MAT 123: Introduction to Calculus

Summer Session 1, 2015

Home

Welcome to the course website for MAT 123: Introduction to Calculus. Please check this website frequently, especially the announcements below and the course schedule availble on the schedule and homework page. These sections will be updated frequently throughout the course. Use the navigation bar above to access other important information about the course.

Announcements

All course announcements will be posted here.

- July 3 - Solutions to the final exam have been posted to the Exams page. The final has been graded. Grades for the final exam and overall course grades have been posted to Blackboard. Course grades will be submitted to the registrar over the weekend.
- June 29 - The solutions to the sample final are posted, along with a practice sheet about limits.
- June 26 - The original Paper Homework 9 assignment sheet has a typo in question 7. The corrected version is now posted.
- June 25 - The topics list for the final and a sample final have been posted to the Exams page. Solutions to the sample final will be posted to the website over the weekend.
- June 24 - Homework 9 is now available on WebAssign. Please also complete the paper supplement to homework 9, available here. Study materials for the final exam will be posted over the next few days.
- June 22 - The schedule has been updated and solutions to Quiz 6 have been posted. Also, make sure you know all of the trig identities listed on this sheet.
- June 17 - The homework due 6/22 includes a paper supplement in addition to the usual WebAssign homework. Print this page and follow the instructions on it.
- June 15 - The Schedule has been updated to include more suggested homework problems. The Exams page has also been updated with some important information about estimated grades.
- June 12 - Solutions to the midterm exam are posted on the Exams page.
- June 10 - The schedule has ben update with suggested homework to accompany today's lecture. Also, if you'd like to learn more about how thinking logarithmically may be innate, you could try listening to this episode of Radiolab, or reading this news article.
- June 8, later - Some more practice problems to prepare for the midterm have been added to the Homework page.
- June 8 - Solutions to the sample midterm have been posted to the Exams page. The sample midterm and midterm topics list have also been updated to remove some topics that we weren't able to cover in class today. The Homework page also lists some practice problems you to help you review exponential and logarithm functions in preparation for the midterm, including a graphing practice sheet. There is no WebAssign homework due this week - study for the midterm instead.
- June 5 - A sample midterm has been posted to the exams section. Solutions to the sample midterm will be posted later in the weekend - you should attempt the sample midterm without looking at the solutions. Also, there WILL NOT be a quiz during class on Monday, June 8. We'll use that extra time to catch up on some material we need to cover.
- June 4 - A list of topics for the Midterm is available on the Exams page, while suggested review

problems for each section are on the Homework page. A sample midterm will be posted to the website tomorrow.

- June 1 - Course schedule is updated to reflect changes, and solutions to Quizzes 1, 2 have been posted.
- May 15 - Course Website Available

MAT 123: Introduction to Calculus

Summer Session 1, 2015

Syllabus

The course syllabus, with schedule, is also available as a PDF. Note that the course schedule in this document will not be updated as the course progresses. The most up-to-date schedule is availble on the Schedule and Homework page.

General Information

Course: MAT 123, Introduction to Calculus
Section: 60
Instructor: Joseph Thurman
Time and Location: MW 9:30am–12:55 pm, Stony Brook Manhattan, Room 312
Instructor email: jthurman AT math.stonybrook.edu

Email and this course website will be the main avenue for communication outside of class. Please check this website frequently so that you are always aware of upcoming assignments and exams. Urgent announcements (e.g., class cancellations) will be emailed to enrolled students using Blackboard. Make sure to check the email address listed in your contact information on Blackboard (most likely "firstname.lastname@stonybrook.edu").

Course Description

The goal of this course is give students the mathematical foundation necessary for future study of calculus. The focus will be on college algebra and trigonometry. From the course catalog:

> Comprehensive preparation for the regular calculus sequences, with introduction to derivatives. Careful development of rational, exponential, logarithmic, and trigonometric functions, and their applications. Asymptotics and limits. Linear approximations, slope and derivatives, detailed curve sketching. General modeling examples.

Office Hours

The instructor will hold office hours after each class meeting from 1:30 - 3:00 pm at Stony Brook Manhattan. The exact location may vary, so ask the staff at the reception desk, who will be able to find the right spot.

Textbook

Title: Precalculus - A Prelude to Calculus (Second Edition)
Author: Sheldon Axler

The textbook is available from the Stony Brook bookstore, although students are encouraged to consider other sources where the book may be available for a lower price.

Students will also be required to purchase an access code to WebAssign, the online homework system we will use in this course. Students purchasing the textbook from the bookstore can purchase a package that includes both the book and access code. The access code can also be purchased separately through the WebAssign system. You will be able to access WebAssign for the first two weeks of the course without purchasing an access code.

Grading Policy

Grades will be assigned based on student performance on quizzes, homework, a midterm exam, and a final exam.

Quizzes: Starting with the second class meeting, each class will begin with a short (approx. 10 minute) quiz covering the material from the previous lecture. There will be no quizzes on exam days. The lowest quiz score will be dropped. Quiz performance will count for **10%** of the overall grade.

Homework: There will be homework assigned at the end of each class meeting. Shorter assignments will be given after Monday lectures, to be completed before the lecture on Wednesday of the same week. Longer assignments will be assigned after each Wednesday lecture, to be completed before the lecture on Monday of the next week. All homework for credit will be completed through the WebAssign online portal. Instructions on using WebAssign are available on the Schedules and Homework page. **Late homework will not be accepted**. Homework performance will count for **20%** of the overall grade.

In addition to the required problems, recommended homework problems will also be assigned to give students more opportunity for practice. These problems will not be graded and do not count for any credit. These recommended practice problems will come from the textbook.

Midterm Exam: There will be an in-class midterm during the first half of class on Wednesday, June 10. Information about the format and content of the exam will be posted to the Quizzes and Exams page in the week before the exam. The midterm will account for **30%** of the overall grade.

Final Exam: There will be an in-class final exam during the final class period on Wednesday, July 1. The exam will be cumulative. Information about the format and content of the exam will be posted to the Quizzes and Exams page in the week before the exam. The final exam will account for **40%** of the overall grade.

Calculator Use

A scientific calculator with basic arithmetic and trigonometric functions may be necessary in this course to complete some homework assignments. A graphing calculator will never be necessary. No calculators of any kind will be allowed during in-class quizzes or exams.

Academic Integrity

Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at stonybrook.edu/uaa/academicjudiciary.

Disability Support Services

If you have a physical, psychological, medical, or learning disability that may impact your course work, please contact Disability Support Services at (631) 632-6748 or studentaffairs.stonybrook.edu/dss/. They will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential.

Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to stonybrook.edu/ehs/fire/disabilities.shtml

MAT 123: Introduction to Calculus

Summer Session 1, 2015

Schedule and Homework

Please check this page often, as it will be frequently updated with homework assignments. The schedule is tentative, and will be adjusted as the class progresses.

All online homework is completed through the WebAssign system. You can log in to WebAssign using Blackboard. Follow the instructions here. A guide to starting with WebAssign is available here. The customer support page for WebAssign is located here.

Class	Topics	Required HW	Suggested HW	Note
5/27	Overview of Course and Syllabus 1.1 - Functions 1.2 - Coordinate Plane and Graphs 1.4 - Composition of Functions	HW 1 due 5/29	1.1 - 13-25, 33-37 1.2 - 13-35 1.4 - 11-21, 31-35	Odd problems only
5/29	1.3 - Function Transformations 1.5 - Inverse Functions	HW 2 due 6/1	1.3 - 17,21,25,29,33,37,55,57 1.5 - 1-9, 15-19 odds	Friday Class Meeting
6/1	1.6 - Graphs and Inverse Functions 2.1 - Lines and Linear Functions 2.2 - Quadratic Functions and Conics	HW 3 due 6/3	1.6 - 1-11 2.1 - 1-19, 29-33 2.2 - 13-21, 29, 35, 39, 41	Odd problems only
6/3	2.2 - Quadratic Functions and Conics, continued 2.4 - Polynomials	HW 4 due 6/8	2.4 - 1-17, 25, 27	Odd problems only
6/8	2.5 - Rational Functions 2.3,3.1 - Exponentials, Logarithms 3.2,3.3 - Logarithm Rules	Study for Midterm	2.5 - 1-7, 33,35 3.1 - 9-27, 49-65 Graphing Practice	Odd problems only
6/10	Midterm Exam 3.2,3.3 - Logarighm Rules, Cont 3.4,3.7 - Exponential Growth	HW 5 Due 6/15	3.3 - 13-31,35,37 3.4 - 21-27	Odd problems only
6/15	3.4,3.7 - Exponential Growth, Cont 4.1 - The Unit Circle 4.2 - Radians 4.3, 4.4 - Trigonometric Functions	HW 6 Due 6/17	3.2 - 25-31 3.4 - 15, 17, 19, 31 3.7 - 1-11 4.1 - 1,3,7-13,35-39 4.2 - 1-10 all, 11-23 odd	Odds unless noted

6/17	4.3, 4.4 - Trigonometric Functions, Cont. 4.5 - Trigonometry of Right Triangles 4.6 - Trigonometric Identities	HW 7 on WebAssign Supplement	4.1 - 41-46 all 4.2 - 37-42 all 4.3 - 1-10 all 4.4 - 7-17 odd 4.5 - 1,3,13-19 odd	WebAssign and Supplement Due 6/22
6/22	4.6 - Trigonometric Identities, cont 5.5 - Double- and Half-Angle Identities 5.6 - Addition and Subtraction Formulas 5.1 - Inverse Trig Functions	HW 8 on Webassign Due 6/24	4.6 - 5-37 odds 5.5 - 11-42 all 5.6 - 13-24 all 5.1 - 1-4 all 5.2 - 23-30 all	Know These Identities
6/24	6.1 - Transformations of Trig Functions Solving Equations with Trig	HW 9 on WebAssign Supplement	6.1 - 1-31 odds	WebAssign and Supplement Due 6/29
6/29	Limits Review for Final	None - Study For Final	Limit Practice Sheet	-
7/1	Final Exam	-	-	-

The schedule was last modified on 10/10/48321 03:32:51

MAT 123: Introduction to Calculus

Summer Session 1, 2015

Quizzes and Exams

Quizzes

- Quiz 1, with solutions
- Quiz 2, with solutions
- Quiz 3, with solutions
- Quiz 4, with solutions
- Quiz 5, with solutions
- Quiz 6, with solutions
- Quiz 7, with solutions
- Quiz 8, with solutions. We did not take this quiz in class due to time constraints, but it should be helpful to look over it for extra practice before the final.

Midterm Exam

You can see a blank copy of the midterm here, and look at the solutions to the midterm here. We won't have time to go over the solutions to the midterm in class, so please read through the solutions carefully and try to understand your mistakes - you will still be responsible for understanding this material on the final exam. For future reference, the topics list for the midterm, the sample midterm, and the solutions to the sample midterm are all still available.

An "Estimated Grade" for each student's performance up to the midterm has been posted to the grade center in Blackboard. This is meant to give you a rough idea of your current standing in the class. This is no guarantee of final grades - for example, if you have a B now as your estimated grade, this does not mean you will get a B as your final grade, especially if you do not perform well on the final exam. This score was computed using all graded work up to and including the midterm: quizzes 1-3, homeworks 1-5, and the midterm. Note that the estimated grade includes all quiz grades, even though the lowest quiz grade will be dropped when computing the final grades. This estimated grade will not be updated, so it will not reflect performance on future homework and quizzes.

Final Exam

A blank copy of the final exam is available here, with solutions here. For reference, you can still see the sample final and the solutions to the sample final. The list of topics covered on the final is here.

The grades for the final and ouverall course grades are posted to Blackboard. If you have any questions about the grading for the course, please email me. Grades will be submitted to the registrar over the weekend.

MAT 123 Sample Final Solutions

1 Conceptual Questions.

This section contains some general questions to help you test your understanding of some of the key concepts we've learned since the midterm. As with the sample midterm, these questions are not necessarily representative of the kind of questions that will appear on the final - the true sample final is in the next section. Note that the final exam will be cumulative, but that these conceptual questions only cover the material we've learned since the midterm.

1. What real-world situations are modeled by exponential growth or decay?

   Population growth and the value of a bank account with compound interest both exhibit exponential growth, while the amount of radioactive material in a sample exhibits exponential decay.

2. How is the number e defined? What makes this number special?

   We defined the number e as the value of the limit $\lim_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n$. This limit arises when considering the value of a bank account with interest compounded n times a year - as n increases to infinity, the value in the account approached P_0e^{rt}, where P_0 is the initial amount in the account and r is the interest rate. In this way, e is related to any process that involves continuous growth.

   Recall that we also saw in class that e is special for some reasons relating to calculus. If you draw a line tangent to the graph of $f(x) = e^x$ at the point $(0, 1)$, that line will have slope 1. We also saw that the area between the graph of the function $f(x) = \frac{1}{x}$ and the x-axis between $x = 1$ and $x = e$ is exactly 1. You'll learn more about the importance of these facts in calculus.

3. How are the sine, cosine, and tangent functions defined using the unit circle? How are these functions defined using right triangles?

   Given an angle θ, we can draw that angle on the unit circle and consider the endpoint of the radius corresponding to that angle. That point will have xy-coordinates $(\cos\theta, \sin\theta)$. The tangent function is then defined as $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

   If θ is an acute angle in a right triangle, then $\sin\theta$ is the ratio of the length of the side opposite the angle to the length of the hypotenuse. The value of $\cos\theta$ is the ratio of the length of the side adjacent to the angle to the length of the hypotenuse, and the value of $\tan(\theta)$ is the ratio of the length of the opposite side to the length of the adjacent side.

4. What are reference angles, and how are they used to evaluate trig functions?

   The reference angle for an angle θ is the angle that the radius corresponding to θ on the unit circle makes with the x-axis, always measured as a positive angle. (For example, the reference angle for $7\pi/6$, and angle in the third quadrant, is $\pi/6$.) We used the greek letter α (alpha) to denote reference angles.

   To use reference angles to evaluate trig functions, we have that the value of a trig function at θ will be the same as the value of the trig function at α, except it may be negative depending on the which quadrant the angle θ lies in. For example, to evaluate $\sin(7\pi/6)$, we have that $\sin(7\pi/6) = \pm\sin(\pi/6) = \pm 1/2$, and we need to decide if the value will be positive or negative. The sine function is negative in the third quadrant, so we have $\sin(7\pi/6) = -1/2$. We used the mnemonic"All Students Take Calculus" to remember which trig functions are positive in which quadrants.

5. State as many trig identities as you can.

   You should know all of the trig identities listed on the reference sheet on the website.

6. What are the domain and range of the trigonometric functions? What are the domain and range of the inverse trigonometric functions?

   Sine and cosine have domain all real numbers, and range $[-1, 1]$. Tangent has domain all real numbers except $\frac{\pi}{2} + \pi k$ for k an integer. Recall that these points are where the cosine function is zero, and correspond to vertical asymptotes on the graph of tangent. The range of the tangent function is all real numbers.

   The sine, cosine, and tangent functions are not invertible, so to define their inverse functions we must restrict their domains. For sine, we restrict the domain to $[-\pi/2, \pi/2]$. Therefore the inverse sine function has domain $[-1, 1]$ and range $[-\pi/2, \pi/2]$. For cosine, we restrict the domain to $[0, \pi]$, so the inverse cosine function has domain $[-1, 1]$ and range $[0, \pi]$. For tangent, we restrict the domain to $(-\pi/2, \pi/2)$. So the inverse tangent function has domain $(-\infty, \infty)$ and range $(-\pi/2, \pi/2)$.

7. Why do mathematicians use radians instead of degrees for angle measure? How do you convert between radian and degree measures?

   Degrees are very arbitrary - there is no natural reason to divide a circle into 360° instead of some other number. Instead, mathematicians use radians, which are related to arc length along the circle. If you draw the angle θ, measured in radians, on the unit circle, the arc length of the sector of the circle swept out by this angle will be exactly θ units long.

   To convert from degrees to radians, multiply the measure of the angle in degrees by $\frac{\pi}{180}$. To convert from radians to degrees, multiply the measure of the angle in radians by $\frac{180}{\pi}$.

8. What are the period, amplitude, phase shift, and center line of a sinusoidal trigonometric function? How do you find these values using a graph? Using a formula?

   The period of the function is the difference between two maximum values of the graph (or two minimum values). The center line occurs halfway between the maximum and minimum values of the function. The amplitude is is the distance from the center line to the maximum (or minimum) value of the function. The phase shift is how far the graph is shifted left or right from the standard sine or cosine graph. Note that all of these values are easily computed from looking at a graph.

   Using a formula, we could have a formula of the type

$$\begin{aligned} f(x) &= a\sin(b(x+d)) + c \\ g(x) &= a\cos(b(x+d)) + c \end{aligned}$$

   In theses formulas, the amplitude is given by $|a|$, the period is $\frac{2\pi}{|b|}$, the phase shift is given by d, and the center line occurs at $y = c$.

2 Sample Final

This sample final is meant to have the same style of questions and the same length and difficulty as the final. However, topics covered on this sample final may not be on the actual final, and there may be topics on the final that do not appear on the sample final. You are responsible for knowing all of the concepts on the topics sheet for the final. There will be no "cheat sheet" or list of formulas included with the final.

You may not be able to complete questions marked with a star ($\star$) until after the lecture on Monday, June 29.

Question 1: Assume that θ is an angle with $\pi/2 < \theta < \pi$ such that $\sin\theta = 4/5$. Evaluate the following trigonometric functions.

(a) $\cos\theta$

Note that θ is in the second quadrant, so $\cos\theta$ will be negative. Using the Pythagorean identity,

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(\frac{4}{5}\right)^2} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

(b) $\sin(\theta/2)$

Since θ is in the second quadrant, $\theta/2$ will be in either the first or the second quadrant. In either case, the sine function is positive in this quadrant. Therefore, using the half-angle identity, we have

$$\sin(\theta/2) = \sqrt{\frac{1-\cos(\theta)}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

(c) $\sin(\theta + \pi/6)$

Using the sum identity for sine, we have

$$\begin{aligned}
\sin(\theta+\pi/6) &= \sin\theta\cos(\pi/6) + \cos\theta\sin(\pi/6) \\
&= \frac{4}{5}\cdot\frac{\sqrt{3}}{2} + \frac{-3}{5}\cdot\frac{1}{2} \\
&= \frac{4\sqrt{3}-3}{10}
\end{aligned}$$

(d) $\cos(2\theta)$

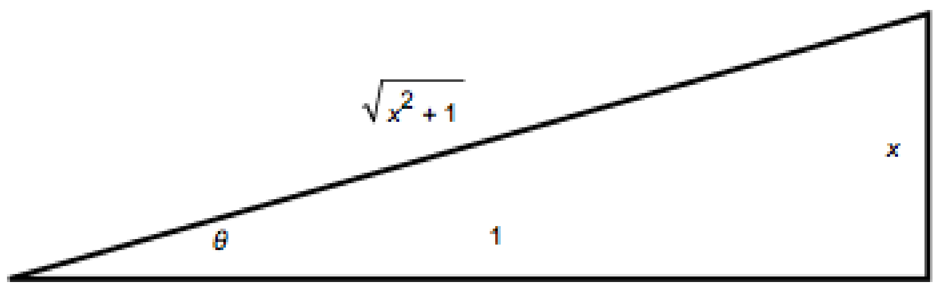
Using the double angle formula for cosine (which has three different forms), we have

$$\begin{aligned}
\cos(2\theta) &= 2\cos^2\theta - 1 \\
&= 2\left(\frac{-3}{5}\right)^2 - 1 \\
&= \frac{18}{25} - 1 \\
&= \frac{-7}{25}
\end{aligned}$$

Question 2: Simplify or evaluate each of the following. Your answers should not include any trigonometric expressions or logarithms.

(a) $\cos(\arctan x)$

Let $\theta = \arctan x$. This means that $\tan\theta = x$, we we draw a right triangle with angle θ such that the side opposite the angle has length x and the side adjacent has length 1. (Recall that the tangent is the ration of opposite side over adjacent side). We then fill in the length of the hypotenuse using the Pythagorean theorem. This gives the following triangle.



