and quiz grades will also be reported on Blackboard. On this webpage, under **Schedule & Homework** you will find the most up-to-date version of the weekly course schedule and the written homework assigments.

Grading Policy

Grades will be computed using the following scheme:

- Recitation (Homework, WebAssign, Quizzes) – 20%
- Each Midterm – 25%
- Final Exam – 30%

Textbook

The course textbook is *Single Variable Calculus (Stony Brook Edition)* , by James Stewart. This is the same book as Stewart's *Concepts and Contexts, 4th edition*, but with a different cover and a lower price. The same book is used by MAT 125, MAT 126, MAT 127, MAT 131 and MAT 132. A copy of the textbook is also available online, in your WebAssign account. You will need to purchase WebAssign access, please see more information at the end of this page.

Homework

You cannot learn calculus without working problems. Expect to spend at least 8 hours a week solving problems; do all of the assigned problems, as well as additional ones to study. Each week, you will be given two sets of problems: one due in Recitation, and one to be completed online (using WebAssign). Your solutions for the paper homework should be written neatly and legibly in grammatically correct mathematical English, and all steps should be clearly outlined.

WebAssign

For online homework, we will be using WebAssign, a web-based system in which you see the problems, submit your answers and/or solutions and get immediate feedback on your work. You will be graded on how many questions you get correct and how many tries it takes you to get the correct answer. Generally, the online assignments will be due on Wednesday morning.

If you are enrolled in MAT 126 you already have a WebAssign account (if your Blackboard username is **x**, then your WebAssign username is **x::app-125**). You do not need a class key. If you have not purchased WebAssign with your textbook or separately you will be prompted to pay-up when you enter the program. You get a free trial for the first two weeks. The best way to enter WebAssign is from the course webpage on **Blackboard.**

If you used WebAssign for MAT 125, you may already have a different WebAssign account (with a different username). You can try to link your WebAssign account for MAT 126 to your old account. This is usually done from the account where the paid multi-term access code was introduced. Instructions on how to do that can be found here: **http://www.webassign.net/manual/student_guide/t_s_using_same_login_different_accounts.htm**

Stony Brook University

NAME: Midterm Solutions

RECITATION #: ____

MAT 126 – Spring 2016 – Midterm 1

February 23, 2016

INSTRUCTIONS – PLEASE READ

- Please turn off your cell phone and put it away.
- ➩ Please write your name and your section number right now.
- ➩ This is a closed book exam. You are NOT allowed to use a calculator or any other electronic device or aid.
- ➩ The midterm has 6 problems worth a total of 100 points. Look over your test packet as soon as the exam begins. If you find any missing pages or problems please ask a proctor for another test booklet.
- ➩ Show your work. To receive full credit, your answers must be neatly written and logically organized. If you need more space, write on the back side of the preceding sheet, but be sure to label your work clearly. You do not need to simplify your answers unless explicitly instructed to do so.
- ➩ Academic integrity is expected of all Stony Brook University students at all times, whether in the presence or absence of members of the faculty.

PROBLEM	SCORE
1.	
2.	
3.	
4.	
5.	
6.	
Total	

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R31	W	5:30-6:23pm	Mariangela Ferraro
R32	M	5:30-6:23pm	Charles Cifarelli
R33	Tu	1:00-1:53pm	Yu Zeng

Problem 1. (17 points)

a) Find the function $f(y)$ such that $f'(y) = e^y + \cos(y)$ and $f(0) = 10$.

$$f(y) = \int f'(y)\,dy = \int (e^y + \cos(y))\,dy = e^y + \sin(y) + c$$

$$f(0) = 10 \Rightarrow e^0 + \sin(0) + c = 10 \Rightarrow c = 9$$

$$f(y) = e^y + \sin(y) + 9$$

b) Let $h(y) = \int\limits_2^y \sqrt{t^3+1}\,dt$. Compute $h(2)$ and $h'(2)$.

$$h(2) = \int_2^2 \sqrt{t^3+1}\,dt = 0$$

$$h'(y) = \frac{d}{dy}\left(\int_2^y \sqrt{t^3+1}\right)dt = \sqrt{y^3+1}$$

$$h'(2) = \sqrt{2^3+1} = 3$$

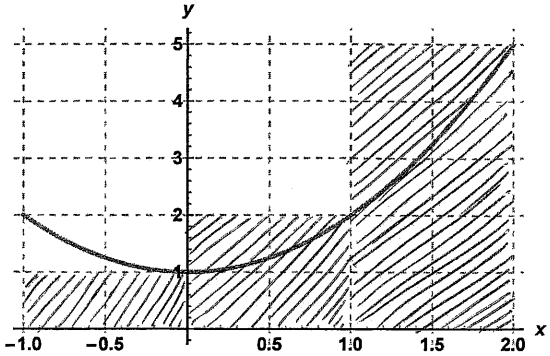
c) Suppose that g is an integrable functions on $[0,3]$, such that $\int_1^3 g(x)dx = 6$, $\int_1^0 g(x)dx = -5$ and $\int_2^3 g(x)dx = 1$. Compute $\int_0^2(-2g(x)+5)dx$.