We then wish to evaluate $\cos(\arctan(x)) = \cos(\theta)$. Since the cosine is the ratio of the adjacent side to the hypotenuse, we have

$$\cos(\arctan(x)) = \cos(\theta) = \frac{1}{\sqrt{x^2+1}}$$

(b) $2\log_5(7) - \log_5(25+220)$

Using the various logarithm rules,

$$\begin{aligned} 2\log_5(7) - \log_5(25+220) &= \log_5(7^2) - \log_5(245) \\ &= \log_5(49) - \log_5(245) \\ &= \log_5\left(\frac{49}{245}\right) \\ &= \log_5\left(\frac{1}{5}\right) \\ &= -1 \end{aligned}$$

since $5^{-1} = \frac{1}{5}$.

(c) $\cos(\arccos(3/4))$

Recalling that $\cos\theta$ and $\arccos\theta$ are inverse functions, the value is $3/4$. (This is using the identity $\cos(\arccos x) = x$ for all x in the interval $[-1, 1]$.)

(d) $\arccos(\cos(3\pi/4))$

Here the relevant identity is that $\arccos(\cos\theta) = \theta$ only for θ in the interval $[0, \pi]$. Since $3\pi/4$ is in this interval, we have the value is $3\pi/4$.

(e) $\log_6(24) + 2\log_6(3)$

Again using logarithm rules,

$$\begin{aligned} \log_6(24) + 2\log_6(3) &= \log_6(24) + \log_6(9) \\ &= \log_6(24 \cdot 9) = \log_6 216 \\ &= 3 \end{aligned}$$

since $6^3 = 216$.

<u>Question 3:</u> Let

$$f(x) = 2x + 3 \qquad g(x) = \frac{x+1}{x-1}$$

Compute each of the following. Simplify your answer as much as possible

(a) $f(g(2))$

We first compute $g(2)$, which is

$$g(2) = \frac{2+1}{2-1} = 3$$

Therefore

$$f(g(2)) = f(3) = 2 \cdot 3 + 3 = 9$$

(b) $(g \circ f)(x)$.

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g\,(2x+3) \\ &= \frac{(2x+3)+1}{(2x+3)-1} \\ &= \frac{2x+4}{2x+2} \\ &= \frac{2(x+2)}{2(x+1)} \\ &= \frac{x+2}{x+1} \end{aligned}$$

(c) Let $h(x) = (g \circ f)(x)$. Is h invertible? If so, find a formula for h^{-1}.

We found in the previous problem that $h(x) = \frac{x+2}{x+1}$. The best way to check if this function is invertible is simply to try to compute the inverse and see if it is possible. To do this, we set $y = \frac{x+2}{x+1}$ and solve for x. This gives

$$\begin{aligned} y &= \frac{x+2}{x+1} \\ (x+1)y &= x+2 \\ xy + y &= x+2 \\ xy - x &= 2 - y \\ x(y-1) &= 2 - y \\ x &= \frac{2-y}{y-1} \end{aligned}$$

Because we were able to solve for x, the function is invertible, and the inverse function is

$$h^{-1}(y) = \frac{2-y}{y-1}$$

Question 4:

(a) Write the equation of a line that passes through the point $(2, 3)$ and is perpendicular to the line that passes through the points $(-5, 1)$ and $(-7, 8)$

The line through the points $(-5, 1)$ and $(-7, 8)$ has slope

$$\frac{8-1}{-7-(-5)} = \frac{7}{-2}$$

Therefore the slope of the line we are trying to find will be $m = \frac{2}{7}$. The equation for a line passing through $(2, 3)$ and with slope $m = \frac{2}{7}$ is

$$y - 3 = \frac{2}{7}(x-2)$$

(b) A colony of bacteria has an initial population of 100, but has grown to 400 after 3 hours. Write a function that gives the population of the colony of bacteria as a function of t, in hours.

The colony has quadrupled in size in 4 hours, with an initial population of 100, so the population function is

$$P(t) = 100 \cdot 4^{t/3}$$

(c) How long will it take for the population to reach 3200?

We need to solve

$$\begin{aligned} 3200 &= 100 \cdot 4^{t/3} \\ 32 &= 4^{t/3} \\ \log_4(32) &= \frac{t}{3} \\ 3\log_4(32) &= t \end{aligned}$$

We can evaluate this logarithm multiple ways. One easy way is to use the change of base formula for logarithms -

$$\begin{aligned} t &= 3\log_4(32) \\ &= 3 \cdot \frac{\log_2(32)}{\log_2(4)} \\ &= 3 \cdot \frac{5}{2} \\ &= \frac{15}{2} \end{aligned}$$

So it will take $15/2 = 7.5$ hours for the population to reach 3200.

Question 5:

(a) Find the zeroes of the function $f(x) = -3x^3 - 18x^2 - 24x$.

Setting equal to zero and factoring,

$$\begin{aligned} -3x^3 - 18x^2 - 24x &= 0 \\ -3x(x^2 + 6x + 8) &= 0 \\ -3x(x+2)(x+4) &= 0 \end{aligned}$$

Setting each factor to 0 gives the solutions $x = 0, -2, -4$.

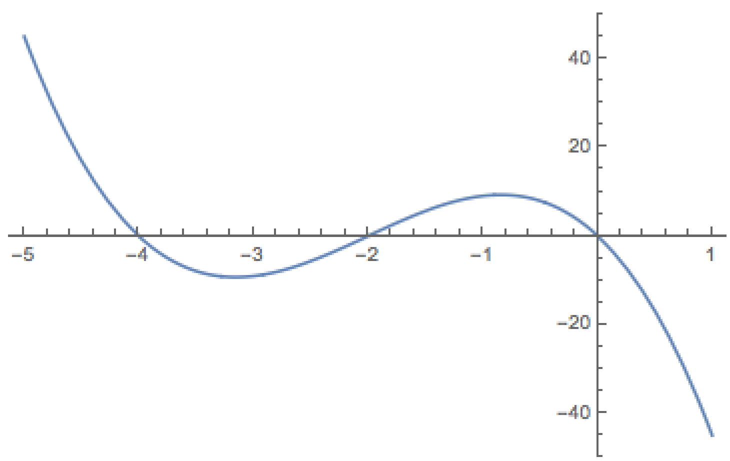
(b) Describe the end behavior of $f(x)$. Write your answer using limit notation.

The function is a polynomial of degree three with a negative leading coefficient. Therefore the end behavior is

$$\lim_{x \to -\infty} f(x) = \infty \qquad \lim_{x \to \infty} f(x) = -\infty$$

(c) Draw a graph of $f(x)$. Your graph should have the proper zeroes, the proper end behavior, and the correct number of "turns."

The graph is shown below. Notice that the graph has two turns, appropriate for a degree 3 function.



(d) Write the quadratic function $g(x) = 4x^2 - 4x + 3$ in vertex form.

Using the shortcut to the vertex form, we have

$$h = \frac{-b}{2a} = \frac{4}{2 \cdot 4} = \frac{1}{2}$$

Evaluating $g(h)$ gives us k, so we have

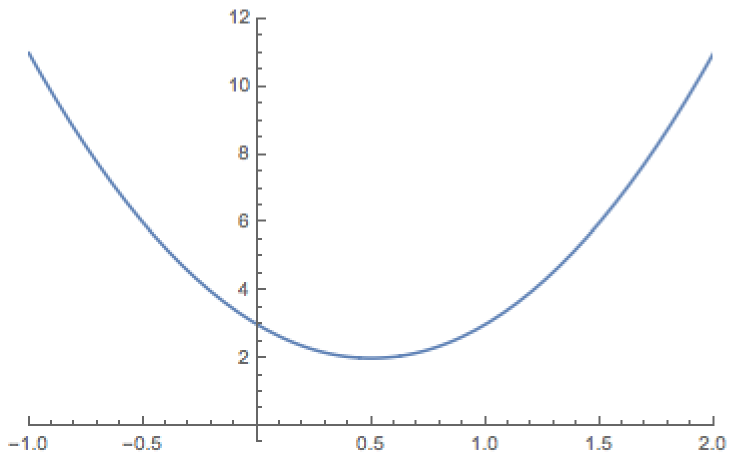
$$g\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 3 = 2$$

Using the vertex form $g(x) = a(x-h)^2 + k$, we have

$$g(x) = 4\left(x - \frac{1}{2}\right)^2 + 2$$

(e) Draw a graph of $g(x)$. Your graph should have the correct vertex and the correct y-intercept.

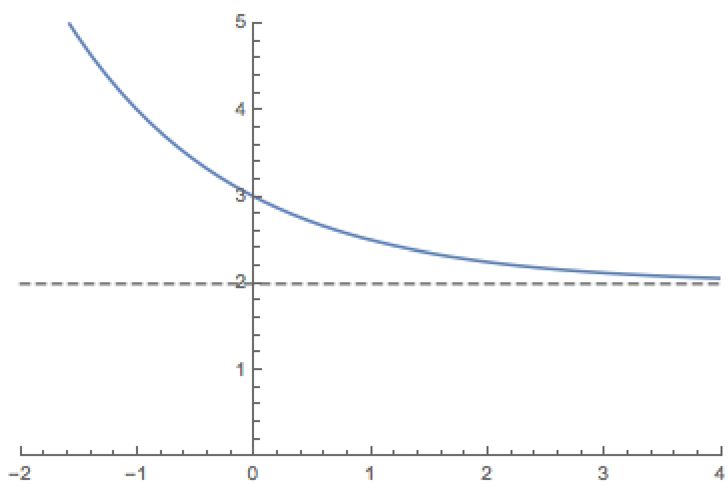
The graph is shown below. Notice the graph has the correct vertex at $\left(\frac{1}{2}, 2\right)$, and has y-intercept at $g(0) = 3$. The graph is obtained from the usual graph of x^2 by shifting right half a unit, stretching vertically by 4, and then shifting up by 2 units.



Question 6: Sketch the graph of each of the following functions. Make sure you graphs include any necessary asymptotes and have the correct domain. Label at least 3 points on each graph.

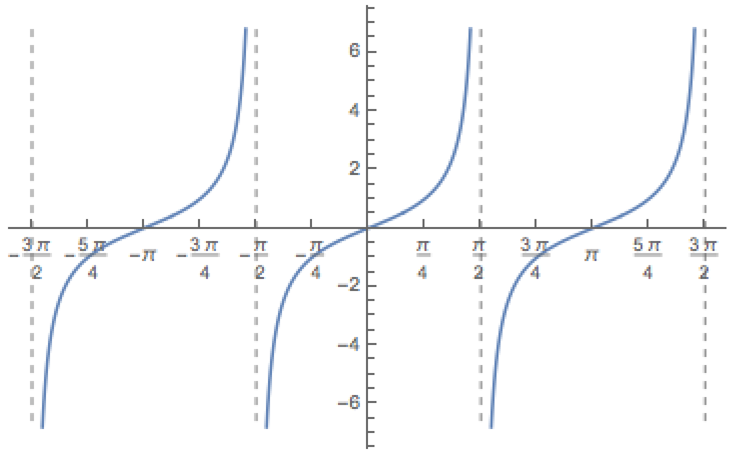
(a) $f(x) = \left(\frac{1}{2}\right)^x + 2$

The graph is below. It is a standard exponential graph in the case $b < 1$, shifted up 2 units. Note the horizontal asymptote at $y = 2$. The easiest points to label are $(0, 3)$ and $(1, 5/2)$.



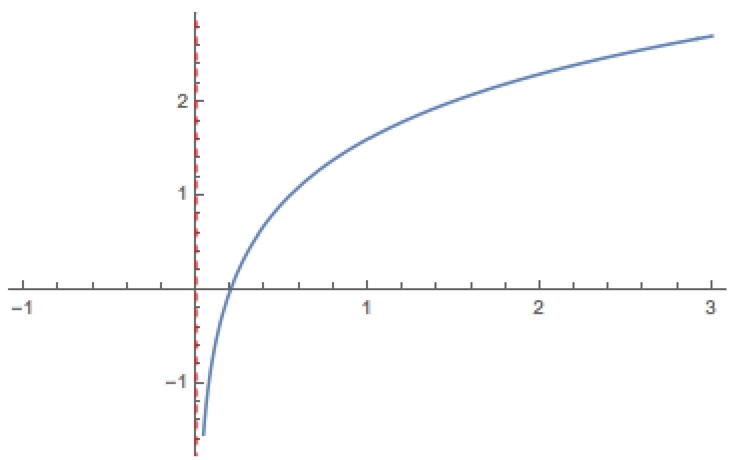
(b) $g(x) = \tan(x)$

The graph is shown below. Recall that the tangent graph has vertical asymptotes at $\pi/2 + k\pi$ for k any integer. The easiest points to plot on the graph are $(-\pi/4, -1), (0, 0)$, and $(\pi/4, 1)$.



(c) $h(x) = \ln(5x)$

This graph is a standard logarithm graph, shrunk horizontally by $1/5$. (You could also write $\ln(5x) = \ln(5) + \ln(x)$, and say that this is the standard logarithm graph shifted up by $\ln(5)$ units. Note that the graph has a vertical asymptote at $x = 0$. The easiest points on the graph to label are $(1/5, 0), (1, \ln(5))$ and $(e/5, 1)$.



Question 7: Consider the function

$$g(x) = -\cos(4x - 1) + 2$$

(a) Identify the period, amplitude, phase shift, and center line of this function.

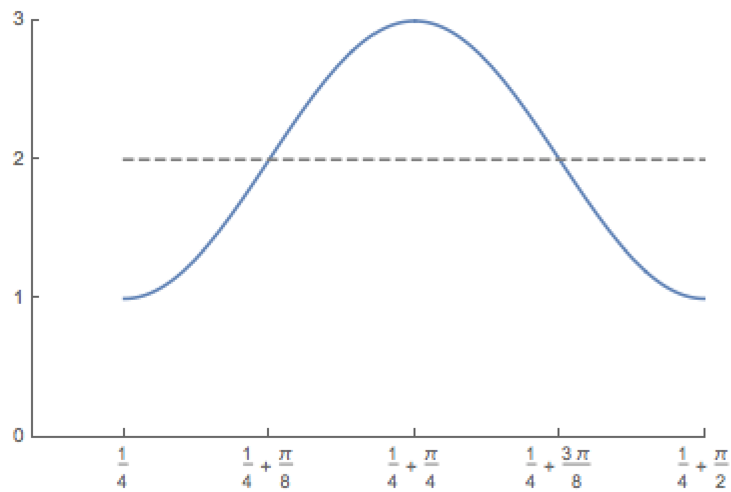
First, we rewrite the equation into the standard form,

$$g(x) = -\cos\left(4(x - \frac{1}{4})\right) + 2$$

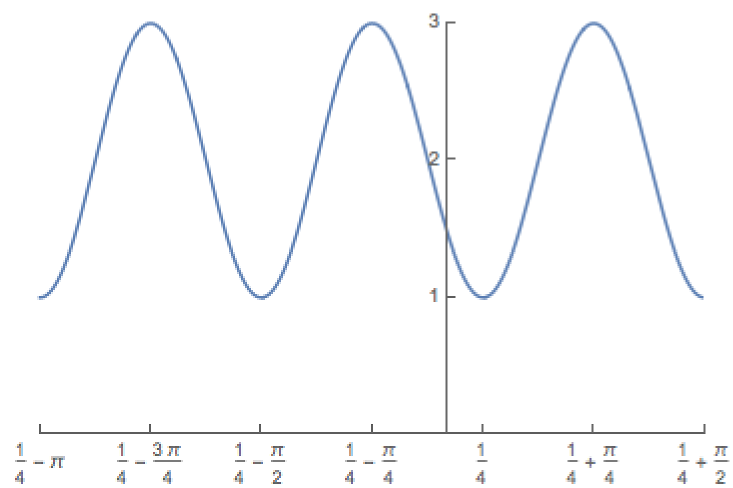
We can then just read the values off from the formula. The period is $\frac{2\pi}{4} = \pi/2$. The amplitude is $|-1| = 1$. The phase shift is $1/4$ unit right. The center line is at $y = 2$.

(b) Using your information from part (a), draw a graph of the function.

Two graphs are shown below. The first shows only one period. The x-axis is labeled starting at 1/4, because of the phase shift right. Then each subsequent marking on the x-axis occurs every $(\pi/2)/4 = \pi/8$ units. The center line is plotted in gray. The graph has the shape of the cosine function, but the negative outside means that is starts below the center line, not above. The amplitude is 1.



The second graph shows more multiple periods of the graph.



Question 8: Solve each of the following equations, or explain why there is no solution.

(a) $\cos(x) = \cos(2x)$

We begin by rewriting the right-hand side using the double angle formula for cosine, which yields

$$\begin{aligned}
\cos(x) &= \cos(2x) \\
\cos(x) &= 2\cos^2 x - 1 \\
0 &= 2\cos^2 x - \cos x - 1 \\
0 &= (2\cos x + 1)(\cos(x) - 1)
\end{aligned}$$

Setting the first fact to zero gives the equation $\cos x = -1/2$. The angles in the interval $[0, 2\pi)$ that solve that equation are $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Setting the second factor to 0 gives the equation $\cos(x) = 1$, and the only angle in $[0, 2\pi)$ that solves that equation is 0. Thus all of the solutions are generated from the angles $0, \frac{2\pi}{3}$ and $\frac{4\pi}{3}$ by adding or subtracting 2π repeatedly, which gives the list

$$\left.\begin{array}{r} 2\pi k \\ \frac{2\pi}{3} + 2\pi k \\ \frac{4\pi}{3} + 2\pi k \end{array}\right\} \text{ where } k = \ldots, -2, -1, 0, 1, 2, \ldots$$

(b) $\ln(x + 1) = 2 - \ln(x - 1)$

We rearrange using logarithm rules, to obtain

$$\begin{aligned} \ln(x + 1) &= 2 - \ln(x - 1) \\ \ln(x + 1) + \ln(x - 1) &= 2 \\ \ln((x + 1)(x - 1)) &= 2 \\ (x + 1)(x - 1) &= e^2 \\ x^2 - 1 &= e^2 \\ x^2 &= e^2 + 1 \\ x &= \pm\sqrt{e^2 + 1} \end{aligned}$$

Notice that $-\sqrt{e^2 + 1}$ is not a solution, since plugging that value in for x would result in taking the logarithm of a negative number, which is not possible. Therefore the only solution is $x = \sqrt{e^2 + 1}$.

(c) $2x^2 + 4x + 3 = 0$

We use the quadratic formula. This gives

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{-4 \pm \sqrt{-8}}{4}$$

Since the discriminant is negative, there are no real solutions.

Question 9: Consider the function

$$f(x) = \frac{4x^3 - x}{x^2 + x - 12}$$

(a) Identify the zeroes of the function.

To find the zeroes of a rational function, we find the zeroes of the numerator. Solving using factoring, we have

$$\begin{aligned} 4x^3 - x &= 0 \\ x(4x^2 - 1) &= 0 \\ x(2x - 1)(2x + 1) &= 0 \end{aligned}$$

So the zeroes of $f(x)$ are $x = 0, 1/2, -1/2$.

(b) What is the domain of this function? Write your answer in interval notation.

The domain will include all points except those x values that make the denominator zero. We first find those values:

$$\begin{aligned} x^2 + x - 12 &= 0 \\ (x-3)(x+4) &= 0 \end{aligned}$$

Therefore the denominator will be zero for $x = 3, -4$, and the domain is all real numbers except $3, -4$. In interval notation, this is

$$(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$$

(c) Describe the end behavior of the function.

The highest degree term on top is $4x^3$, while the highest degree term on bottom is x^2. Therefore $f(x)$ will have the same end behavior as the function

$$\frac{4x^3}{x^2} = 4x$$

Thus function approaches ∞ as x gets large and approaches $-\infty$ as x gets small, so the end behavior of the function will be

$$\lim_{x\to\infty} f(x) = \infty \qquad \lim_{x\to-\infty} f(x) = \infty$$

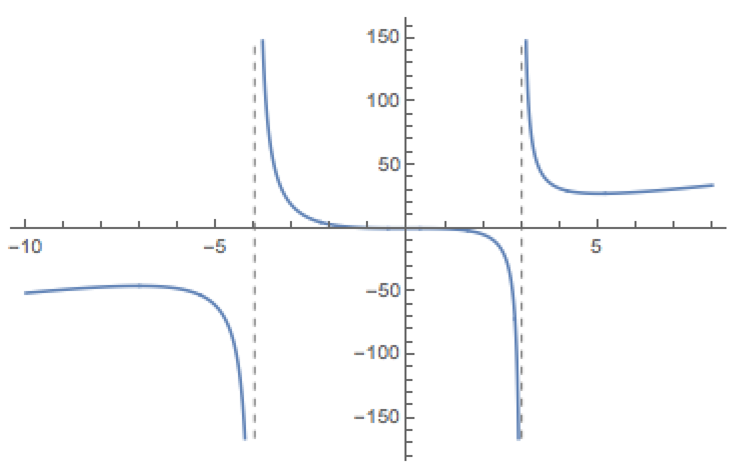
(d) Identify the y-intercept of the function.

We find the y-intercept by plugging 0 into the function. Observe that $f(0) = 0$, so that is the y-intercept.

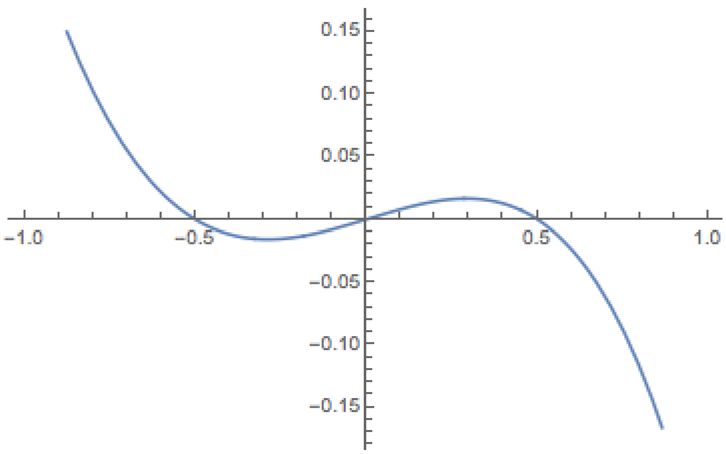
(e) Using the information from the previous parts of this question, draw a graph of the function. Your graph should have the proper domain, asymptotes, zeroes, and y-intercept. To help you graph the function, here are some values of the function at various points.

$$f(-5) = \frac{-495}{8} \qquad f(-2) = 3 \qquad f(2) = -5 \qquad f(4) = \frac{63}{2}$$

Two graphs are shown below. The first graph shows the large-scale behavior of the function. Note that it has vertical asymptotes at $x = -4, 3$, the two points that are not in the domain of the function, and that there is no horizontal asymptotes.



This second graph zooms in to the function around $x = 0$, so that the zeroes can be seen. Note that it has zeroes at $-1/2, 0, 1/2$.



<u>Question 10:</u> A scientist is using carbon-14 dating to find the age of a fossil. She estimates that the fossil contains only 10% of the carbon-14 that was originally in the sample. Carbon-14 has a half life of 5730 years. How old is the fossil? Write your answer using a logarithm.

Letting A_0 represent the original amount of carbon-14 in the fossil, we can write the amount A of carbon-14 in the fossil as a function of time, t, in years, as

$$A(t) = A_0 \left(\frac{1}{2}\right)^{t/5730}$$

We want to find the time when the amount is equal to 10% of the original amount, that is, find the time when the amount is $(0.1)A_0$. So we solve the equation

$$\begin{aligned}
(0.1)A_0 &= A_0 \left(\frac{1}{2}\right)^{t/5730} \\
(0.1) &= \left(\frac{1}{2}\right)^{t/5730} \\
\log_{1/2}(0.1) &= \frac{t}{5730} \\
5730 \cdot \log_{1/2}(0.1) &= t
\end{aligned}$$

Thus the fossil's age is $5730 \cdot \log_{1/2}(0.1)$ years.

You would not be able to find a decimal value of that number on a test, but the actual value of this number is

$$5730 \cdot \log_{1/2}(0.1) = 19,034$$

<u>Question 11:</u> Verify the following trigonometric identity, the triple angle formula for sine -

$$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$$

(*Hint:* Use the sum and double angle formulas after writing $\sin(3\theta) = \sin(\theta + 2\theta)$).

$$\begin{aligned}
\sin(3\theta) &= \sin(\theta + 2\theta) \\
&= \sin\theta\cos(2\theta) + \cos\theta\sin(2\theta) \\
&= \sin\theta(1 - 2\sin^2\theta) + \cos\theta(2\sin\theta\cos\theta) \\
&= \sin\theta(1 - 2\sin^2\theta) + 2\sin\theta\cos^2\theta \\
&= \sin\theta(1 - 2\sin^2\theta) + 2\sin\theta(1 - \sin^2\theta) \\
&= \sin\theta - 2\sin^3\theta + 2\sin\theta - 2\sin^3\theta \\
&= 3\sin\theta - 4\sin^3\theta
\end{aligned}$$

($\star$) **Question 12:** Compute the following limits.

(a) $\displaystyle\lim_{x\to 1} \frac{x^2 + x - 2}{x - 1}$

If we try to evaluate the limit by simply plugging in 1 for x, we'll get $\frac{0}{0}$. This means that both the numerator and the denominator have a common factor of $(x - 1)$, which we can cancel. This gives

$$\begin{aligned}
\lim_{x\to 1} \frac{x^2 + x - 2}{x - 1} &= \lim_{x\to 1} \frac{(x + 2)(x - 1)}{x - 1} \\
&= \lim_{x\to 1}(x + 2)
\end{aligned}$$

No we can simplfy plug in $x = 1$, which yields 3, so

$$\lim_{x\to 1} \frac{x^2 + x - 2}{x - 1} = 3$$

(b) $\displaystyle\lim_{x\to\infty} \frac{x^2 + x - 2}{x - 1}$

Recall that this limit is asking us to find the end behavior as x gets large. Dividing both the top and bottom by x, we have

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^2 + x - 2}{x - 1} &= \lim_{x\to\infty} \frac{x^2 + x - 2}{x - 1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \lim_{x\to\infty} \frac{x + 1 - \frac{2}{x}}{1 - \frac{1}{x}}
\end{aligned}$$

Both $\frac{1}{x}$ and $\frac{2}{x}$ go to zero as $x \to \infty$, so we have that this limit is

$$\lim_{x\to\infty} \frac{x + 1}{1} = \lim_{x\to\infty} x + 1 = \infty$$

So the final answer is

$$\lim_{x\to\infty} \frac{x^2 + x - 2}{x - 1} = \infty$$

(c) $\lim_{x\to 2}(x^2 - 2x + 1)$

In this case, we're taking the limit of a polynomial, which is a continuous function. Therefore we can just plug in the value $x = 2$, to get

$$\lim_{x\to 2}(x^2 - 2x + 1) = 2^2 - 2(2) + 1 = 1$$

(d) $\lim_{h\to 0} \dfrac{(3+h)^2 - 3^2}{h}$

If we try to plug in $h = 0$, we again get $\frac{0}{0}$. To evaluate, we simplify the top, yielding

$$\begin{aligned}
\lim_{h\to 0} \frac{(3+h)^2 - 3^2}{h} &= \lim_{h\to 0} \frac{9 + 6h + h^2 - 9}{h} \\
&= \lim_{h\to 0} \frac{6h + h^2}{h} \\
&= \lim_{h\to 0}(6 + h) = 6
\end{aligned}$$

so

$$\lim_{h\to 0} \frac{(3+h)^2 - 3^2}{h} = 6$$

MAT 123 - Limits Practice

Try computing the following limits. The answers are given to help you check your work, but you should be able to explain why each answer is correct. This sheet is just for your practice - you do not need to turn in any work to be graded.

1. $\displaystyle\lim_{x\to 3} \frac{x^3 - x^2 - 6x}{x^2 - 2x - 3} = \frac{15}{4}$

2. $\displaystyle\lim_{x\to 2} \frac{x^3 - x^2 - 6x}{x^2 - 2x - 3} = \frac{8}{3}$

3. $\displaystyle\lim_{x\to -\infty} \frac{3x^3 - 12}{4x^3 + 3} = \frac{3}{4}$

4. $\displaystyle\lim_{x\to \pi/2} \frac{\cot x}{\cos x} = 1$

5. $\displaystyle\lim_{x\to 1} \left(x^3 + 2x + 12\right) = 15$

6. $\displaystyle\lim_{h\to 0} \frac{(3+h)^3 - 3^3}{h} = 27$

7. $\displaystyle\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = 3x^2$

8. $\displaystyle\lim_{x\to 0^+} \sqrt{x} = 0$

9. $\displaystyle\lim_{x\to 0} \tan(x) = 0$

Name: ______________________________

MAT 123 - HW 9 Supplement

For each of the functions below, identify the amplitude, period, phase shift, and center line for the graph. Then draw one period of the graph, making sure to clearly label the axes of you graph.

1. $f(x) = -2\sin\left(\frac{x}{2} + \frac{\pi}{6}\right)$
2. $g(x) = \sin(-\pi x + \pi) - 1$
3. $h(x) = 7\cos(2\pi x) - 3/2$

For each of the equations below, find all of the possible solutions (there will be infinitely many).

4. $4\tan\theta + 2 = 2\tan\theta$
5. $4\sin^2\theta - 1 = 0$
6. $\cos^2 x + \cos x = \sin^2 x$
7. $2\cos(x)\sin(5x) + 2\cos(x) + \sin(5x) + 1 = 0$

Write your answers to these questions on separate sheets of paper, showing all of your work. Staple all of this together, with this page as the first page, and turn this assignment in at the start of class on June 29.

MAT 123 - Trig Identities

You will be responsible for knowing the following trigonometric identities.

1. The Pythagorean identities
   (i) $\sin^2\theta + \cos^2\theta = 1$
   (ii) $\tan^2\theta + 1 = \sec^2\theta$
   (iii) $1 + \cot^2\theta = \csc^2\theta$

2. The Even-Odd Identities
   (i) $\sin(-\theta) = -\sin(\theta)$ (sine is odd)
   (ii) $\cos(-\theta) = \cos(\theta)$ (cosine is even)
   (iii) $\tan(-\theta) = -\tan(\theta)$ (tangent is odd)

3. The Complementary Angles Identities
   (i) $\sin(\theta) = \cos(\frac{\pi}{2} - \theta)$
   (ii) $\cos(\theta) = \sin(\frac{\pi}{2} - \theta)$
   (iii) $\tan(\theta) = \frac{1}{\tan(\frac{\pi}{2} - \theta)}$

4. The Periodic Identities
   (i) $\sin(\theta) = \sin(\theta + 2\pi)$
   (ii) $\cos(\theta) = \cos(\theta + 2\pi)$
   (iii) $\tan(\theta) = \tan(\theta + \pi)$

5. The Sum and Difference Identities
   (i) $\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$
   (ii) $\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

6. The Double Angle Identities
   (i) $\sin(2\theta) = 2\sin\theta\cos\theta$
   (ii) $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

7. The Power-Reducing Identities
   (i) $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$
   (ii) $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

8. The Half-Angle Identities
   (i) $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$
   (ii) $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$

Name: ______________________________

MAT 123 - HW 7 Supplement

For each of the following angles, you should

(a) Draw the angle on the unit circle.

(b) Identify the reference angle, and show the reference angle in your drawing for part (a).

(c) Compute $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.

Write your answers to these questions on separate sheets of paper. Make sure your answers are legible and your graphs are understandable. Staple all of this together, with this page as the first page, and turn this assignment in at the start of class on June 22.

1. $\theta = -\frac{4\pi}{3}$

2. $\theta = \frac{13\pi}{2}$

3. $\theta = \frac{11\pi}{6}$

4. $\theta = -\frac{9\pi}{4}$

5. $\theta = -375\pi$

6. $\theta = \frac{2\pi}{3}$

Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to `http://www.stonybrook.edu/ehs/fire/disabilities.shtml`.

Tentative Course Schedule

Below is a tentative schedule for the course. For the most up-to-date schedule, which will include homework assignments, see the course web page. The numbers refer to sections of the textbook.

Date	**Topics Covered**
5/27	• Overview of Course and Syllabus • 1.1 - Functions • 1.2 - Coordinate Plane and Graphs • 1.4 - Composition of Functions
5/29	Note: Correction Day. Friday 5/29 follows a Monday schedule • 1.3 - Function Transformations and Graphs • 1.5, 1.6 - Inverse Functions
6/1	• 2.1 - Lines and Linear Functions • 2.2 - Quadratic Functions and Conics
6/3	• 2.4 - Polynomials • 2.5 - Rational Functions
6/8	• 2.3, 3.1 - Exponential and Logarithmic Functions • 3.2, 3.3 - Logarithm Rules
6/10	• Midterm Exam • 3.4, 3.7 - Exponential Growth
6/15	• 4.1 - The Unit Circle • 4.2 - Radians • 4.3, 4.4 - Trigonometric Functions
6/17	• 4.5 - Trigonometry of Right Triangles • 4.6 - Trigonometric Identities
6/22	• 5.1 - Inverse Trigonometric Functions • 5.2 - Inverse Trigonometric Identities
6/24	• 5.5 - Double- and Half-Angle Identities • 6.1 - Transformations of Trigonometric Functions
6/29	• What is Calculus? • Review for Final Exam
7/1	Final Exam

MAT 123 - Graphing Practice

The book doesn't have many practice problems to help you graph rational, exponential, and logarithm functions, so try to graph the following functions on your own. Include as much information in your graphs as you can - proper domain and range, x and y intercepts, vertical asymptotes, horizontal asymptotes, end behavior, and a few labeled points as well.

To check your answers, use WolframAlpha. Go to `http://www.wolframalpha.com`, type in "graph", and then type in the function you'd like to graph. For example, to see a graph of $k(x)$, you could type

```
graph (2x^2-4x+2)/(x^2+2x+1)
```

into the box to produce a graph. Note the parentheses, which are necessary for order of operations.

(a) $f(x) = 3^x$

(b) $g(x) = -(2^x) + 1$

(c) $h(x) = \log_4(x)$

(d) $j(x) = \dfrac{x^2 - 5x + 6}{x^3 + 5x^2 + 4x}$

(e) $k(x) = \dfrac{2x^2 - 4x + 2}{x^2 + 2x + 1}$

(f) $\ell(x) = \dfrac{x^3 - 4x}{2x - 1}$

Name: ______________________________

MAT 123 Quiz 1 - June 29

Question 1: Let $f(x) = \frac{x+1}{2x-1}$ and let $g(x) = \sqrt{x}$.

(a) Write the formula for $(f \circ g)(x)$.

(b) Write the formula for $(g \circ f)(x)$

(c) What is the domain of the function $\left(\frac{g}{f}\right)(x)$?

Name: SOLUTION

MAT 123 Quiz 1 - June 29

Question 1: Let $f(x) = \frac{x+1}{2x-1}$ and let $g(x) = \sqrt{x}$.

(a) Write the formula for $(f \circ g)(x)$.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(\sqrt{x}\right) \\ &= \frac{\sqrt{x}+1}{2\sqrt{x}-1}\end{aligned}$$

You can stop here for full credit - you didn't have to simplify on this quiz. If you want, though, you can remove the square root from the denominator as follows –

$$\begin{aligned}\frac{\sqrt{x}+1}{2\sqrt{x}-1} &= \frac{\sqrt{x}+1}{2\sqrt{x}-1} \cdot \frac{2\sqrt{x}+1}{2\sqrt{x}+1} \\ &= \frac{(\sqrt{x}+1)(2\sqrt{x}+1)}{(2\sqrt{x}-1)(2\sqrt{x}+1)} \\ &= \frac{(\sqrt{x}+1)(2\sqrt{x}+1)}{4x-1} \\ &= \frac{2x+3\sqrt{x}+1}{4x-1}\end{aligned}$$

Here we simplified the denominator using the difference of squares formula, which says that for any numbers a, b, we have

$$a^2 - b^2 = (a-b)(a+b)$$

(b) Write the formula for $(g \circ f)(x)$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g\left(\frac{x+1}{2x-1}\right) \\ &= \sqrt{\frac{x+1}{2x-1}}\end{aligned}$$

(c) What is the domain of the function $\left(\frac{g}{f}\right)(x)$?

First, we'll figure out the domains of f and g. For f, the only problem we might have would be that we can't divide by zero. So we can't plug in any x such that $2x - 1 = 0$. Solving that equation, we have that we can't plug in $x = 1/2$. So the domain of f is all real numbers except $1/2$. In interval notation, that is

$$(-\infty, 1/2) \cup (1/2, \infty)$$

For $g(x) = \sqrt{x}$, recall that we can't take the square root of a negative number. Therefore the domain of g is all numbers greater than or equal to 0. In interval notation, this is

$$[0, \infty)$$

Finally, remember that the domain of g/f is the set of all numbers x that are in both the domain of f and the domain of g, except for any numbers that would make $f(x) = 0$ (we can't divide by zero). We notice that $f(x) = 0$ when the numerator is zero, that is, when $x = -1$. This point is already not in the domain, though, since $x = -1$ is not in the domain of g.

Therefore the domain of g/f is exactly the set of all numbers that are in both the domain of f and the domain of g. Comparing the domains of f and g, this will be the set of all numbers that are bigger than or equal to zero, except for 1/2. In interval notation, this is

$$[0, 1/2) \cup (1/2, \infty)$$

For almost full credit, you could have written that

$$\begin{aligned} \left(\frac{g}{f}\right)(x) &= \frac{\sqrt{x}}{\frac{x+1}{2x-1}} \\ \text{"} &= \text{"} \frac{\sqrt{x}(2x-1)}{(x+1)} \end{aligned}$$

This equals sign is not correct, because you can plug $x = 1/2$ to the expression on the second line and get out a number (it will be 0), but you cannot plug in $x = 1/2$ into the expression on the first line (you'll end up dividing by zero). Still, if you wrote that

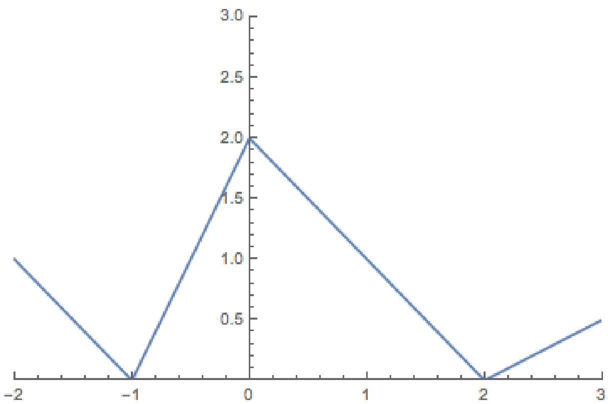
$$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{x}(2x-1)}{(x+1)}$$

and used that to show that the domain of that function is $[0, \infty)$, then you'll get almost full credit.

Name: ______________________________

MAT 123 Quiz 2 - June 1

Question 1: The graph of some function $f(x)$ is shown below.