$$\int_0^2 g(x)\,dx = \int_0^1 g(x)\,dx + \int_1^2 g(x)\,dx = -\int_1^0 g(x)\,dx + \int_1^3 g(x)\,dx - \int_2^3 g(x)\,dx$$

$$= -(-5) + 6 - 1 = 10$$

$$\int_0^2 (-2g(x)+5)\,dx = -2\int_0^2 g(x)\,dx + \int_0^2 5\,dx = -20 + 10 = -10$$

Problem 2. (20 points) Consider the function $f(x) = x^2 + 1$, defined on the interval $[-1, 2]$.



a) Approximate the area between the graph of f and the x-axis using 3 right hand rectangles with equal widths.

$$\Delta x = \frac{2-(-1)}{3} = \frac{3}{3} = 1$$

$$A \simeq \left(f(0) + f(1) + f(2)\right)\Delta x = (1+2+5)\cdot 1 = 8$$

b) Write a formula for a Riemann Sum with n right hand rectangles.

$$R_n = \sum_{k=1}^{n} f(-1 + k\Delta x)\Delta x \qquad \text{where } \Delta x = \frac{3}{n}$$

$$R_n = \sum_{k=1}^{n} \left(\left(-1 + 3\frac{k}{n}\right)^2 + 1\right)\cdot\frac{3}{n}$$

c) Evaluate the limit of the Riemann Sum from part (b) as $n \to \infty$, either using integrals, or by direct computation, using the formula $\sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$.

<u>Sol 1</u>

$$\lim_{n\to\infty} R_n = \int_{-1}^{2} f(x)\,dx = \int_{-1}^{2} (x^2+1)\,dx = \frac{x^3}{3} + x \Big|_{-1}^{2}$$

$$= \left(\frac{8}{3} + 2\right) - \left(\frac{-1}{3} - 1\right) = \frac{9}{3} + 3 = 6$$

Sol 2

$$R_n = \sum_{k=1}^{n} \frac{6}{n} + \sum_{K=1}^{n} \frac{27K^2}{n^3} - \sum_{K=1}^{n} 18\frac{K}{n^2}$$

$$R_n = 6\cdot\frac{\cancel{n}}{\cancel{n}} + \frac{27}{n^3}\sum_{K=1}^{n} K^2 - \frac{18}{n^2}\sum_{K=1}^{n} K$$

$$= 6 + \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{18}{n^2}\,\frac{n(n+1)}{2}$$

$$= 6 + \frac{9(n+1)(2n+1)}{2n^2} - 9\,\frac{n+1}{n}$$

$$= 6 + 9\left(\frac{2n^2+3n+1}{2n^2}\right) - 9\left(1+\frac{1}{n}\right)$$

$$= 6 + 9\left(1+\frac{3}{2n}+\frac{1}{2n^2}\right) - 9\left(1+\frac{1}{n}\right) = 6 + 9\left(\frac{1}{2n}+\frac{1}{2n^2}\right)$$

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} 6 + 9\left(\underset{\substack{\downarrow\\ 0}}{\frac{1}{2n}} + \underset{\substack{\downarrow\\ 0}}{\frac{1}{2n^2}}\right) = 6.$$

Problem 3. (21 points) Evaluate the following expressions:

a) $\int_2^5 (6x^2 + 4x - 1)\, dx$ $= \frac{6x^3}{3} + \frac{4x^2}{2} - x \Big|_2^5 = 2x^3 + 2x^2 - x \Big|_2^5$

$= (250 + 50 - 5) - (16 + 8 - 2)$

$= 295 - 22 = 273$

b) $\int \frac{\sin^2(x)}{\sec(x) - \sec(x)\cos^2(x)}\, dx$ $= \int \frac{\sin^2 x}{\sec x\,(1 - \cos^2 x)}\, dx =$

$= \int \frac{\sin^2 x}{\sec x \cdot \sin^2 x}\, dx = \int \frac{1}{\sec x}\, dx = \int \cos x$

$= \sin x + c$

c) $\frac{d}{dx}\left(\int_{2x}^{x^5} \tan(\ln(t^2))\, dt\right)$ $= \tan(\ln(x^5)^2) \cdot (x^5)' - \tan(\ln(2x)^2) \cdot (2x)'$

$= \tan(\ln(x^{10})) \cdot 5x^4 - \tan(\ln(4x^2)) \cdot 2$

Problem 4. (14 points) Calculate the following integrals, using the appropriate substitution:

a) $\int \frac{1}{x(\ln x)^2}\,dx \quad = \int \frac{1}{u^2}\,du = \int u^{-2}du = \cancel{\int}$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \frac{u^{-2+1}}{-2+1} + c = -\frac{1}{u} + c$$

$$= -\frac{1}{\ln x} + c$$

b) $\int_0^{\sqrt[3]{\pi}} x^2 \sin(x^3)\,dx \quad = \int_0^{\pi} \frac{1}{3} \sin(u)\,du = -\frac{\cos(u)}{3}\Big|_0^{\pi}$