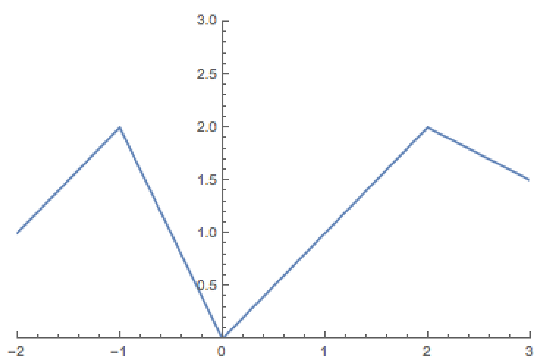
Match the functions listed below to their graphs. (Pay attention to the labels on the axes)

(i) $g(x) = f(2x)$ Graph ____________

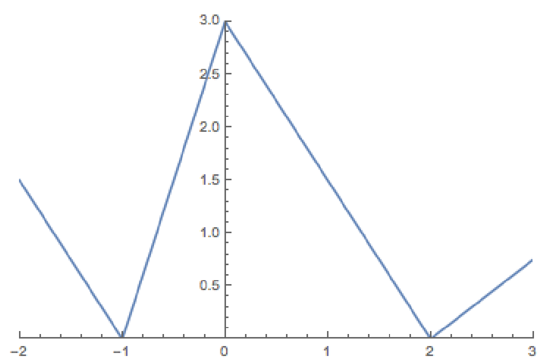
(ii) $h(x) = -f(x) + 2$ Graph ____________

(iii) $j(x) = f(x-1)$ Graph ____________

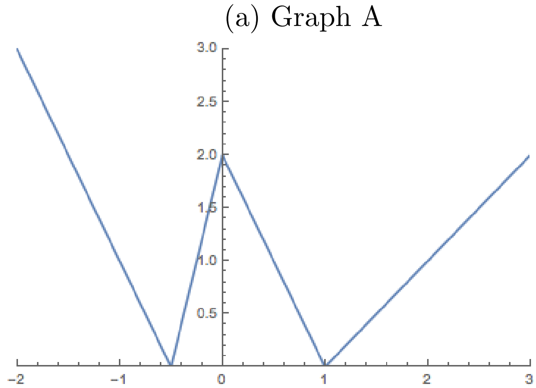
(iv) $k(x) = \frac{3}{2}f(x)$ Graph ____________



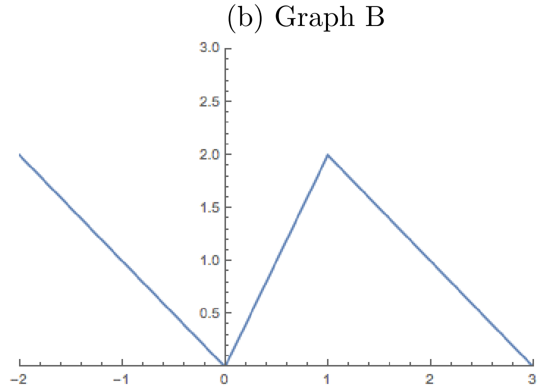
(a) Graph A



(b) Graph B



(c) Graph C

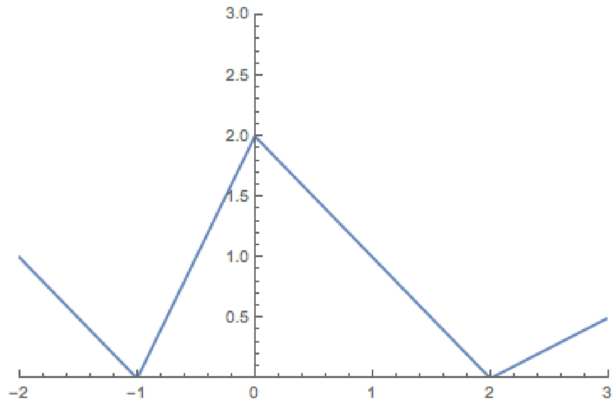


(d) Graph D

Question 2: The function $f(x) = \frac{x+1}{2x-3} + 1$ is invertible. Find a formula for the inverse function f^{-1}. What is the domain of f^{-1}? What is the range of f?

MAT 123 Quiz 2 Solutions

Question 1: The graph of some function $f(x)$ is shown below.



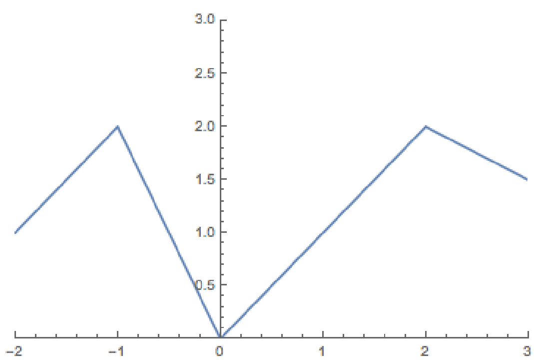
Match the functions listed below to their graphs. (Pay attention to the labels on the axes)

(i) $g(x) = f(2x)$ Graph C

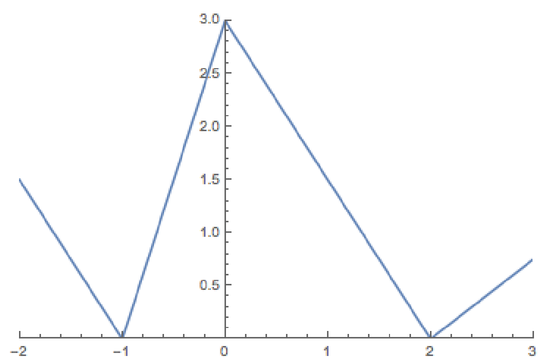
(ii) $h(x) = -f(x) + 2$ Graph A

(iii) $j(x) = f(x-1)$ Graph D

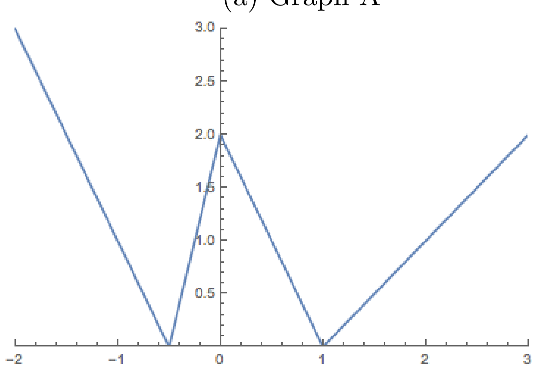
(iv) $k(x) = \frac{3}{2}f(x)$ Graph B



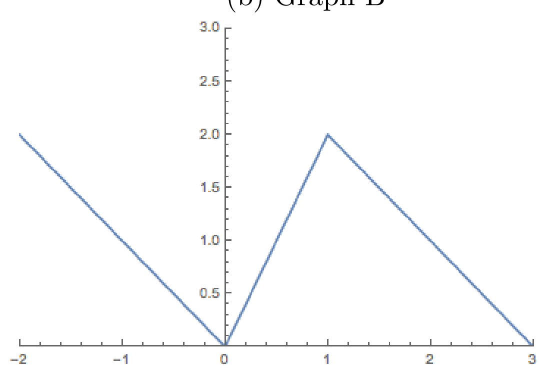
(a) Graph A



(b) Graph B



(c) Graph C



(d) Graph D

The graph of $g(x)$ is obtained from a horizontal shrink of the original graph, which gives Graph C. The graph of $h(x)$ is obtained by reflecting the graph of $f(x)$ over the x-axis and shifting up 2 units, which gives graph A. The graph of $j(x)$ is the graph of $f(x)$ shifted right 1 unit, giving graph D. Finally the graph of $k(x)$ is obtained by stretching the graph of $f(x)$ vertically, which yields graph B.

Question 2: The function $f(x) = \frac{x+1}{2x-3} + 1$ is invertible. Find a formula for the inverse function f^{-1}. What is the domain of f^{-1}? What is the range of f?

To find the inverse, we solve the equation $y = \frac{x+1}{2x-3} + 1$ for x:

$$\begin{aligned} y &= \frac{x+1}{2x-3} + 1 \\ y - 1 &= \frac{x+1}{2x-3} \\ (2x-3)(y-1) &= x+1 \\ 2xy - 2x - 3y + 3 &= x+1 \\ 2xy - 3x &= 3y - 2 \\ x(2y-3) &= 3y-2 \\ x &= \frac{3y-2}{2y-3} \end{aligned}$$

Therefore the inverse function is $f^{-1}(y) = \dfrac{3y-2}{2y-3}$, or you could also have written $f^{-1}(x) = \dfrac{3x-2}{2x-3}$.

We find the domain of $f^{-1}(y)$ as usual. We can plug any number into f^{-1} except for those that would result in dividing by zero. So we can include all y except for those such that $2y - 3 = 0$, or all y except for $y = \frac{3}{2}$. The domain of f^{-1} is therefore all real numbers except for $\frac{3}{2}$, or, in interval notation,

$$\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$$

Finally, recall that the domain of f^{-1} and the range of f are the same.

Name: ____________________

MAT 123 Quiz 3 - June 3

Question 1: Write the equation of the line that is perpendicular to the line $y = \frac{-2}{3}x + 4$ and passes through the point $(-2, -5)$.

Question 2: Consider the function $f(x) = 3x^2 + 12x + 7$.

(a) Complete the square to write $f(x)$ in vertex form.

(b) Using your answer from part (a), draw a graph of $f(x)$. Be sure to label the vertex on the graph, and plot the y-intercept.

MAT 123 Quiz 3 Solutions

Question 1: Write the equation of the line that is perpendicular to the line $y = \frac{-2}{3}x + 4$ and passes through the point $(-2, -5)$.

To write the equation of a line, we need to know a point on that line and the slope of the line. We are given a point on our line - $(-2, -5)$ - so all we need to figure out is the slope.

We our told that our line is perpendicular to the line $y = \frac{-2}{3}x + 4$, which has a slope of $\frac{-2}{3}$. Using the fact that perpendicular lines have slopes that are negative reciprocals, we have that the slope of our line is $m = \frac{3}{2}$.

Our answer is then to write the equation of a line with slope $m = \frac{3}{2}$ through the point $(-2, -5)$. Using the point-slope form for the equation of a line, we have that our equation is

$$y - (-5) = \frac{3}{2}(x - (-2))$$

or

$$y + 5 = \frac{3}{2}(x + 2)$$

You can stop here for full credit. If you like, though, you could write your answer in slope-intercept form by manipulating the above equation:

$$\begin{aligned} y + 5 &= \frac{3}{2}(x + 2) \\ y + 5 &= \frac{3}{2}x + 3 \\ y &= \frac{3}{2}x - 2 \end{aligned}$$

Question 2: Consider the function $f(x) = 3x^2 + 12x + 7$.

(a) Complete the square to write $f(x)$ in vertex form.

We begin by factoring out the 3 from the first two terms, yielding

$$f(x) = 3(x^2 + 4x) + 7$$

Using the formula for completing the square, we have that

$$x^2 + 4x = (x + 2)^2 - 4$$

Substituting this into our expression gives that

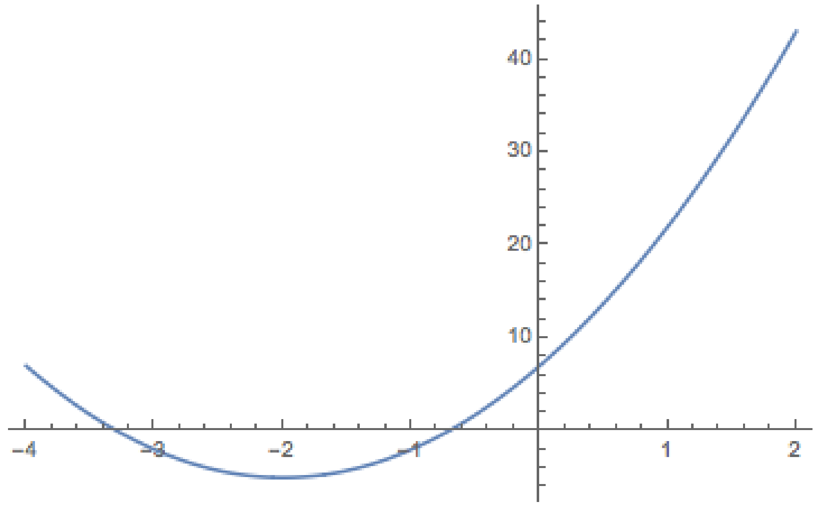
$$\begin{aligned} f(x) &= 3(x^2 + 4x) + 7 \\ &= 3\left((x + 2)^2 - 4\right) + 7 \\ &= 3(x + 2)^2 - 12 + 7 \\ &= 3(x + 2)^2 - 5 \end{aligned}$$

So the function, written in vertex form, is

$$f(x) = 3(x + 2)^2 - 5$$

(b) Using your answer from part (a), draw a graph of $f(x)$. Be sure to label the vertex on the graph and the y-intercept.

The graph of $f(x) = 3(x+2)^2 - 5$ will is obtained from the graph of x^2 by shifting left 2 units, stretching vertically by 3, then shifting down 5 units. The vertex is at $(-2, -5)$. The graph is



Note that this graph has its vertex at (-2,-5). It also has the appropriate y-intercept. The y-intercept is the value of the function when we plug in $x = 0$, which we can compute is 7, and this graph crosses the y-axis at 7.

Name: ________________________________

MAT 123 Quiz 4 - June 15

Question 1: Alice invests \$5,000 in a bank that pays 8% interest, compounded quarterly.

(a) Write a function that give the amount of money in Alice's bank account as a function of the number of years t since her initial deposit.

(b) How long will it take for the balance in Alice's account to double? How long will it take for the balance in Alice's account to quadruple? Write your answers using logarithms.

Question 2: Simplify the expression

$$\log_5(250) - \frac{\log_7(5) + \log_7(2)}{\log_7(5)}$$

Your answer should be an integer.

MAT 123 Quiz 4 Solutions

Question 1: Alice invests \$5,000 in a bank that pays 8% interest, compounded quarterly.

(a) Write a function that give the amount of money in Alice's bank account as a function of the number of years t since her initial deposit.

We'll use the compound interest formula, which is

$$P(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$$

Here $P(t)$ is the amount of money after t years, P_0 is the initial amount, r is the interest rate (written as a decimal), and n is the number of times per year that interest is compounded. Plugging in the given information that $P_0 = 5000, r = 0.08$, and $n = 4$, we have

$$P(t) = 5000\left(1+\frac{0.08}{4}\right)^{4t}$$

(b) How long will it take for the balance in Alice's account to double? How long will it take for the balance in Alice's account to quadruple? Write your answers using logarithms.

Double the initial deposit is \$10000, so we are solving the equation

$$\begin{aligned} 10000 &= 5000\left(1+\frac{0.08}{4}\right)^{4t} \\ 2 &= (1.02)^{4t} \\ \log_{1.02}(2) &= 4t \\ t &= \frac{1}{4}\log_{1.02}(2) \end{aligned}$$

Using a calculator, one can show that this is approximately 8.75 years.

Quadruple the initial deposit is \$20000, we we are solving the equation

$$\begin{aligned} 20000 &= 5000\left(1+\frac{0.08}{4}\right)^{4t} \\ 4 &= (1.02)^{4t} \\ \log_{1.02}(4) &= 4t \\ t &= \frac{1}{4}\log_{1.02}(4) \end{aligned}$$

We can notice using logarithm rules that this number is in fact

$$t = \frac{1}{4}\log_{1.02}(4) = \frac{1}{4}\log_{1.02}(2^2) = \frac{2}{4}\log_{1.02}(2) = \frac{1}{2}\log_{1.02}(2)$$

Which is twice the time it took Alice's money to double. This makes sense - we can see from the equations above that the amount of time it takes for the initial amount of money to double is actually independent of the amount of the initial deposit. Thus if it takes about 8.75 years to double once, it will take $8.75 + 8.75 = 17.5$ years for Alice's money to double twice (that is, quadruple).

Question 2: Simplify the expression

$$\log_5(250) - \frac{\log_7(5) + \log_7(2)}{\log_7(5)}$$

Your answer should be an integer.

First, we use a logarithm rule to combine the two logarithms in the numerator of the fraction. We have that $\log_7(5) + \log_7(2) = \log_7(5 \cdot 2) = \log_7(10)$. Thus the expression above is the same as

$$\log_5(250) - \frac{\log_7(10)}{\log_7(5)}$$

Next, we use the change of base formula to write

$$\log_5(250) - \log_5(10)$$

Combining these into one logarithm gives

$$\log_5(250) - \log_5(10) = \log_5\left(\frac{250}{10}\right) = \log_5(25) = 2$$

Name: ____________________

MAT 123 Quiz 5 - June 17

Question 1: The half-life of sodium-24 is approximately 15 hours. (Recall that this means that the amount of sodium-14 in a given sample will decrease by 1/2 every 15 hours). A scientist is studying a sample that initially contains 10 grams of sodium-24.

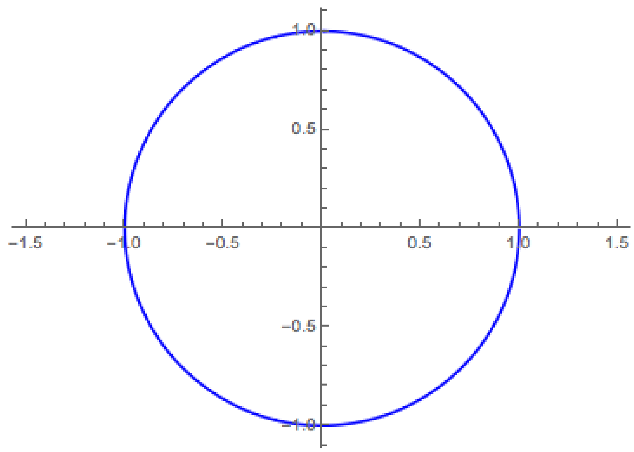
(a) Write a function for the amount of sodium-24 in the sample, A, as a function of the number of hours, t, that have passed since the initial measurement was made.

(b) How many hours will it take for the amount of sodium-24 in the sample to reach 1 gram? Write your answer as an exact number, using logarithms.

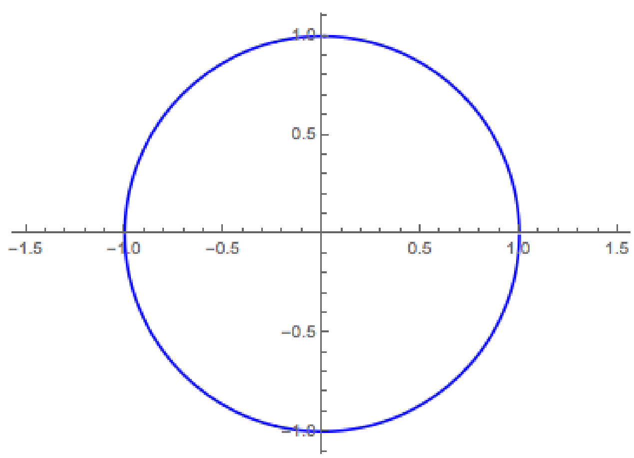
(c) (2 pts bonus) Estimate the value of the number in part (b) by counting approximately how many half-lives must pass for the sample to reach 1 g.

<u>Question 2:</u> Draw a radius corresponding to each angle on the unit circle.

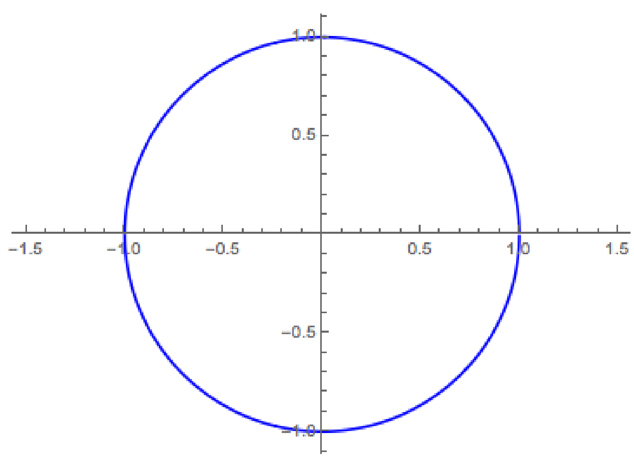
(a) $\theta = \dfrac{7\pi}{6}$



(b) $\theta = -\dfrac{\pi}{4}$



(c) $\theta = 5\pi$



MAT 123 Quiz 5 Solutions

Question 1: The half-life of sodium-24 is approximately 15 hours. (Recall that this means that the amount of sodium-14 in a given sample will decrease by 1/2 every 15 hours). A scientist is studying a sample that initially contains 10 grams of sodium-24.

(a) Write a function for the amount of sodium-24 in the sample, A, as a function of the number of hours, t, that have passed since the initial measurement was made.

The initial amount is 10 grams, and the amount halves every 15 hours. Therefore the function is

$$A(t) = 10\left(\frac{1}{2}\right)^{t/15}$$

(b) How many hours will it take for the amount of sodium-24 in the sample to reach 1 gram? Write your answer as an exact number, using logarithms.

Solving the equation for the value of t that gives an output of 1, we have

$$\begin{aligned} 1 &= 10\left(\frac{1}{2}\right)^{t/15} \\ \frac{1}{10} &= \left(\frac{1}{2}\right)^{t/15} \\ \log_{1/2}\left(\frac{1}{10}\right) = \frac{t}{15} & \\ t &= 15\log_{1/2}\left(\frac{1}{10}\right) \end{aligned}$$

The exact number is therefore $15\log_{1/2}\left(\frac{1}{10}\right)$ hours.

(c) (2 pts bonus) Estimate the value of the number in part (b) by counting approximately how many half-lives must pass for the sample to reach 1 g.

We count the number of times we must halve 10 in order to get approximately one. Repeatedly halving 10 gives the list of numbers

$$10, 5, 2.5, 1.25, 0.625$$

Our value of 1 is between 1.25 and 0.625, which are obtained from halving 10 three and four times, respectively. Note that 1 is closer to 1.25, so we should expect the value of the number to be a little more that 3 half-lives. Each half life is 15 hours, so three half-lives is 45 hours, so we can guess that perhaps it will take approximately 48 hours for the sample to decrease to 1 gram. A similar answer (for example, 47, 49) would also get full credit.

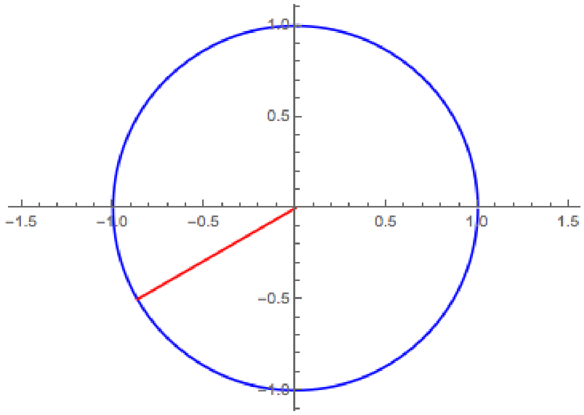
To see how good our guess is, we can find using a calculator that

$$15\log_{1/2}\left(\frac{1}{10}\right) \approx 49.829,$$

so a guess of 48 hours is not far off.

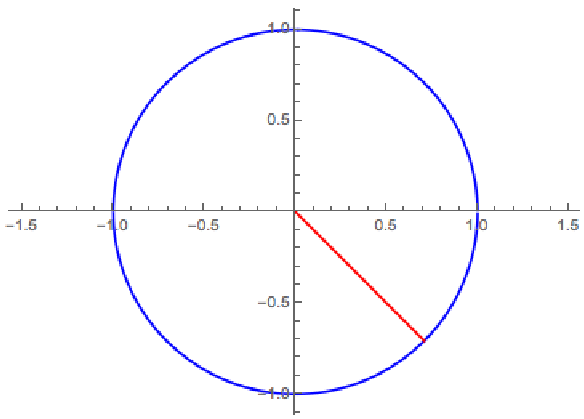
Question 2: Draw a radius corresponding to each angle on the unit circle.

(a) $\theta = \dfrac{7\pi}{6}$



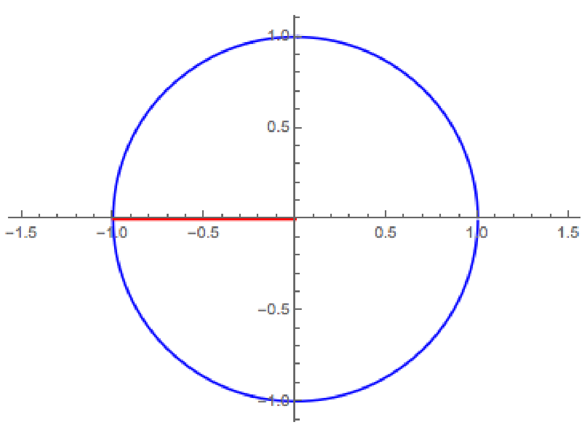
The angle that this radius makes with the negative x-axis is $\pi/6$, or $30°$.

(b) $\theta = -\dfrac{\pi}{4}$



Recall that $\pi/4$ radians is the same as $45°$. Since the angle is negative, we go clockwise from the positive x-axis.

(c) $\theta = 5\pi$



Recall that a given radius may be described by many different angle measures, which differ by adding or subtracting 2π (once around the circle). So the angle measures $5\pi, 5\pi - 2\pi = 3\pi$, and $3\pi - 2\pi = \pi$ all describe the same radius. Thus we can draw the angle 5π just as we draw the angle π, which we recall is halfway around the circle (or $180°$).

Name: ________________________________

MAT 123 Quiz 6 - June 22

Question 1: Fill in the table below with the values of $\sin\theta, \cos\theta$, and $\tan\theta$ for the given values of θ.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$							
$\cos\theta$							
$\tan\theta$							

MAT 123 Quiz 6 Solutions

Question 1: Fill in the table below with the values of $\sin\theta, \cos\theta$, and $\tan\theta$ for the given values of θ.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef	0	undef

Name: ____________________________________

MAT 123 Quiz 7 - June 24

Question 1: Let θ be an angle in the interval $[0, \pi/2]$ such that $\sin(\theta) = \frac{1}{4}$. Evaluate the following

(a) $\cos\theta$

(b) $\cos(2\theta)$

(c) $\sin(\theta + \arccos(1/\sqrt{2}))$

MAT 123 Quiz 7 Solutions

Question 1: Let θ be an angle in the interval $[0, \pi/2]$ such that $\sin(\theta) = \frac{1}{4}$. Evaluate the following

(a) $\cos\theta$

Note that $\cos\theta$ will be positive, since θ is in the first quadrant. Using the Pythagorean identity, we have

$$\sin^2\theta + \cos^2\theta = 1 \Leftrightarrow \cos\theta = \sqrt{1 - \sin^2(\theta)}$$

so

$$\begin{aligned}
\cos(\theta) &= \sqrt{1 - \left(\frac{1}{4}\right)^2} \\
&= \sqrt{1 - \frac{1}{16}} \\
&= \sqrt{\frac{15}{16}} \\
&= \frac{\sqrt{15}}{4}
\end{aligned}$$

(b) $\cos(2\theta)$

We use the double angle formula for cosine -

$$\begin{aligned}
\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\
&= \left(\frac{\sqrt{15}}{4}\right)^2 - \left(\frac{1}{4}\right)^2 \\
&= \frac{15}{16} - \frac{1}{16} \\
&= \frac{14}{16} = \frac{7}{8}
\end{aligned}$$

(c) $\sin(\theta + \arccos(1/\sqrt{2}))$

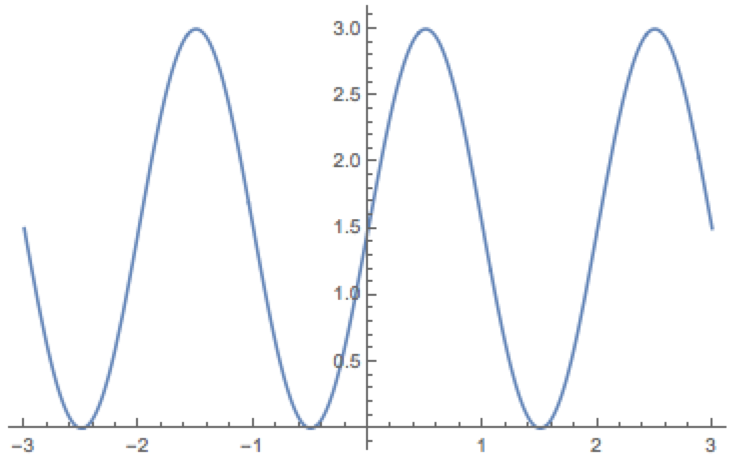
First, we observe that $\arccos(1/\sqrt{2}) = \frac{\pi}{4}$. So we are being asked to evaluate $\sin(\theta + \pi/4)$. Using the sum formula for sine,

$$\begin{aligned}
\sin(\theta + \pi/4) &= \sin(\theta)\cos\left(\frac{\pi}{4}\right) + \cos(\theta)\sin\left(\frac{\pi}{4}\right) \\
&= \frac{1}{4} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{15}}{4} \cdot \frac{1}{\sqrt{2}} \\
&= \frac{1 + \sqrt{15}}{4\sqrt{2}}
\end{aligned}$$

Name: ________________________________

MAT 123 Quiz 8 - June 29

Question 1: Shown below is a graph of a function of the form $f(x) = a\sin\big(b(x+d)\big) + c$.



(a) Using the graph, find the period, amplitude, phase shift, and center line of the graph.

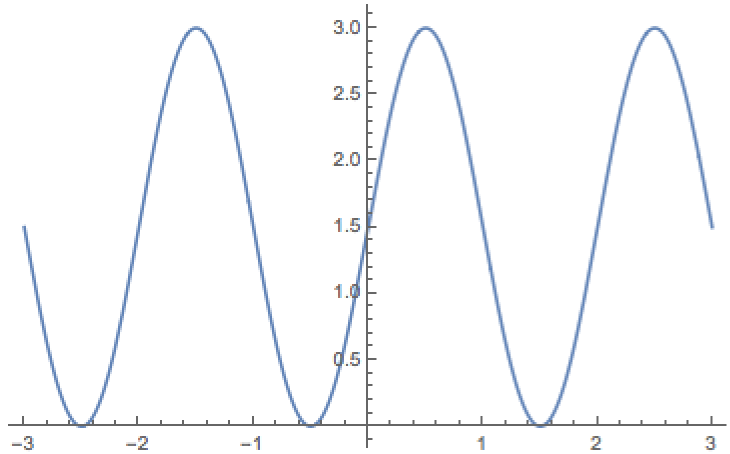
(b) Using your answer from part (a), find the values of the constants a, b, c, and d.

Question 2: Find all solutions to the following equation.

$$2\sin^2 x + 3\cos x - 3 = 0$$

MAT 123 Quiz 8 Solutions

Question 1: Shown below is a graph of a function of the form $f(x) = a\sin\big(b(x+d)\big) + c$.



(a) Using the graph, find the period, amplitude, phase shift, and center line of the graph.

We can see from the graph that the function has a maximum value of 3 and a minimum value of 0. The center line will therefore be halfway between those, at $y = 3/2$. The amplitude will be the distance from the center line to either the maximum or minimum value, so in this case will also be $3/2$.

To find the period, we can observe that the distance between each maximum point in the graph is 2 units. Therefore the graph will have a period of 2 units. Finally, the graph crosses the center line when $x = 0$, just like the graph of $\sin x$, so there is no phase shift in this graph.

(b) Using your answer from part (a), find the values of the constants a, b, c, and d.

The center line occurs at $y = c$, so we have $c = 3/2$. The amplitude is the absolute value of a, so we have that $a = \pm 3/2$. Starting at $x = 0$, we notice that the graph increases, just like the regular $\sin x$ graph, and so we'll have that a is positive. Thus $a = 3/2$. There is no phase shift, so $d = 0$. Finally, the period is $2 = \frac{2\pi}{b}$, so we have that $b = \pi$.

Putting all of this information together, the graph shown is the graph of

$$f(x) = \frac{3}{2}\sin(\pi x) + \frac{3}{2}$$

Note that there are other possible answers to this question, if you pick a different value for the phase shift (which could be any multiple of π), which may require you to change the sign of a.

Question 2: Find all solutions to the following equation.

$$2\sin^2 x + 3\cos x - 3 = 0$$

We begin by using the Pythagorean identity to replace the $\sin^2 x$ with terms involving $\cos x$. The Pythagorean identity can be rearrange to read $\sin^2 x = 1 - \cos^2 x$, so replacing $\sin^2 x$ gives the new equation

$$\begin{aligned} 2(1-\cos^2 x) + 3\cos x - 3 &= 0 \\ 2 - 2\cos^2 x + 3\cos x - 3 &= 0 \\ -2\cos^2 x + 3\cos x - 1 &= 0 \\ (-2\cos x + 1)(\cos x - 1) &= 0 \end{aligned}$$

With the equation factored, we set each factor equal to zero. For the first, we have

$$-2\cos x + 1 = 0 \Leftrightarrow \cos x = \frac{1}{2}$$

So we look for all angles x in the interval $[0, 2\pi)$ that have $\cos x = \frac{1}{2}$. This gives $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$. Setting the second factor equal to zero, we look for all angles x in the interval $[0, 2\pi)$ that have $\cos x = 1$, which gives only $x = 0$. Thus all solutions to this equation will be generated from $x = 0, \frac{\pi}{3}$, and $\frac{5\pi}{3}$ by repeatedly adding or subtracting 2π, so the entire list of solutions is

$$2\pi k, \quad \frac{\pi}{3} + 2\pi k, \quad \frac{5\pi}{3} + 2\pi k \qquad \text{where } k = \ldots, -2, -1, 0, 1, 2, \ldots$$

MAT 123 Midterm Exam

Name: ____

- Please read all instructions before beginning, and do not open the exam until you are told to do so.
- Put away all notes, books, calculators, etc. before beginning the exam, and place them under your desk. The only items on your desk should be this exam, pencils/pens, and an eraser.
- This exam has 6 questions, each with multiple parts. The point value for each question is shown below.
- The final page of the exam is left blank for scratch paper. You may tear this page from the exam booklet, but do not remove any other pages.
- You have 90 minutes to complete the exam.

Question	Points	Score
1	16	
2	7	
3	13	
4	5	
5	9	
6	15	
Total	65	

Question 1: *(16 pts)* Consider the function

$$f(x) = \frac{x^2 - 2x + 1}{3x^2 - 27}$$

(a) What is the domain of this function? Write your answer using interval notation.

(b) What are the zeros of this function?

(c) Describe the end behavior of this function.

(d) Find the y-intercept of the graph.

(e) Using the information from the previous parts of this problem, sketch a graph of this function. Your graph should include all asymptotes and the x and y intercepts. Some values of the function are computed below to help you graph the function -

$$f(-4) = \frac{25}{21} \qquad f(-2) = \frac{-3}{5} \qquad f(2) = \frac{-1}{15} \qquad f(4) = \frac{3}{7}$$

Question 2: *(7 pts)* Consider the points $A = (-1, -4), B = (7, -1)$ and $C = (1, -10)$ in the xy-plane.

(a) Write an equation for the line that passes through the points B and C.

(b) Write an equation for the line that is parallel to the line from part (a) and passes through the point A.

(c) Write an equation for the line that is perpendicular to the line from part (a) and passes through the point A.

Question 3: *(13 pts)* Let $f(x) = 3^x$ and let $g(x) = 2x + 1$.

(a) Write a formula for $h(x) = (g \circ f)(x)$. Using that formula, describe in words how the graph of $h(x)$ can be obtained from the graph of $f(x)$.

(b) Draw the graph of $f(x) = 3^x$. Label at least two points on the graph. On the same set of axes, draw the graph of $h(x)$, and label at least two points on the graph.

(c) If you've drawn the graph of $h(x)$ correctly, you should notices that $h(x)$ is invertible. Draw the graph of $h^{-1}(x)$. Label at least two points on the graph, and make sure your graph has the proper domain and range.

(d) Find a formula for $h^{-1}(x)$. (Your formula should involve a logarithm.)

Question 4: *(5 pts)* Let $M(t) = -2t^2 - 8t - 13$.

(a) Write the equation for this quadratic in vertex form.

(b) Does this function have a maximum value? If so, what is it? Does this function have a minimum value? If so, what is it?

Question 5: *(9 pts)* Solve the following equations, or explain why there is no solution.

(a) $\log_3\left(\frac{1}{9}\right) = \log_x 25$

(b) $2x^2 - 4x - 10 = 20$

(c) $3x^2 + 5x + 3 = 0$

Question 6: *(15 pts)* Consider the polynomial $p(x)$ defined by

$$p(x) = \frac{-1}{2}(x^2 - 4)(x^2 + 1)$$

(a) Find the zeros of this polynomial.

(b) Identify the degree and leading coefficient of $p(x)$.

(c) Describe the end behavior of this polynomial.

(d) Find the values of $p(-1)$ and $p(1)$.

(e) Plot a graph of the polynomial. Your graph should have the proper x and y intercepts, end behavior, and number of "turns," and should pass through the points you found in part (d).

SCRATCH PAPER

MAT 123 Midterm Exam Solutions

Question 1: Consider the function

$$f(x) = \frac{x^2 - 2x + 1}{3x^2 - 27}$$

(a) *(3 pts)* What is the domain of this function? Write your answer using interval notation.

As this is a rational function, the domain will be all real numbers except those that make the denominator zero. The denominator factors as

$$3x^2 - 27 = 3(x^2 - 9) = 3(x - 3)(x + 3)$$

so the denominator will equal zero for $x = \pm 3$. So the domain of the function is all real numbers except ± 3, which, in interval notation, is

$$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

(b) *(3 pts)* What are the zeros of this function?

The zeroes of a rational function are the same as the zeroes of the numerator, provided that they are in the domain. The numerator factors as

$$x^2 - 2x + 1 = (x - 1)^2$$

so the only zero of the numerator is $x = 1$ (which is a repeated root). Note that this is in the domain.

(c) *(4 pts)* Describe the end behavior of this function.

The end behavior is determined by the highest degree terms on top and bottom, so as x gets very large or very small, the values of $f(x)$ will be close to the values of

$$\frac{x^2}{3x^2} = \frac{1}{3}$$

So the end behavior is

$$\lim_{x \to \infty} f(x) = \frac{1}{3} \qquad \lim_{x \to -\infty} f(x) = \frac{1}{3}$$

(Note that this means the function will have a horizontal asymptote at $y = \frac{1}{3}$).

(d) *(1 pt)* Find the y-intercept of the graph.

Recall that the y-intercept is given by the function's value at $x = 0$, so we compute

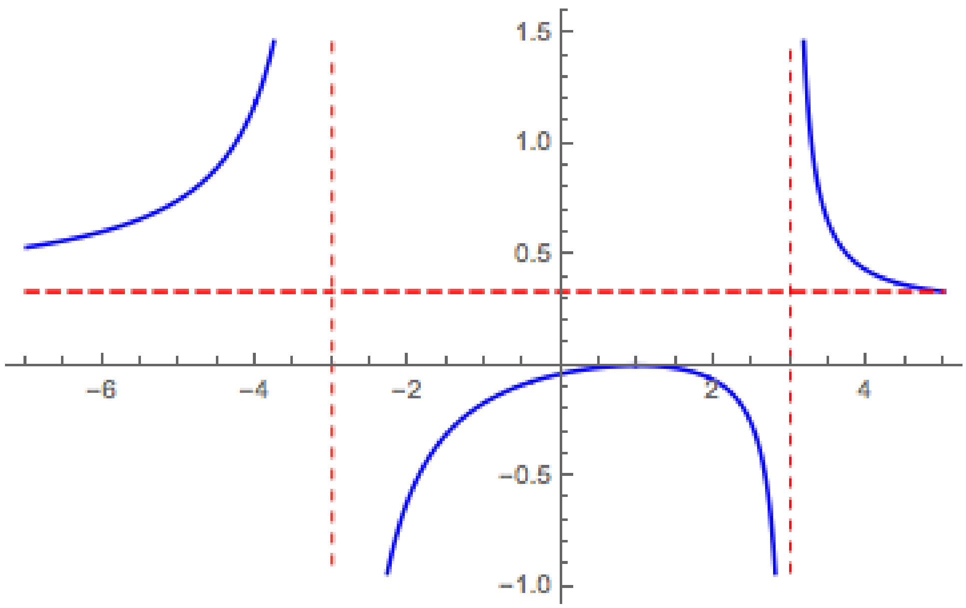
$$f(0) = \frac{0^2 - 2 \cdot 0 + 1}{3 \cdot 0^2 - 27} = \frac{1}{-27}$$

So the y-intercept is at $\frac{-1}{27}$.

(e) *(5 pts)* Using the information from the previous parts of this problem, sketch a graph of this function. Your graph should include all asymptotes and the x and y intercepts. Some values of the function are computed below to help you graph the function -

$$f(-4) = \frac{25}{21} \qquad f(-2) = \frac{-3}{5} \qquad f(2) = \frac{-1}{15} \qquad f(4) = \frac{3}{7}$$

Parts (a) and (b) together tell us that the graph will have vertical asymptotes at $x = -3$ and $x = 3$. Part (c) tells us that the graph will have a horizontal asymptote at $y = \frac{1}{3}$. We then can plot the x- and $y-$intercepts, and finally use the points given above to help us draw each part of the graph. The graph is shown below - the function is graphed in blue, and the asymptotes are graphed in dotted red lines.



Question 2: Consider the points $A = (-1, -4), B = (7, -1)$ and $C = (1, -10)$ in the xy-plane.

(a) *(3 pts)* Write an equation for the line that passes through the points B and C.