$u = x^3$

$du = 3x^2 dx$

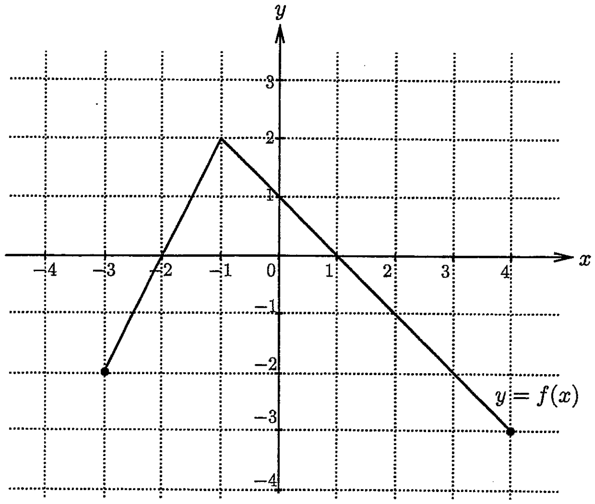
$x = 0 \Rightarrow u = 0$

$x = \sqrt[3]{\pi} \Rightarrow u = \pi$

$$= \frac{(-\cos(\pi)) - (-\cos 0)}{3}$$

$$= \frac{-(-1) - (-1)}{3} = \frac{2}{3}$$

Problem 5. (20 points) Consider the function $f(x)$ graphed below:



Now define a **new** function $F(x) = \int_{-3}^{x} f(t)\,dt$ on the interval $[-3, 4]$.

a) Compute $F(-3)$, $F(1)$ and $F(4)$.

$F(-3) = \int_{-3}^{-3} f(t)\,dt = 0$.

$F(1) = \frac{2 \cdot 3}{2} - \frac{2 \cdot 1}{2} = 2$

$F(2) = F(1) + \int_{1}^{4} f(t)\,dt$

$= 2 - \frac{3 \cdot 3}{2} = -\frac{5}{2}$

b) Where is F increasing? Where is F decreasing?

$F'(x) = \frac{d}{dx}\left(\int_{-3}^{x} f(t)\,dt\right) = f(x)$. by FTC

F is increasing when f is positive, that is on $[-2, 1]$.

F is decreasing when $f(x) < 0$, that is on $[-3, -2] \cup [1, 4]$.

c) Where is F concave-up? Where is F concave-down?

$F''(x) = f'(x)$

F is concave-up when $F''(x) > 0$, that is when $f'(x) > 0$, or equivalently when f is increasing (its slope is positive).

So F is concave-up on $[-3, -1]$ and concave-down on $[-1, 4]$.

Problem 6. (8 points) Determine whether the following statements are true or false. Circle your response and give a brief explanation (a reason why it's true or an example where it fails).

a) TRUE [✓ FALSE] Suppose that f and g are two integrable functions on $[0,1]$. Then

$$\int_0^1 f(x)g(x)dx = \int_0^1 f(x)dx \int_0^1 g(x)dx.$$

Take for example $f(x) = x$ and $g(x) = x$

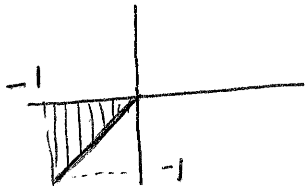
$$\int_0^1 f(x)g(x)\,dx = \int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$\int_0^1 f(x)dx \int_0^1 g(x)\,dx = \left(\int_0^1 x\,dx\right)^2 = \left(\frac{x^2}{2}\Big|_0^1\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{1}{3} \neq \frac{1}{4}$$

b) TRUE [✓ FALSE] Let f be an integrable functions on $[a,b]$. The definite integral $\int_a^b f(x)dx$ represents the area of the region enclosed between the graph of the function and the x-axis.

Assume f is negative. Take for example $f(x) = x$ on $[-1, 0]$.



Then $\int_{-1}^{0} f(x)\,dx = \int_{-1}^{0} x\,dx = \frac{x^2}{2}\Big|_{-1}^{0} = -\frac{1}{2}$

However, the area of the region between the graph of f and the x-axis is positive

$$A = \frac{1 \cdot 1}{2} = \frac{1}{2}$$

So in this case $\int_{-1}^{0} f(x)\,dx = -A$

Stony Brook
University

NAME: Midterm Solutions

RECITATION #: ____

MAT 126 – Spring 2016 – Midterm 2

April 6, 2016

INSTRUCTIONS – PLEASE READ

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Some trigonometric formulas that might be useful:

$\sin^2(x) + \cos^2(x) = 1$ $\qquad$ $\sin(2x) = 2\sin(x)\cos(x)$

$\tan^2(x) = \sec^2(x) - 1$ $\qquad$ $\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