Using the slope formula, the line will have slope

$$m = \frac{-10 - (-1)}{1 - 7} = \frac{-9}{-6} = \frac{3}{2}$$

Any of the following answers would then be acceptable. The first two are the point-slope form, using the points B and C respectively, and the third is the same line written in slope-intercept form.

$$\begin{aligned} y + 1 &= \frac{3}{2}(x - 7) \\ y + 10 &= \frac{3}{2}(x - 1) \\ y &= \frac{3}{2}x - \frac{23}{2} \end{aligned}$$

(b) *(2 pts)* Write an equation for the line that is parallel to the line from part (a) and passes through the point A.

Since the line we are trying to write is parallel to the line from part (a), it will have the same slope, $m = \frac{3}{2}$. Using point-slope form with this slope and the point A gives the equation.

$$y + 4 = \frac{3}{2}(x + 1)$$

In slope-intercept form the equation is

$$y = \frac{3}{2}x - \frac{5}{2}$$

(c) *(2 pts)* Write an equation for the line that is perpendicular to the line from part (a) and passes through the point A.

Since the line we are trying to write is perpendicular to the line from part (a), it will have slope, $m = \frac{-2}{3}$. Using point-slope form with this slope and the point A gives the equation.

$$y + 4 = \frac{-2}{3}(x + 1)$$

In slope-intercept form the equation is

$$y = \frac{-2}{3}x - \frac{14}{3}$$

Question 3: Let $f(x) = 3^x$ and let $g(x) = 2x + 1$.

(a) *(3 pts)* Write a formula for $h(x) = (g \circ f)(x)$. Using that formula, describe in words how the graph of $h(x)$ can be obtained from the graph of $f(x)$.

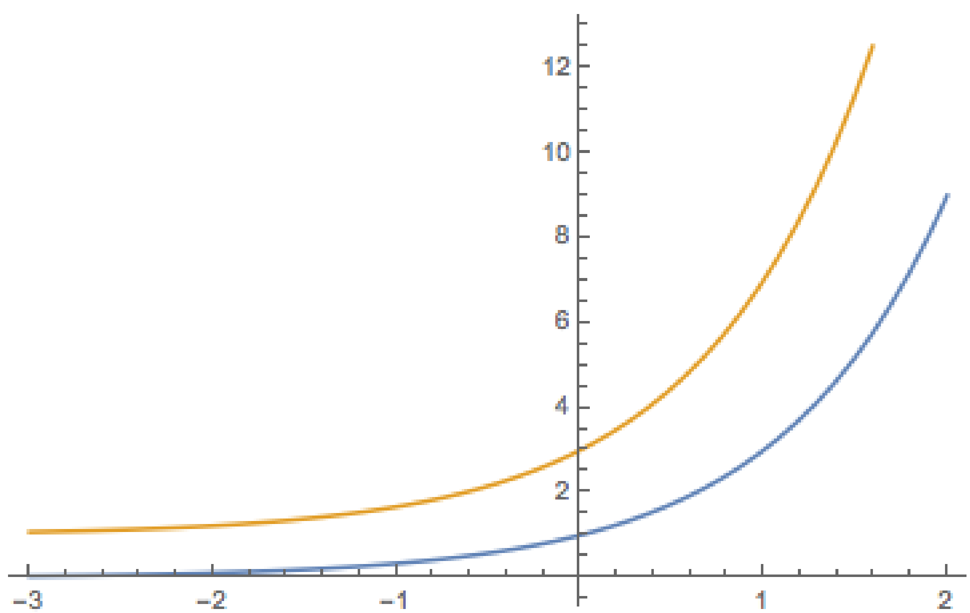
$$\begin{aligned} h(x) &= (g \circ f)(x) = g(f(x)) \\ &= g(3^x) \\ &= 2 \cdot 3^x + 1 \end{aligned}$$

Interpreting this using graph transformations, the graph of $h(x)$ is obtained from the graph of $f(x)$ by a vertical stretch by 2, followed by a shift up one unit.

(b) *(4 pts)* Draw the graph of $f(x) = 3^x$. Label at least two points on the graph. On the same set of axes, draw the graph of $h(x)$, and label at least two points on the graph.

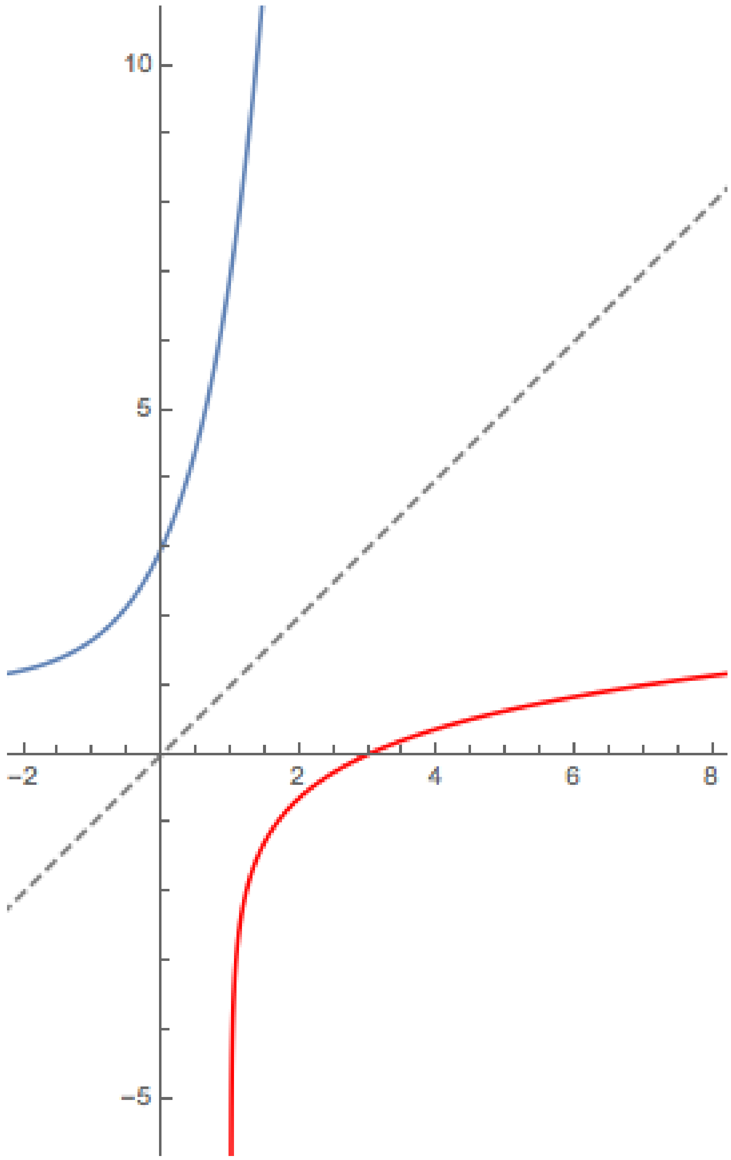
The graph of $f(x) = 3^x$ has the standard shape of an exponential graph. It is show in blue below. Notice that the graph has a horizontal asymptote on the left at $y = 0$, and that the graph passes through the points $(0, 1)$ and $(1, 3)$.

The graph of $h(x)$ is shown in gold. Notice that it has a horizontal asymptote on the left at $y = 1$, which is shifted up 1 unit from the asymptote of the graph of f. It passes through the points $(0, 3)$ and $(1, 7)$.



(c) *(3 pts)* If you've drawn the graph of $h(x)$ correctly, you should notices that $h(x)$ is invertible. Draw the graph of $h^{-1}(x)$. Label at least two points on the graph, and make sure your graph has the proper domain and range.

Recall that the graph of the inverse function is obtained by reflecting the graph of the original function over the line $y = x$. As the original function has a horizontal asymptote at $y = 1$, the inverse function will have a vertical asymptote at $x = 1$. The original function passes through the points $(0, 3)$ and $(1, 7)$, so the new function will pass through the points $(3, 0)$ and $(7, 1)$. The graphs of both $h(x)$ and $h^{-1}(x)$ are shown below. The original graph $h(x)$ is in blue, the inverse $h^{-1}(x)$ is in red, and the line $y = x$ is shown for clarity.



(d) *(3 pts)* Find a formula for $h^{-1}(x)$. (Your formula should involve a logarithm.)

Writing $y = 2 \cdot 3^x + 1$, we solve that equation for x to find the inverse function. This gives

$$\begin{aligned} y &= 2 \cdot 3^x + 1 \\ y - 1 &= 2 \cdot 3^x \\ \frac{y-1}{2} &= 3^x \\ \log_3\left(\frac{y-1}{2}\right) &= x \end{aligned}$$

So the inverse function is

$$f^{-1}(y) = \log_3\left(\frac{y-1}{2}\right)$$

Question 4: Let $M(t) = -2t^2 - 8t - 13$.

(a) *(3 pts)* Write the equation for this quadratic in vertex form.

We'll use the shortcut method to write this in vertex form, instead of completing the square. Using that $h = \frac{-b}{2a}$, we have

$$h = \frac{-b}{2a} = \frac{-(-8)}{2(-2)} = \frac{8}{-4} = -2$$

The value of k is then $M(h)$, which we find is

$$k = M(h) = -2 \cdot (-2)^2 - 8(-2) - 13 = -8 + 16 - 13 = -5$$

So in vertex form, the equation is

$$M(t) = -2(t+2)^2 - 5$$

(b) *(2 pts)* Does this function have a maximum value? If so, what is it? Does this function have a minimum value? If so, what is it?

The minimum or maximum value of a quadratic function always happens at the vertex. If a is positive, the parabola opens up and the vertex is the minimum value. If a is negative, the parabola opens down and the vertex is the maximum value. In this case, we have $a = -2$, so the parabola opens up and we have a maximum, but no minimum. The maximum value is then the y-coordinate of the vertex, which is -5.

Question 5: Solve the following equations, or explain why there is no solution.

(a) *(3 pts)* $\log_3\left(\frac{1}{9}\right) = \log_x 25$

On the left hand side of this equation, we observe that

$$\log_3\left(\frac{1}{9}\right) = -2$$

since

$$3^{-2} = \frac{1}{9}$$

So the equation can be rewritten as

$$-2 = \log_x 25$$

Rewriting this as an exponential equation, we want to find x such that

$$x^{-2} = 25$$

Taking both sides to the $-1/2$ power gives

$$x = 25^{-1/2} = \frac{1}{\sqrt{25}} = \frac{1}{5}$$

So the solution is $x = \frac{1}{5}$.

(b) *(3 pts)* $2x^2 - 4x - 10 = 20$

First, we rearrange the equation to

$$2x^2 - 4x - 30 = 0$$

We then notice that we can factor out a 2, yielding

$$2(x^2 - 2x - 15) = 0$$

Finally, we can factor the quadratic as

$$2(x-5)(x+3) = 0$$

Setting each factor to zero gives the solutions $x = 5, -3$.

(c) *(3 pts)* $3x^2 + 5x + 3 = 0$

This equation does not factor, so we have to use the quadratic formula. The solutions are then

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3} \\ &= \frac{-5 \pm \sqrt{-11}}{6} \end{aligned}$$

We can't take the square root of a negative number, so this equation has no solutions.

Question 6: Consider the polynomial $p(x)$ defined by

$$p(x) = \frac{-1}{2}(x^2 - 4)(x^2 + 1)$$

(a) *(3 pts)* Find the zeros of this polynomial.

Recall that we find the zeroes of a polynomial by factoring and setting each factor equal to 0. Luckily this polynomial is already factored, so setting each factor equal to zero gives the equations $x^2 - 4 = 0$ and $x^2 + 1 = 0$. Solving the first, we have

$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

Attempting to solve the second gives

$$\begin{aligned} x^2 + 1 &= 0 \\ x^2 &= -1 \end{aligned}$$

No real number squares to a negative number, so this has no solutions. Thus the only zeroes for our polynomial are $x = \pm 2$.

(b) *(3 pts)* Identify the degree and leading coefficient of $p(x)$.

To find the degree and leading coefficient, we need to multiply out the factors on the right hand side to write the polynomial in standard form. This gives

$$\begin{aligned} p(x) &= \frac{-1}{2}(x^4 - 3x^2 - 4) \\ &= \frac{-1}{2}x^4 + \frac{3}{2}x^2 + 2 \end{aligned}$$

The degree is 4, and the leading coefficient it $\frac{-1}{2}$.

(c) *(3 pts)* Describe the end behavior of this polynomial.

Recall that we only look at the highest degree term to determine the end behavior. If we plug a very large number into the highest degree term $\frac{-1}{2}x^4$, taking that number to the fourth power will give a very large positive number, which will give a large negative number when multiplied by the negative leading coefficient.. Thus we have

$$\lim_{x \to \infty} p(x) = -\infty$$

If we plug a very large negative number into $\frac{-1}{2}x^4$, taking that number to an even power will give a very large positive number, which will give a large negative number when multiplied by the negative leading coefficient.. Thus we have

$$\lim_{x \to -\infty} p(x) = -\infty$$

(d) *(2 pts)* Find the values of $p(-1)$ and $p(1)$.

Computing these values gives

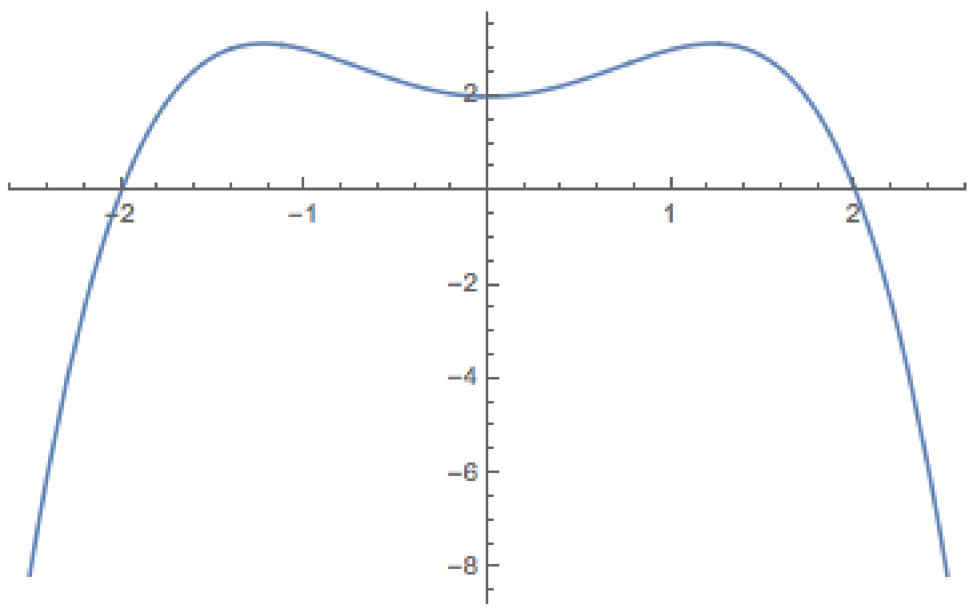
$$\begin{aligned} p(-1) &= \frac{-1}{2}((-1)^2 - 4)((-1)^2 + 1) \\ &= \frac{-1}{2}(-3)(2) \\ &= 3 \end{aligned}$$

and

$$\begin{aligned} p(-1) &= \frac{-1}{2}((1)^2 - 4)((1)^2 + 1) \\ &= \frac{-1}{2}(-3)(2) \\ &= 3 \end{aligned}$$

(e) *(4 pts)* Plot a graph of the polynomial. Your graph should have the proper x and y intercepts, end behavior, and number of "turns," and should pass through the points you found in part (d).

We have already figured out the x-intercepts (at $x = \pm 2$) and the end behavior, plus the points from part (d). The y-intercept is the value of the function at zero, which we can compute is $p(0) = 2$. Finally, our polynomial is degree 4, so should have no more than three "turns." Putting this together in a graph gives



MAT 123 - Topics for Midterm Exam

The midterm exam will be held in class on 6/10. The following topics will be covered. An approximate reference to the textbook is given for each topic, although not all material we have discussed is in the textbook.

- Functions - Definition of a function. Different ways to describe a function, including using words, formulas, tables, and graphs. Domain and range. Finding the domain of a function, given a formula. Interval notation for writing domains and ranges. (Section 1.1)
- Graphing Functions - The coordinate plane and plotting ordered pairs. Graph of a function is set of all pairs $(x, f(x))$ for x in the domain of the function. Using a graph to find values of a function. Using a graph to solve equations of the form $f(x) = b$ for b a fixed number. Determining domain and range of a function from a graph. The vertical line test. (Section 1.2)
- Ways to combine functions - addition, subtraction, multiplication, division, and composition. Finding the domain of the resulting functions. (Section 1.4)
- Graphing function transformations - changing a graph of a function via translations, stretching/shrinking, and reflection, both vertically and horizontally. Understand both in terms of graphs (e.g., how to translate a graph) and in terms of formulas (e.g., $g(x) = f(x) + 2$ is the graph of $f(x)$ shifted up two units.). Domain and range of transformed functions. Even and odd functions (Section 1.3)
- Inverse functions - One-to-one functions. Deciding if a function is invertible. Definition of inverse function. Computing inverse functions. Restricting the domain of a function to make it invertible. Domain and range of inverse functions. Composition and inverse functions. (Section 1.5)
- Obtaining graph of inverse function by reflection. Horizontal line test. Increasing and decreasing functions (Section 1.6)
- Linear functions - Slope formula. Point-slope form for the equation of a line. Slope-intercept form for the equation of a line. Definition of a linear function. Slopes of parallel and perpendicular lines. (Section 2.1)
- Quadratic Functions - Definition of a quadratic function. Completing the square. The quadratic formula to find roots of a quadratic function. The vertex form of a quadratic equation. Graphing quadratic functions using the vertex form and graph transformations. Finding maximum and minimum values of a quadratic function. (Section 2.2)
- Polynomial Functions - Definition of polynomial. Degree of polynomial. Addition, subtraction, multiplication, and division of polynomials. Degree and leading coefficient. Zeroes of a polynomial. Relationship between zeros of a polynomial and factors of the polynomial. End behavior of polynomials. General shape of graphs of polynomials (Section 2.4)
- Rational Functions - Definition of rational functions. Domain of rational functions. End behavior of rational functions. Zeroes of rational functions. Vertical asymptotes. Graphing rational functions. (Section 2.5)

- Exponential Functions - Definition of exponential functions. Graph of exponential functions. Properties of exponents. (Sections 2.3 and 3.1)
- Logarithms - Definition of logarithm as inverse function for exponential. Graph of the logarithm function. (Section 3.1)

Note that logarithm rules (Sections 3.2, 3.3) were introduced in class on 6/8, but will not be on the midterm exam.

MAT 123 Sample Midterm

1 Conceptual Questions.

This section contains some general questions to help you test your understanding of some of the key concepts we've learned so far. These questions are not necessarily representative of the kind of questions that will appear on the midterm - the true sample midterm is in the next section.

1. Can the graph of a function have more than one y-intercept? Can the graph of a function have more than one x-intercept? Explain your answers.
2. What information do you need to know in order to write the equation of a line?
3. A hiker starts climbing a mountain at 8:00 am, reaches the summit at 1:00 pm, and then climbs back down to arrive back at her starting point at 4:00 pm. Let A represent the altitude of the hiker, in feet, and let t represent the time after 8:00 am, in hours. We can consider A as a function of t, but cannot consider t as a function of A. Explain why.
4. Explain the difference between the Vertical Line Test and the Horizontal Line Test. What is each used for?
5. Explain why an increasing or decreasing function is always one-to-one.
6. Consider a line in the xy-plane. Is the line necessarily the graph of a function? Is a linear function always one-to-one?
7. How do you find the domain of a composite function?
8. Is $(f \circ g) = (g \circ f)$ for any pair of functions? Is $(f + g) = (g + f)$ for any pair of functions?
9. Explain in words how the graph of each of the following compares to the graph of the function $f(x)$: $f(x) + 2, f(x) - 2, f(x + 2), f(x - 2), f(2x), 2f(x), f(x/2), f(x)/2, f(-x), -f(x)$.
10. Give an example of a function that is neither even nor odd.
11. What is the quadratic formula that gives the solutions to the equation $ax^2 + bx + c = 0$? How does the value of $b^2 - 4ac$ change the behavior of the solutions?
12. Why does the end behavior of a polynomial function only depend on the term with the highest degree?
13. Why can a polynomial of degree n have no more than n roots? (Consider factors)

2 Sample Midterm

This sample midterm is meant to have the same style of questions and the same length and difficulty as the midterm. However, topics covered on this sample midterm may not be on the actual midterm, and there may be topics on the midterm that do not appear on the sample midterm. You are responsible for knowing all of the concepts on the topics sheet for the midterm. There will be no "cheat sheet" or list of formulas included with the midterm.