Problem 1. (18 points) Compute the following integrals:

a) $\displaystyle\int_0^{\pi/2} x^2\cos(2x)\,dx$

Sol: Use integration by parts twice

$$\int_0^{\pi/2} x^2\cos(2x)dx = \int_0^{\pi/2} x^2\left(\frac{\sin(2x)}{2}\right)'dx = x^2\frac{\sin(2x)}{2}\Big|_0^{\pi/2} - \int_0^{\pi/2} 2x\cdot\frac{\sin(2x)}{2}\,dx$$

$$\boxed{\begin{array}{ll} u = x^2 & dv = \cos(2x)dx \\ du = 2x\,dx & v = \frac{\sin(2x)}{2} \end{array}}$$

$$= \underbrace{\left(\frac{\pi}{2}\right)^2\frac{\sin(\pi)}{2} - 0^2\frac{\sin(0)}{2}}_{0} + \int_0^{\pi/2} x\left(-\sin(2x)\right)dx.$$

$$\int_0^{\pi/2} x(-\sin(2x))dx = x\,\frac{\cos(2x)}{2}\Big|_0^{\pi/2} - \int_0^{\pi/2}\frac{\cos(2x)}{2}\,dx = \left(\frac{\pi}{4}\cos(\pi) - 0\right) - \frac{\sin(2x)}{4}\Big|_0^{\pi/2}.$$

$$\boxed{\begin{array}{ll} u = x & dv = -\sin(2x)dx \\ du = 1\cdot dx & v = \frac{\cos(2x)}{2} \end{array}}$$

$$= -\frac{\pi}{4} - \underbrace{\left(\frac{\sin(\pi) - \sin(0)}{4}\right)}_{0} = \boxed{-\frac{\pi}{4}}$$

b) $\displaystyle\int \tan^3(x)\sec(x)\,dx$

Sol: $\int \tan^3(x)\sec(x)\,dx = \int \tan^2 x \cdot \tan x\sec x\,dx = \int(\sec^2 x - 1)\tan x\sec x\,dx$

Use the substitution $u = \sec x$

$du = \tan x\sec x\,dx$

$$= \int(u^2 - 1)\,du = \frac{u^3}{3} - u + c = \frac{\sec^3(x)}{3} - \sec(x) + c$$

Problem 2. (18 points) Compute the following integrals:

a) $\int \frac{5x^2 - x + 2}{(x^2+1)(x-1)}\, dx$

Sol:

$$\frac{5x^2-x+2}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} \Rightarrow 5x^2-x+2 = (x-1)(Bx+C) + A(x^2+1)$$

$$\underline{x=1}: \quad 5\cdot 1^2 - 1 + 2 = 0 + A(1^2+1) \Rightarrow 6 = 2A \Rightarrow A = 3$$

$$5x^2-x+2 = x^2(A+B) + x(C-B) + (A-C) \Rightarrow \begin{cases} 5 = A+B \\ -1 = C-B \\ 2 = A-C \end{cases} \Rightarrow \begin{cases} B = 2 \\ C = 1 \end{cases}$$

Therefore

$$\int \frac{5x^2-x+2}{(x^2+1)(x-1)}\, dx = 3\int \frac{1}{x-1}\, dx + \int \frac{2x}{x^2+1}\, dx + \int \frac{1}{x^2+1}\, dx$$

$$= 3 \ln|x-1| + \ln(x^2+1) + \tan^{-1}(x^2+1) + c$$

$$\int \frac{2x}{x^2+1}\, dx \underset{\boxed{\begin{matrix} u = x^2+1 \\ du = 2x\,dx \end{matrix}}}{=\!=} \int \frac{du}{u} = \ln|u| + c = \ln(x^2+1) + c$$

b) $\int e^x \sin x \, dx$

Use Integration by parts twice:

Sol 1:

$$\left.\begin{aligned} \int e^x \sin x\, dx &= \int (e^x)' \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx \\ \int e^x \cos x\, dx &= \int (e^x)' \cos x\, dx = e^x \cos x - \int e^x(-\sin x)\, dx \end{aligned}\right| \Rightarrow$$

$$\Rightarrow \int e^x \sin dx = e^x \sin x - e^x \cos x - \int e^x \sin x\, dx \Rightarrow \int e^x \sin x\, dx = \frac{e^x \sin x - e^x \cos x}{2} + C$$

Sol 2:

$$\int e^x \sin x\, dx = \int e^x (-\cos x)'\, dx = e^x(-\cos x) - \int e^x(-\cos x)\, dx = -e^x \cos x + \int e^x \cos x\, dx$$

$$\int e^x \cos x\, dx = \int e^x (\sin x)'\, dx = e^x \sin x - \int e^x \sin x\, dx$$

$$\Rightarrow \int e^x \sin x\, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\, dx$$

$$\Rightarrow \int e^x \sin x\, dx = \frac{-e^x \cos x + e^x \sin x}{2} + C$$

Problem 3. (15 points) Evaluate the integral $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$, for $-3 \leq x \leq 3$. Simplify your final answer.

Sol: Use the trigonometric substitution $x = 3\sin\theta$

$$dx = 3\cos\theta\, d\theta$$

$$\sqrt{9-x^2} = \sqrt{9-9\sin^2\theta} = \sqrt{9\cos^2\theta} = 3|\cos\theta|$$

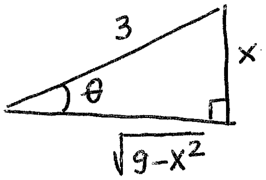
$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx = \int \frac{9\sin^2\theta}{3\cos\theta} \cdot 3\cos\theta\, d\theta = 9\int \sin^2\theta\, d\theta$$

To compute the integral, use the fact that $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$

$$= 9\int \frac{1-\cos 2\theta}{2}\, d\theta = \frac{9}{2}\theta - \frac{9}{2} \cdot \frac{\sin(2\theta)}{2} + C = \frac{9}{2}\theta - \frac{9}{4}\sin(2\theta) + C$$

$$= \frac{9}{2}\theta - \frac{9}{4}\, 2\sin\theta\cos\theta \cancel{d\theta} + C = \frac{9}{2}(\theta - \sin\theta\cos\theta) + C$$

$$x = 3\sin\theta \implies \theta = \sin^{-1}\left(\frac{x}{3}\right)$$



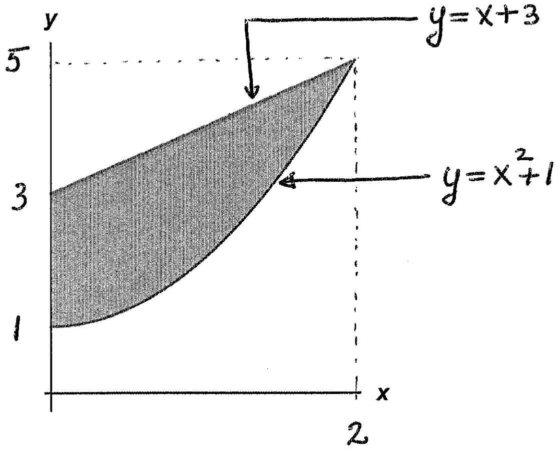
We build a right triangle with an angle θ with $\sin\theta = \frac{x}{3}$ by labeling the opposite side x and hypotenuse 3

$$\left.\begin{aligned} \sin\theta &= \frac{x}{3} \\ \cos\theta &= \frac{\sqrt{9-x^2}}{3} \end{aligned}\right| \implies \sin(2\theta) = 2\sin\theta\cos(\theta) = \frac{2\sqrt{9-x^2}\cdot x}{9}$$

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx = \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{\sqrt{9-x^2}\cdot x}{2} + C$$

Problem 4. (25 points) The region R in the first quadrant bounded by $y = x^2 + 1$ and $y = x + 3$ is shown to the right.

a) Identify the two curves on the figure and label their points of intersection and the y-intercepts.



$$x^2+1 = x+3 \Rightarrow x^2-x-2=0$$

$$\Rightarrow x^2+x-2x-2=0 \Rightarrow x(x+1)-2(x+1)=0$$

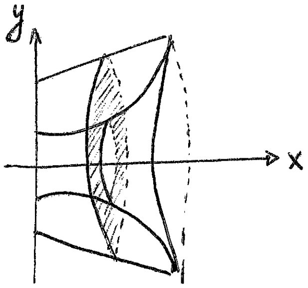
$$\Rightarrow (x-2)(x+1)=0 \Rightarrow x=-1 \text{ or } x=2$$

b) Find the area of the region R.

$$\text{Area} = \int_0^2 \left((x+3)-(x^2+1)\right) dx = \int_0^2 \left(x-x^2+2\right) dx = \frac{x^2}{2}-\frac{x^3}{3}+2x \Big|_0^2$$

$$= \left(\frac{4}{2}-\frac{8}{3}+4\right)-0 = 6-\frac{8}{3} = \frac{10}{3}$$

c) Find the volume of the solid of revolution that results when R is revolved about the x-axis.



A typical cross-section is a washer with outer radius $R = x+3$ and inner radius $r = x^2+1$. The area of the washer is

$$A(x) = \pi(R^2-r^2) = \pi\left((x+3)^2-(x^2+1)^2\right)$$

$$\text{Volume} = \pi\int_0^2 \left((x+3)^2-(x^2+1)^2\right) dx = \pi\int_0^2 (x^2+6x+9)-(x^4+2x^2+1)\, dx$$

$$= \pi\int_0^2 (-x^4-x^2+6x+8)\, dx = \pi\left(-\frac{x^5}{5}-\frac{x^3}{3}+3x^2+8x\right)\Big|_0^2$$

$$= \pi\left(-\frac{32}{5}-\frac{8}{3}+12+16\right) = \pi\left(-\frac{136}{15}+28\right) = \pi\,\frac{284}{15}$$