You may not be able to complete questions marked with a star ($\star$) until after the lecture on Monday, June 8.

Question 1: Consider the function $f(x) = x^2 + x - 6$.

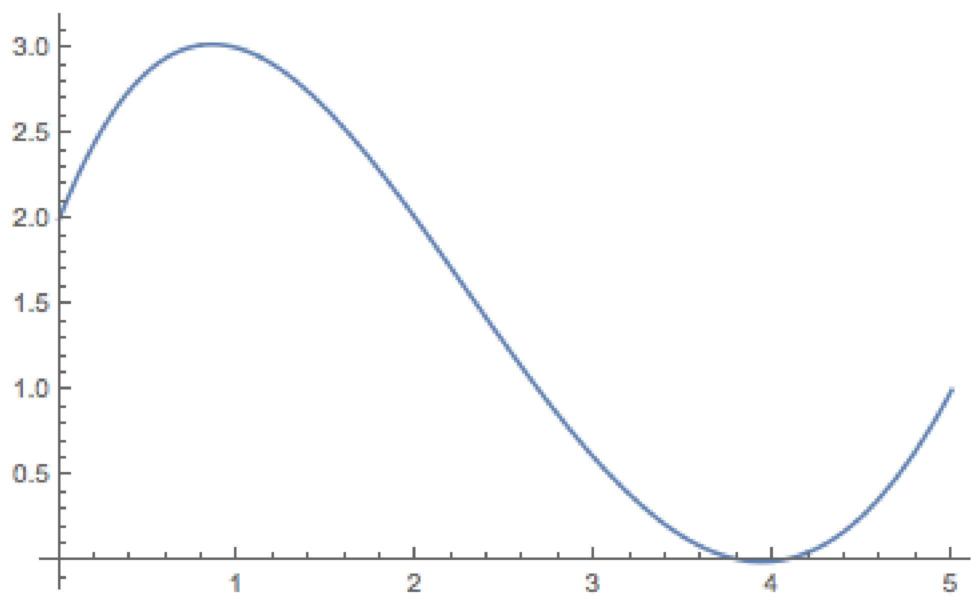
(a) Find the roots of $f(x)$. Use whatever method you prefer.

(b) Write the equation for $f(x)$ in vertex form.

(c) Using your answer from part (b), describe how the graph of $f(x)$ compares to the graph of the $y = x^2$.

(d) Draw the graph of $f(x)$. Be sure to label the vertex, x-intercepts, and y-intercepts.

Question 2: Consider the graph of a function $f(x)$ below. Note the labels on the axes.



(a) What are the domain and range of f? Write your answer in interval notation.

(b) On what intervals is the function increasing and decreasing? Write your answer in interval notation.

(c) Using the graph, estimate the value of $f(3)$. Then, estimate the values of all x such that $f(x) = 2$.

(d) Let g be the function obtained from f by restricting f to the largest interval in the domain on which f is decreasing. Draw the graph of g^{-1}. Make sure that your graph has the proper domain and range.

Question 3:

(a) Write an equation for a line that passes through the points $(2, 5)$ and $(6, 13)$, using point-slope form.

(b) Write the equation for the line from part (a) in slope-intercept form.

(c) Write the equation for the line that is perpendicular to the line you found in part (a) and passes through the point $(-2, 2)$.

Question 4: Let $f(x) = 3x^2 + 2x - 1$, $g(x) = -x^3 - 1$.

(a) Write a formula for $(f \circ g)(x)$. Simplify as much as possible.

(b) The function $p(x) = (f \circ g)(x)$ is a polynomial. What is the degree of this polynomial? What is the leading coefficient?

(c) Describe the end behavior of $p(x)$.

Question 5: Let $f(x) = x^2 + 8x + 12$ and $g(x) = x^3 - x$.

(a) Find the zeros of f.

(b) Find the zeros of g.

(c) ($\star$) Consider the rational function $h(x) = \frac{f(x)}{g(x)}$. Write the domain of this function, using interval notation. Find the zeroes of this function.

(d) ($\star$) Draw a graph of $h(x)$. Label all asymptotes and intercepts.

($\star$) **Question 6:**

(a) You won't need to know how to do this type of problem for the midterm. Simplify the following expression. It may help you to know that $3^3 = 27, 3^4 = 81, 3^5 = 243$, and $3^6 = 729$.

$$\log_3(54) + \log_3(18) - 2\log_3(2)$$

(b) Draw a graph of the function $f(x) = \log_2 x$. Be sure to label at least two points on the graph. What are the domain and range of this function?

($\star$) **Question 7:** A sample of bacteria is grown in a lab. The sample begins with 100 individuals, and quadruples every hour. The population of the bacteria, P, is a function of time, t, in hours, given by

$$P(t) = 100 \cdot 4^t$$

(a) Draw a graph of this function. Be sure to label at least two points on the graph.

(b) How many hours will it take for the population to reach 2000? Write your answer using a logarithm.

MAT 123 Sample Midterm Solutions

1 Conceptual Questions.

1. Can the graph of a function have more than one y-intercept? Can the graph of a function have more than one x-intercept? Explain your answers.

   A function cannot have more than one y-intercept, as the y-intercept is simply the value of the function at 0, and functions have a unique output for a given input. A graph can have many x-intercepts, or none, as x-intercepts are places where the function's value is 0. Consider the graphs of the functions $x^2, x^2 - 1$, and $x^2 + 1$.

2. What information do you need to know in order to write the equation of a line?

   The slope and a point on the line, or two points on the line.

3. A hiker starts climbing a mountain at 8:00 am, reaches the summit at 1:00 pm, and then climbs back down to arrive back at her starting point at 4:00 pm. Let A represent the altitude of the hiker, in feet, and let t represent the time after 8:00 am, in hours. We can consider A as a function of t, but cannot consider t as a function of A. Explain why.

   A is a function of t because for any time there is a unique corresponding height. However, to any height we can associate two different times (the time the height is reached while the hiker climbs up and the time when the hiker climbs down). Since a function must have a unique output for every input, t cannot be a function of A.

4. Explain the difference between the Vertical Line Test and the Horizontal Line Test. What is each used for?

   The vertical line test is used to check if a curve in the plane is the graph of a function. It is a graph of a function if any vertical line drawn in the plane crosses the curve at most one time. The horizontal line test is used to check if a function is one-to-one (invertible) using its graph. It is one-to-one if any horizontal line drawn in the plane crosses the graph at most one time.

5. Explain why an increasing or decreasing function is always one-to-one.

   If a function is increasing, that means that if $a < b$, then $f(a) < f(b)$. For any two different numbers x_1, x_2 in the domain, one is always less than the other - let's say that $x_1 < x_2$. Then $f(x_1) < f(x_2)$. In particular, $f(x_1) \neq f(x_2)$. Thus if $x_1 \neq x_2$, we have that $f(x_1) \neq f(x_2)$. Recall that this means that the function f is one-to-one, as it means that two different inputs to the function always give two different outputs. A similar argument works for decreasing functions.

6. Consider a line in the xy-plane. Is the line necessarily the graph of a function? Is a linear function always one-to-one?

   It is not necessarily a function - a vertical line is not the graph of a function, as it clearly does not pass the vertical line test. Any non-vertical line is the graph of a function, though. A linear function is not necessarily one-to-one. Constant functions are linear. However, any non-constant linear function is one-to-one.

7. How do you define the domain of a composite function?

To find the domain of $f \circ g$, begin by finding the domain of g. Then, remove every number x from that domain such that $g(x)$ is not in the domain of f. The domain of g with these numbers removed is the domain of $f \circ g$.

8. Is $(f \circ g) = (g \circ f)$ for every pair of functions? Is $(f+g) = (g+f)$ for every pair of functions?

   No - you've seen some examples in the homework. Yes. For any number x in the domain, we have that $(f+g)(x) = f(x)+g(x) = g(x)+f(x) = (g+f)(x)$, so the functions $f+g$ and $g+f$ are the same.

9. Explain in words how the graph of each of the following compares to the graph of the function $f(x)$: $f(x)+2, f(x)-2, f(x+2), f(x-2), f(2x), 2f(x), f(x/2), f(x)/2, f(-x), -f(x)$.

   Shifted up 2 units. Shifted down 2 units. Shifted left 2 units. Shifted right 2 units. Shrunk horizontally by half. Stretched vertically by 2. Stretched horizontally by 2. Shrunk vertically by 1/2. Reflected over y-axis. Reflected over x-axis.

10. Give an example of a function that is neither even nor odd.

    Many possible answers. For example, $f(x) = (x-1)^2$.

11. What is the quadratic formula that gives the solutions to the equation $ax^2+bx+c=0$? How does the value of b^2-4ac change the behavior of the solutions?

    The formula is

    $$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

    The quantity b^2-4ac is called the "discriminant" of the equation. If it is positive, the equation will have two different real number solutions. If the discriminant is 0, the equation will have a single real number solution, called a "repeated" solution. If the discriminant is negative, the equation has no real solutions.

12. Why does the end behavior of a polynomial function only depend on the term with the highest degree?

    When considering the end behavior of a polynomial, we consider the values we get out of the polynomial when we put in extremely large positive or negative numbers. When we put large numbers into the polynomial, terms with higher powers of x will be so much larger than terms with lower powers of x that the lower-power terms won't make very much difference in the value of the function. Therefore the contribution of the term with highest degree will be much bigger than the contributions from the other terms, so it is the only term that we need to consider.

13. Why can a polynomial of degree n have no more than n roots? (Consider factors)

    Recall that if r is a root of a polynomial p, that is, if $p(r) = 0$, then we have that $(x-r)$ is a factor of p. Thus if a polynomial has m roots, it has m factors that are degree-one polynomials. This means a polynomial with m roots has degree at least m. Thus if a polynomial of degree n had more than n roots, it would have to be a polynomial of degree bigger than n, but that doesn't make sense.

2 Sample Midterm

Question 1: Consider the function $f(x) = x^2 + x - 6$.

(a) Find the roots of $f(x)$. Use whatever method you prefer.

This equation can be factored. We have

$$0 = x^2 + x - 6 = (x - 2)(x + 3)$$

so the roots are $x = 2, -3$.

(b) Write the equation for $f(x)$ in vertex form.

Remember that the vertex form is obtained by completing the square. Using the "shortcut" method to complete the square, we know that the vertex will have x-coordinate

$$h = \frac{-b}{2a} = \frac{-1}{2 \cdot 1} = \frac{-1}{2}$$

The y-coordinate for the vertex is

$$k = f(h) = f(-1/2) = (-1/2)^2 - 1/2 - 6 = \frac{-25}{4}$$

The vertex form is therefore

$$\begin{aligned} f(x) &= a(x - h)^2 + k \\ f(x) &= \left(x + \frac{1}{2}\right)^2 - \frac{25}{4} \end{aligned}$$

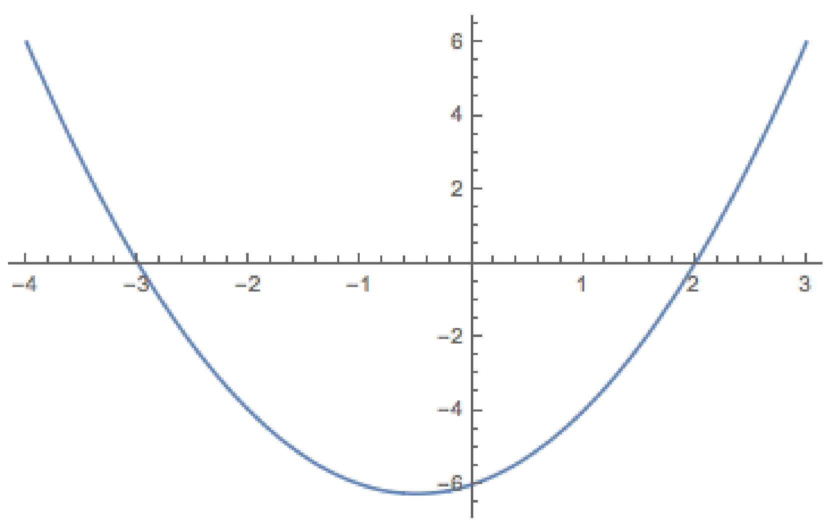
You can check this answer by multiplying out the right hand side.

(c) Using your answer from part (b), describe how the graph of $f(x)$ compares to the graph of the $y = x^2$.

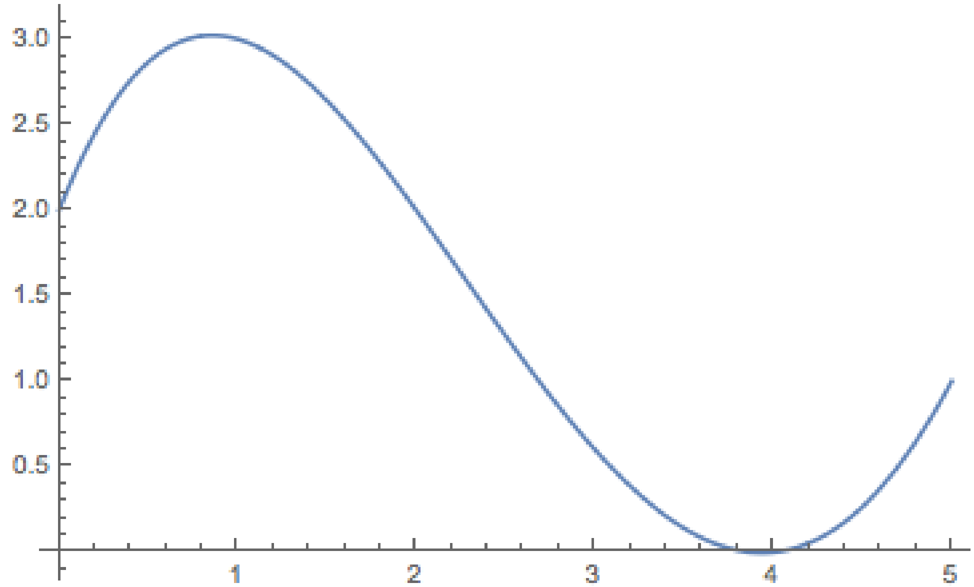
The graph of $f(x)$ is obtained by shifting the graph of x^2 left 1/2 unit and down 25/4 units.

(d) Draw the graph of $f(x)$. Be sure to label the vertex, x-intercepts, and y-intercepts.

The graph is below. Looking on the axes of the graphs, you can see that the graph has the proper x-intercepts at $2, -3$, the proper y-intercept at -6, and the correct vertex. Your graph should be labeled, either along the axes or at each point, to show where the intercepts and vertex are.



Question 2: Consider the graph of a function $f(x)$ below. Note the labels on the axes.



(a) What are the domain and range of f? Write your answer in interval notation.

The domain is $[0, 5]$. The range is $[0, 3]$

(b) On what intervals is the function increasing and decreasing? Write your answer in interval notation.

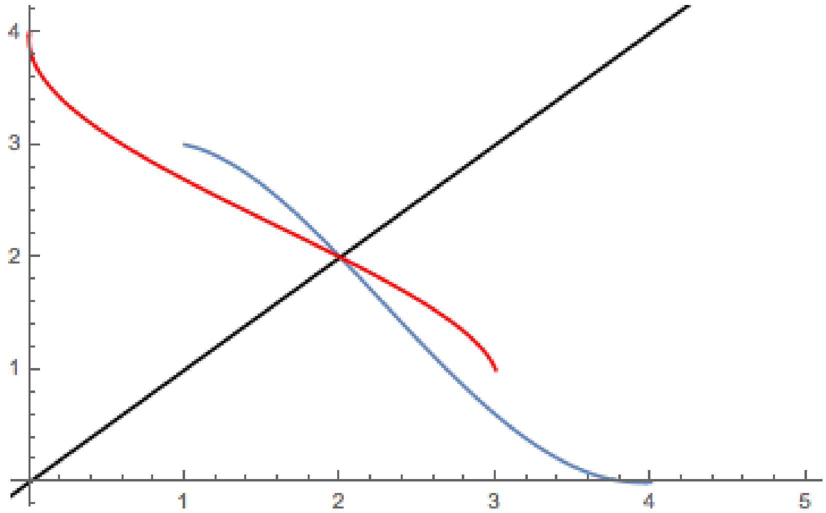
The function is increasing on the intervals $(0, 1)$ and $(4, 5)$. It is decreasing on the interval $(1, 4)$.

(c) Using the graph, estimate the value of $f(3)$. Then, estimate the values of all x such that $f(x) = 2$.

The y-coordinate of the point on the graph with x-coordinate 3 is approximately 1/2, so we have $f(3) = 1/2$. The horizontal line at $y = 2$ crosses the graph at points with x-coordinates 0 and 2, so we have that $f(0) = 2$ and $f(2) = 2$.

(d) Let g be the function obtained from f by restricting f to the largest interval in the domain on which f is decreasing. Draw the graph of g^{-1}. Make sure that your graph has the proper domain and range.

Function g has domain $(1, 4)$, and range $(0, 3)$, so g^{-1} has domain $(0, 3)$ and range $(1, 4)$. Its graph is obtained via reflection about the line $y = x$. The graphs of both g and g^{-1} are shown below - the graph of g is in blue, the graph of g^{-1} is in red, and the line $y = x$ is shown in black.



Question 3:

(a) Write an equation for a line that passes through the points $(2, 5)$ and $(6, 13)$, using point-slope form.

The slope of the line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{13 - 5}{6 - 2} = \frac{8}{4} = 2$$

Using the point-slope formula with slope $m = 2$ and point $(x_0, y_0) = (2, 5)$, we have

$$y - 5 = 2(x - 2)$$

If you wrote the equation with the other point, you would have

$$y - 13 = 2(x - 6)$$

(b) Write the equation for the line from part (a) in slope-intercept form.

To write in slope-intercept form, we simply re-arrange the equation

$$\begin{aligned} y - 5 &= 2(x - 2) \\ y - 5 &= 2x - 4 \\ y &= 2x + 1 \end{aligned}$$

If you began with the other equation, you would have

$$\begin{aligned} y - 13 &= 2(x - 6) \\ y - 13 &= 2x - 12 \\ y &= 2x + 1 \end{aligned}$$

Notice that these are the same.

(c) Write the equation for the line that is perpendicular to the line you found in part (a) and passes through the point $(-2, 2)$.

Recalling the relationship between slopes of perpendicular lines, we have that the new line we are trying to find will have slope $m = \frac{-1}{2}$. Using the point-slope form, its equation is

$$y - 2 = \frac{-1}{2}(x + 2)$$

Question 4: Let $f(x) = 3x^2 + 2x - 1$, $g(x) = -x^3 - 1$.

(a) Write a formula for $(f \circ g)(x)$. Simplify as much as possible.

$$\begin{aligned}
(f \circ g)(x) &= f(g(x)) = f(-x^3 - 1) \\
&= 3(-x^3 - 1)^2 + 2(-x^3 - 1) - 1 \\
&= 3(x^6 + 2x^3 + 1) - 2x^3 - 2 - 1 \\
&= 3x^6 + 6x^3 + 3 - 2x^3 - 3 \\
&= 3x^6 + 4x^3
\end{aligned}$$

(b) The function $p(x) = (f \circ g)(x)$ is a polynomial. What is the degree of this polynomial? What is the leading coefficient?

The degree (highest power of x) is 6. The leading coefficient (the coefficient on the highest degree term) is 3.

(c) Describe the end behavior of $p(x)$.

Recall that end behavior only depends on the degree and leading coefficient. If we plug in a very large number, taking it to the sixth power gives a very large positive number, and multiplying it by the positive leading coefficient makes the output positive. Thus plugging a very large positive number into p gives a very large positive value, and we have

$$\lim_{x \to \infty} p(x) = \infty$$

If we plug in a very negative number, taking it to an even power will make it a very large positive number, which will remain positive after multiplying by the leading coefficient. Therefore

$$\lim_{x \to -\infty} p(x) = \infty$$

Question 5: Let $f(x) = x^2 + 8x + 12$ and $g(x) = x^3 - x$.

(a) Find the zeros of f.