Problem 5. (16 points) Evaluate the following improper integrals or explain why they diverge. Simply writing "converges" or "diverges" with no explanation or work will result in no credit.

a) $\int_{-1}^{2} \frac{1}{x^{\frac{2}{3}}}\,dx$ The function $\frac{1}{x^{2/3}}$ is continuous on $[-1,0) \cup (0,2]$ and discontinuous at $x=0$.

$$\int_{-1}^{2} \frac{1}{x^{2/3}}\,dx = \int_{-1}^{0} \frac{1}{x^{2/3}}\,dx + \int_{0}^{2} \frac{1}{x^{2/3}}\,dx = \boxed{3+3\sqrt[3]{2}}$$

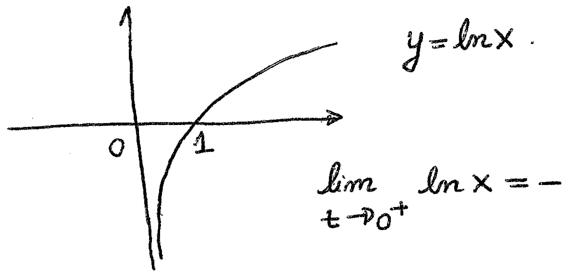
$$\int_{-1}^{0} \frac{1}{x^{2/3}}\,dx = \lim_{t\to 0^-} \int_{-1}^{t} \frac{1}{x^{2/3}}\,dx = \lim_{t\to 0^-} \int_{-1}^{t} x^{-2/3}dx = \lim_{t\to 0^-} \frac{x^{-2/3+1}}{-2/3+1}\Big|_{-1}^{t}$$

$$= \lim_{t\to 0^-} 3x^{1/3}\Big|_{-1}^{t} = \lim_{t\to 0^-} 3t^{1/3} - 3(-1)^{1/3} = 0 - 3\sqrt[3]{-1} = 0+3 = \boxed{3}$$

$$\int_{0}^{2} \frac{1}{x^{2/3}}\,dx = \lim_{t\to 0^+} \int_{t}^{2} \frac{1}{x^{2/3}}\,dx = \lim_{t\to 0^+} 3x^{\frac{1}{3}}\Big|_{t}^{2} = \lim_{t\to 0^+} 3\cdot 2^{\frac{1}{3}} - 3\cdot t^{\frac{1}{3}}$$

$$= 3\sqrt[3]{2} - 3\cdot\sqrt[3]{0} = \boxed{3\sqrt[3]{2}}$$

b) $\int_0^1 \ln(x)\,dx = \lim_{t\to 0^+} \int_t^1 \ln x\,dx = \boxed{-1}$



$\lim_{t\to 0^+} \ln x = -\infty$

Use Integration by parts:

$$\int_t^1 \ln x\,dx = \int_t^1 1\cdot \ln x\,dx$$

$$= x\ln x\Big|_t^1 - \int_t^1 x\cdot\frac{1}{x}\,dx = (1\cdot\ln 1 - t\cdot\ln t) - \int_t^1 1\,dx$$

$$= -t\ln t - \cancel{\int} x\Big|_t^1 = -t\ln t - (1-t)$$

$$\lim_{t\to 0^+}\int_t^1 \ln x\,dx = \lim_{t\to 0^+} -t\ln t - 1 + t = \underbrace{\left(\lim_{t\to 0^+} -t\ln t\right)}_{0} + \underbrace{\left(\lim_{t\to 0^+}(-1+t)\right)}_{-1} = \boxed{-1}$$

Use l'Hospital to compute $\lim_{t\to 0^+} -t\ln t = \lim_{t\to 0^+}\frac{\ln t}{-\frac{1}{t}} = \frac{-\infty}{-\infty}$

$$= \lim_{t\to 0^+}\frac{(\ln t)'}{\left(-\frac{1}{t}\right)'} = \lim_{t\to 0^+}\frac{\frac{1}{t}}{\frac{1}{t^2}} = \lim_{t\to 0^+} t = \boxed{0}$$

Problem 6. (8 points) Determine whether the following statements are true or false. Circle your response and give a brief explanation (a reason why it's true or an example where it fails).

a) TRUE [✓ FALSE] Suppose that f is a continuous function on $(-\infty, \infty)$. Then

$$\int_{-\infty}^{+\infty} f(x)\, dx = \lim_{R \to +\infty} \int_{-R}^{R} f(x)\, dx.$$

Sol: Assume for example that $f(x) = x$. $\Rightarrow \int_{-R}^{R} x\,dx = \frac{x^2}{2}\Big|_{-R}^{R} = \frac{R^2 - (-R)^2}{2} = 0$

$$\Rightarrow \lim_{R \to \infty} \int_{-R}^{R} x\,dx = 0.$$

However $\int_{-\infty}^{\infty} x\,dx$ is divergent, because $\int_{-\infty}^{\infty} x\,dx = \int_{-\infty}^{0} x\,dx + \int_{0}^{\infty} x\,dx$ and $\int_{-\infty}^{\infty} x\,dx$ converges if and only if both $\int_{-\infty}^{0} x\,dx$ and $\int_{0}^{\infty} x\,dx$ converge.

$$\int_{0}^{\infty} x\,dx = \lim_{R \to \infty} \int_{0}^{R} x\,dx = \lim_{R \to \infty} \frac{x^2}{2}\Big|_{0}^{R} = \lim_{R \to \infty} \frac{R^2}{2} = \infty$$

$\Rightarrow \int_{0}^{\infty} x\,dx$ diverges, so $\int_{-\infty}^{\infty} x\,dx$ diverges as well.

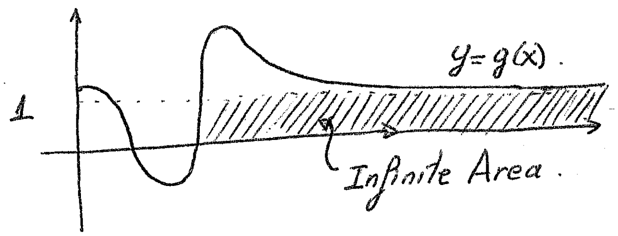
Some Correct Rules: $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{a} f(x)dx + \int_{a}^{\infty} f(x)\,dx$ where a is any real number

$$= \lim_{R_1 \to -\infty} \lim_{R_2 \to \infty} \int_{R_1}^{R_2} f(x)dx = \lim_{R_1 \to -\infty} \int_{R_1}^{\infty} f(x)dx = \ldots$$

b) [✓ TRUE] FALSE Let g be a continuous function on $[0, +\infty)$ such that $\lim_{x \to +\infty} g(x) = 1$.

Then the improper integral $\int_0^{+\infty} g(x)\, dx$ is always divergent.

Sol: Since $\lim_{x \to \infty} g(x) = 1$, for x large enough we have $g(x) \simeq 1$,



Assume for the moment that $g(x) = 1$.

$$\int_0^{\infty} 1\,dx = \lim_{R \to \infty} \int_0^{R} 1\,dx = \lim_{R \to \infty} x\Big|_0^R$$

$$= \lim_{R \to \infty} R - 0 = \infty$$

For any general function $g(x)$, since g is continuous and $\lim_{x \to \infty} g(x) = 1$, there exists some R large such that for all $x > R$ we have $g(x) \simeq 1 > 0.9$

$$\text{So } \int_0^{\infty} g(x)dx = \underbrace{\int_0^{R} g(x)dx}_{\substack{\text{this is not improper} \\ \text{since } g \text{ is continuous on } [0,R]}} + \int_R^{\infty} g(x)dx \longrightarrow \text{this is divergent, since}$$

$$\int_R^{\infty} g(x)dx > \int_R^{\infty} 0.9\,dx = \infty$$

MAT 126 CALCULUS B – SPRING 2016
SUPPLEMENTAL INFORMATION

Synopsis. The goal of this course is to extend your knowledge from Differential Calculus to Integral Calculus. You will develop a deeper understanding of Calculus and learn how to apply these concepts in a variety of areas. MAT 126 is a continuation of MAT 125, covering: definite and indefinite integrals, the Fundamental Theorem of Calculus, symbolic and numeric methods of integration, area under a curve, volumes, complex numbers. Applications of integrals will also be discussed.

Textbook. The course textbook is *Single Variable Calculus (Stony Brook Edition)* , by James Stewart. This is the same book as Stewart's *Concepts and Contexts, 4th edition*, but with a different cover and a lower price. The same book is used by MAT 125, MAT 126, MAT 127, MAT 131 and MAT 132.

Reading. The textbook is intended to be read. Read the assigned sections corresponding to the homework assignments. This will greatly increase your comprehension, and enable you to ask questions in class. Furthermore, the lectures will not always be able to cover all of the material for which you will be responsible. You also have access to the online version of the textbook from your WebAssign account.

Blackboard. Almost all course administration will take place on Blackboard:

`https://blackboard.stonybrook.edu/`

All course announcements will be posted on Blackboard. Your exam, homework and quiz grades will also be reported on Blackboard. We will maintain a front-end webpage for the course, with the tentative weekly schedule:

`http://math.stonybrook.edu/~rtanase/calcb/`

Problem sets. You cannot learn calculus without working problems. Expect to spend at least 8 hours a week solving problems; do all of the assigned problems, as well as additional ones to study. Each week, you will be given two sets of problems, one due in class, and one to be completed online.

- Your weekly written homework will be due in Recitation, starting the week of February 1. It will be graded and returned to you in Recitation in the following week. **Late homework will not be accepted!** You may work together on your problem sets, and you are encouraged to do so. However, you must write up your solutions to the problem sets by yourself. They should be written neatly and legibly in grammatically correct mathematical English. All steps should be clearly explained.
- For online homework, we will be using WebAssign, a web-based system in which you see the problems, submit your answers and/or solutions and get immediate feedback on your work. You will be graded on how many questions you get correct and how many tries it takes you to get the correct answer. If you are enrolled in MAT 126 you already have a WebAssign account. You do not need a class key or any other code. If you haven't purchased WebAssign with your textbook or separately you will be prompted to "pay-up" when you enter the program. You get a free trial for the first two weeks. The best way to enter WebAssign is from the course webpage on Blackboard. Generally, the online assignments will be due on Wednesday morning.

The lowest three scores will be dropped when computing the HW component of your final grade.

Quizzes. There will be (about) four or five unannounced quizzes in recitation over the course of the semester, based on the homework assignment and the reading for that week. The quizzes also provide a good practice for the midterms.

Exams. There will be two **evening** midterms and one final exam. If you have a conflict with any scheduled academic activity, please let me know as soon as possible, so that we may work out this conflict. As per university's regulations, we cannot change the dates of the exams.

- Midterm 1 – Tuesday, February 23, 8:45 - 10:15 pm.
- Midterm 2 – Wednesday, April 6, 8:45 - 10:15 pm.
- Final Exam – Wednesday, May 11, 11:15am - 1:45pm.

Grading policy. Grades will be computed using the following point break-down:

- Each midterm: 25%;
- Final exam: 30%;
- Recitation grade: 20%.

The recitation grade is based on your scores on in-class quizzes and online and written homework. In particular, your homework will count for 75% of the points in your recitation grade.

Prerequisites. C or higher in MAT 125 or 131 or 141 or AMS 151 or level 6 in the mathematics placement examination.

QPS Objective. A C or better in MAT 126 fulfills the Master Quantitative Problem Solving (QPS) objective.

Academic integrity. As always, you are expected to abide by the Stony Brook Code of Academic Integrity. Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at

`http://www.stonybrook.edu/uaa/academicjudiciary.`

Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, and/or inhibits students' ability to learn.

Support outside of lecture. Please feel free to come to our office hours to ask questions about the material we are studying or about the homework assignments. The schedule of the office hours will soon be updated on the Stony Brook webpage:

`http://www.math.stonybrook.edu/office-hours.`

The **Math Learning Center**, in Math S-240A, is also there for you to get help with Calculus. It is staffed most days and some evenings; your lecturer or TA may hold some of his or her office hours there. A schedule should be posted outside the room and at the Math Undergraduate Office.

DSS advisory. If you have a physical, psychological, medical, or learning disability that may affect your course work, please contact Disability Support Services (DSS) office: ECC (Educational Communications Center) Building, room 128, telephone (631) 632-6748/TDD. DSS will determine with you what accommodations are necessary and appropriate. Arrangements should be made early in the semester (before the first exam) so that your needs can be accommodated. All information and documentation of disability is confidential. Students requiring emergency evacuation are encouraged to discuss their needs with their professors and DSS. For procedures and information, go to the following web site `http://www.ehs.sunysb.edu` and search Fire safety and Evacuation and Disabilities.

Lectures and Recitations.

LEC 01	MWF	10:00am-10:53am	Earth&Space 001	Joseph Adams
R01	F	1:00pm-1:53pm	Library W4530	Jaroslaw Jaracz
R02	Tu	4:00pm-4:53pm	Library W4530	Charles Cifarelli
R03	Tu	1:00pm-1:53pm	Library W4530	Jaroslaw Jaracz
R04	Th	8:30am-9:23am	Library W4530	Alaa Abd-El-Hafez
R05	M	1:00pm-1:53pm	Library W4530	Thomas Rico
R06	M	9:00am-9:53am	Mathematics P131	Zhuang Tao
R07	W	11:00am-11:53am	Library W4530	Dyi-Shing Ou
LEC 02	TuTh	2:30pm-3:50pm	Javits Lectr 110	Raluca Tanase*
R08	Tu	4:00pm-4:53pm	Earth&Space 183	Gaurish Telang
R09	Tu	1:00pm-1:53pm	Library E4310	Yuan Gao
R10	Th	1:00pm-1:53pm	Library W4530	Alaa Abd-El-Hafez
R11	F	1:00pm-1:53pm	Library E4310	Ruijie Yang
R12	W	12:00pm-12:53pm	Earth&Space 183	Christopher Ianzano
R13	M	10:00am-10:53am	Library W4525	Zhuang Tao
R14	M	12:00pm-12:53pm	Library E4320	Thomas Rico
LEC 03	MW	4:00pm-5:20pm	Earth&Space 001	David Kahn
R15	W	9:00am-9:53am	Mathematics P131	Ruijie Yang
R16	Tu	10:00am-10:53am	Library W4535	Nicholas Valente
R17	W	10:00am-10:53am	Library N3063	Nicholas Valente
R18	Th	4:00pm-4:53pm	Library N4000	Gaurish Telang
R31	W	5:30pm-6:23pm	Physics P130	Mariangela Ferraro
R32	M	5:30pm-6:23pm	Library W4535	Charles Cifarelli
R33	Tu	1:00pm-1:53pm	Earth&Space 181	Yu Zeng