We will find the zeroes by factoring. Observe that

$$\begin{aligned}
f(x) &= x^2 + 8x + 12 \\
&= (x + 2)(x + 6)
\end{aligned}$$

Setting equal to zero, we have that $(x + 2)(x + 6) = 0$ implies that $x = -2, -6$.

(b) Find the zeros of g.

Again we find zeroes by factoring. We have

$$\begin{aligned} g(x) &= x^3 - x \\ &= x(x^2 - 1) \\ &= x(x-1)(x+1) \end{aligned}$$

The zeros are $x = -1, 0, 1$.

(c) ($\star$) Consider the rational function $h(x) = \frac{f(x)}{g(x)}$. Write the domain of this function, using interval notation. Find the zeroes of this function.

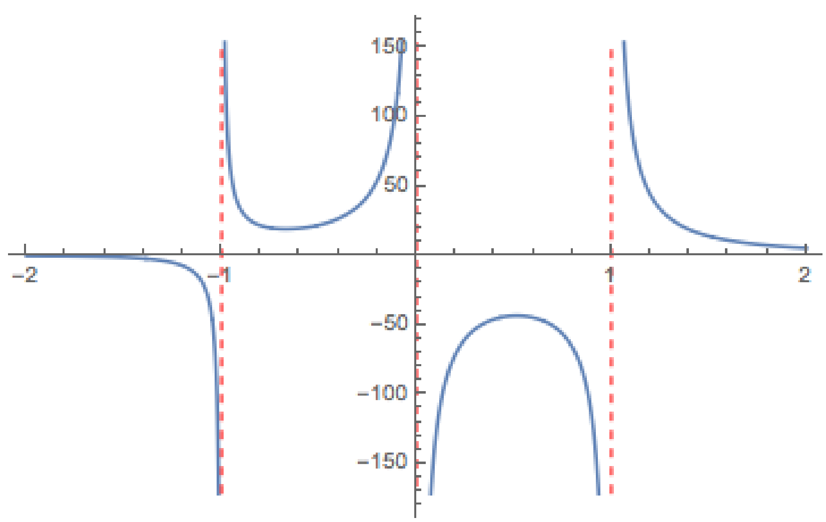
The domain will be all real numbers except those x that make the denominator zero. These are the zeroes of g, which we found in the previous part to be $-1, 0, 1$. Therefore, the domain is

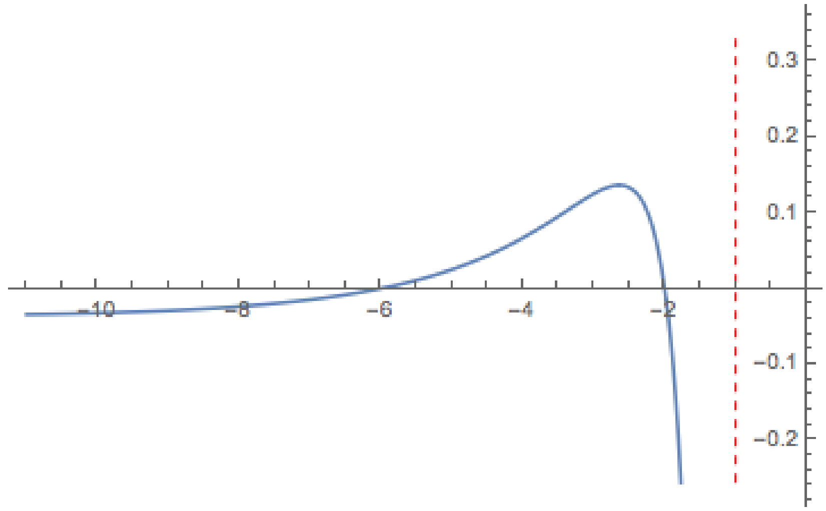
$$(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$$

The zeroes of a rational function are the zeroes of the numerator, assuming those values are in the domain. Therefore the zeroes of h are the zeroes of f, which we have already found are $x = -2, -6$. Notice that both of these values are in the domain.

(d) ($\star$) Draw a graph of $h(x)$. Label all asymptotes and intercepts.

The graph will have a horizontal asymptote at $y = 0$ and vertical asymptotes at $x = -1, 0, 1$. Two graphs are shown below - the first shows the $x-$values on the interval $[-2, 2]$ so you can see how the middle part of the graph looks. The second shows the x-values from $[-11, 0]$ so you can see how the two zeros change the behavior of the graph as it approaches the horizontal asymptote. On both graphs, the vertical asymptotes are graphed in a dotted red line.





(⋆) **Question 6:**

(a) You won't need to know how to do this type of problem for the midterm. Simplify the following expression. It may help you to know that $3^3 = 27, 3^4 = 81, 3^5 = 243,$, and $3^6 = 729$.

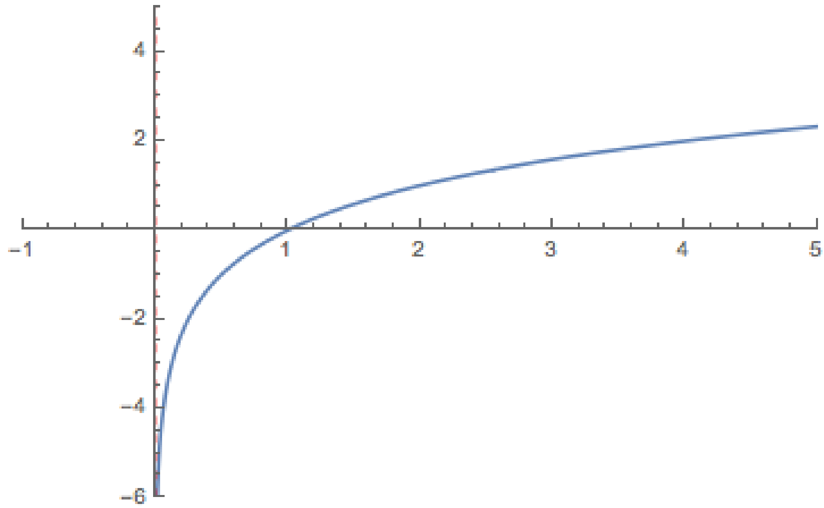
$$\log_3(54) + \log_3(18) - 2\log_3(2)$$

Using logarithm rules, we have that

$$\begin{aligned}
\log_3(54) + \log_3(18) - 2\log_3(2) &= \log_3(54 \cdot 18) - 2\log_3(2) \\
&= \log_3(54 \cdot 18) - \log_3(4) \\
&= \log_3\left(\frac{54 \cdot 18}{4}\right) \\
&= \log_3(27 \cdot 9) \\
&= \log_2(243) = 5
\end{aligned}$$

(b) Draw a graph of the function $f(x) = \log_2 x$. Be sure to label at least two points on the graph. What are the domain and range of this function?

The domain is $(0, \infty)$ The range is $(-\infty, \infty)$. The graph is shown belong - the easiest points to label are that the graph passes through $(1, 0)$ and $(2, 1)$. All of this information can be obtained by remembering that $\log_2 x$ is the inverse function of 2^x.

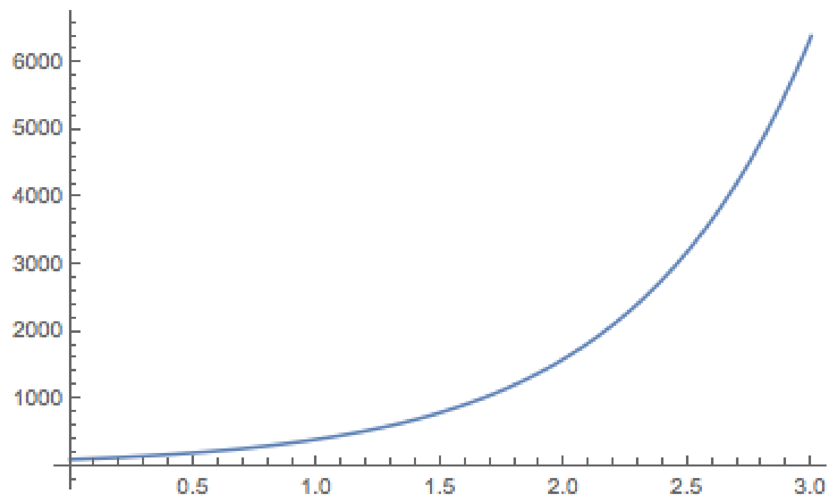


($\star$) **Question 7:** A sample of bacteria is grown in a lab. The sample begins with 100 individuals, and quadruples every hour. The population of the bacteria, P, is a function of time, t, in hours, given by

$$P(t) = 100 \cdot 4^t$$

(a) Draw a graph of this function. Be sure to label at least two points on the graph.

The graph is shown below. The easiest points to label are $(0, 100)$ and $(1, 400)$.



(b) How many hours will it take for the population to reach 2000? Write your answer using a logarithm.

We are being asked to find the value of t such that $P(t) = 2000$, so we have

$$\begin{aligned} 2000 &= 100 \cdot 4^t \\ 20 &= 4^t \\ t &= \log_4(20) \end{aligned}$$

Without a calculator you cannot give a more precise answer (although you should be able to see that $\log_4(20)$ is between 2 and 3, since $4^2 = 16 < 20 < 64 = 4^3$). The approximate value is

$$\log_4(20) \approx 2.161$$

MAT 123 Final Exam

Name: ____

- Please read all instructions before beginning, and do not open the exam until you are told to do so.
- Put away all notes, books, calculators, etc. before beginning the exam, and place them under your desk. The only items on your desk should be this exam, pencils/pens, and an eraser.
- This exam has twelve (12) questions, each with multiple parts. The point value for each question is shown below.
- The final page of the exam is left blank for scratch paper. You may tear this page from the exam booklet, but do not remove any other pages.
- You have 3 hours and 25 minutes to complete the exam.

Question	Points	Score
1	15	
2	10	
3	11	
4	6	
5	8	
6	8	
7	16	
8	12	
9	10	
10	13	
11	12	
12	11	
Total	132	

Question 1: *(15 pts)* Consider the quadratic function

$$f(x) = \frac{-x^2}{3} - \frac{2x}{3} + \frac{8}{3}$$

(a) State the domain of this function, in interval notation.

(b) Find the zeroes of this function.

(c) As given, $f(x)$ is written in standard form. Write the function in vertex form.

(d) Using your answer in part (c), describe how the graph for $f(x)$ can be obtained from the graph of the function $g(x) = x^2$ using graph transformations (stretching, shifting, reflections).

(e) Draw a graph of the function $f(x)$. On your graph, label the zeroes, the vertex, and the y-intercept.

(f) State the range of $f(x)$, in interval notation.

Question 2: *(10 pts)* Solve each of the following equations, or explain why there is no solution.

(a) $3 \cdot 4^w = e^2$

(b) $\log_2(x-1) - \log_2(x+1) = -1$

(c) $\sin^2(2x) + \sin(x)\cos x = 0$

Question 3: *(11 pts)* Simplify each of the following trigonometric expressions. Your answers should not include any trigonometric functions.

(a) $\tan\left(\frac{11\pi}{6}\right)$

(b) $\cos\left(\frac{-3\pi}{4}\right)$

(c) $\arcsin(\sin(2\pi))$

(d) $\arccos\left(\frac{1}{2}\right) - \arcsin(-1)$

(e) $\tan(\arcsin(x))$.

Question 4: *(6 pts)* Let $f(x)$ be a function such that $f(0) = 20$ and $f(2) = 10$.

(a) Assume that $f(x)$ is a linear function. Write a formula for $f(x)$.

(b) Now assume that $f(x)$ is an exponential function. Write a formula for $f(x)$.

Question 5: *(8 pts)* Consider the function

$$g(x) = 2\sin(\pi x) - 1$$

(a) Identify the period, amplitude, phase shift, and center line of this function.

(b) Using your information from part (a), draw a graph of the function.

Question 6: (*8 pts*) A scientist is studying a sample of a radioactive material. At time $t = 0$, the mass of the radioactive material is 480 grams. After 150 days, the mass of the sample is 60 grams. Assume that the radioactive sample undergoes exponential decay.

(a) Write a formula for the function $a(t)$ that gives the amount a of radioactive material present in the sample after t days.

(b) Find the half-life of the radioactive material.

(c) Find the time at which 7.5 grams of material remain.

Question 7: (*16 pts*) Consider the function

$$f(x) = \frac{2x^2 - 3x + 1}{x^2 - x - 2}$$

(a) What is the domain of this function? Write your answer in interval notation.

(b) Identify the zeroes of the function.

(c) Describe the end behavior of the function. Write your answer using limit notation.

(d) Identify the y-intercept of the function.

(e) Using the information from the previous parts of this question, draw a graph of the function. Your graph should have the proper domain, asymptotes, zeroes, and y-intercept, and should pass through the following points

$$f(-2) = \frac{15}{4} \qquad f(3/2) = \frac{-4}{5} \qquad f(3) = \frac{5}{2}$$

Question 8: (*12 pts*) Sketch the graph of each of the following functions. Make sure your graphs include any necessary asymptotes and have the correct domain. Label at least 3 points on each graph.

(a) $f(x) = 4^x - 1$

(b) $g(x) = \log_3(x - 3)$

(c) $h(x) = -(x - 2)^3 + 1$

Question 9: (*10 pts*) Let

$$f(x) = \frac{1}{x-1} \qquad g(x) = 2x^2 - 2 \qquad h(x) = \frac{2x}{x^2+1}$$

Compute each of the following. Simplify your answer as much as possible

(a) $f(f(3))$

(b) $(f \circ h)(x)$

(c) $\dfrac{g(x+1) - g(1)}{x}$, assuming $x \neq 0$.

Question 10: (*13 pts*) Assume that θ is an angle with $0 < \theta < \pi/2$ such that $\sin\theta = \dfrac{15}{17}$. Evaluate the following trigonometric functions. It may be helpful for you to know that $15^2 = 225$ and $17^2 = 289$.

(a) $\cos\theta$

(b) $\cos(2\theta)$

(c) $\sin(\theta + \pi/6)$

(d) $\sin(\theta/2)$

Question 11: (*12 pts*) Evaluate each of the following limits.

(a) $\displaystyle\lim_{x \to 7} \frac{x^2 - 5x - 14}{x^2 - 6x - 7}$

(b) $\displaystyle\lim_{x \to 4} \frac{x^2 - 5x - 14}{x^2 - 6x - 7}$

(c) $\displaystyle \lim_{x \to -\infty} \frac{x^3 - 3x^2 + 2}{4x^3 + 2x - 7}$

(d) $\displaystyle \lim_{x \to 0} \frac{\cos(2x) - 1}{\cos(x) - 1}$ (*Hint*: Use an identity)

<u>Question 12:</u> (*11 pts*)

(a) Let x and y be numbers such that $\log_2(x) = 3$ and $\log_2(y) = 1/2$. Evaluate $\log_4(xy^3)$. Notice that this is a logarithm with base 4, not base 2. Your answer should be a fraction.

(b) Using trigonometric identities, simplify the following expression. Your answer should include only one trigonometric function.

$$\frac{(\sin\theta + \tan\theta)^2 + \cos^2\theta - \sec^2\theta}{\tan\theta}$$

(c) Let $f(x) = \log_4(2x - 1) - \log_4(x + 3)$. This function is invertible. Find $f^{-1}(y)$.

MAT 123 Final Exam Solutions

Question 1: *(15 pts)* Consider the quadratic function

$$f(x) = \frac{-x^2}{3} - \frac{2x}{3} + \frac{8}{3}$$

(a) *(1 pt)* State the domain of this function, in interval notation.

This is a polynomial function, therefore its domain is all real numbers. In interval notation, this is $(-\infty, \infty)$.

(b) *(3 pts)* Find the zeroes of this function.

We find the zeroes by factoring.

$$\begin{aligned} \frac{-x^2}{3} - \frac{2x}{3} + \frac{8}{3} &= 0 \\ \frac{-1}{3}\left(x^2 + 2x - 8\right) &= 0 \\ \frac{-1}{3}(x+4)(x-2) &= 0 \end{aligned}$$

Setting each factor equal to zero, we have that $x = -4$ or $x = 2$.

(c) *(3 pts)* As given, $f(x)$ is written in standard form. Write the function in vertex form.

The x-coordinate of the vertex is given by

$$h = \frac{-b}{2a} = \frac{\frac{2}{3}}{2 \cdot \frac{-1}{3}} = -1$$

The y-coordinate of the vertex is then

$$k = f(h) = f(-1) = \frac{-1}{3} + \frac{2}{3} + \frac{8}{3} = \frac{9}{3} = 3$$

The equation in vertex form is therefore

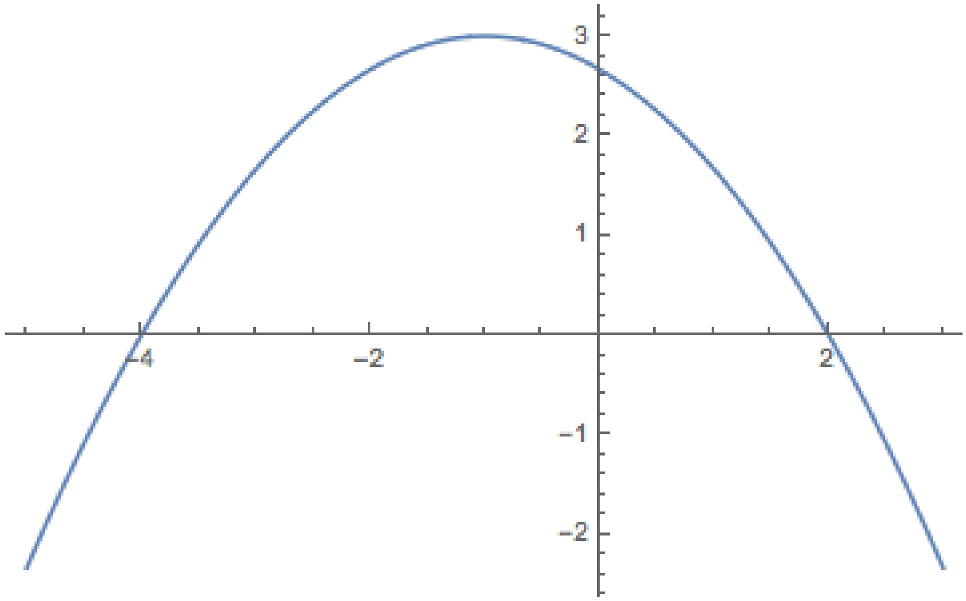
$$f(x) = \frac{-1}{3}(x+1)^2 + 3$$

(d) *(3 pts)* Using your answer in part (c), describe how the graph for $f(x)$ can be obtained from the graph of the function $g(x) = x^2$ using graph transformations (stretching, shifting, reflections).

The graph is obtained from the usual graph of x^2 by shifting left one unit, shrinking vertically by 1/3, reflecting the graph over the x-axis, and then shifting the graph up 3 units, in that order.

(e) *(4 pts)*Draw a graph of the function $f(x)$. On your graph, label the zeroes, the vertex, and the y-intercept.

The graph is shown below. Note that it is a parabola, with vertex at $(-1, 3)$, opening down, and with zeroes at $x = -4, 2$. The y-intercept is at $8/3$.



(f) *(1 pt)* State the range of $f(x)$, in interval notation.

From the graph, we see that the function has y values that include all real numbers less than or equal to 3. In interval notation, this is $(-\infty, 3]$.

Question 2: *(10 pts)* Solve each of the following equations, or explain why there is no solution.

(a) *(2 pts)* $3 \cdot 4^w = e^2$

$$\begin{aligned} 3 \cdot 4^w &= e^2 \\ 4^w &= \frac{e^2}{3} \\ w &= \log_4\left(\frac{e^2}{3}\right) \end{aligned}$$

(b) *(3 pts)* $\log_2(x-1) - \log_2(x+1) = -1$

$$\begin{aligned} \log_2(x-1) - \log_2(x+1) &= -1 \\ \log_2\left(\frac{x-1}{x+1}\right) &= -1 \\ \frac{x-1}{x+1} &= 2^{-1} \\ \frac{x-1}{x+1} &= \frac{1}{2} \\ 2(x-1) &= x+1 \\ 2x-2 &= x+1 \\ x &= 3 \end{aligned}$$

(c) *(5 pts)* $\sin^2(2x) + \sin(x)\cos x = 0$

$$\begin{aligned} \sin^2(2x) + \sin x \cos x &= 0 \\ \sin^2(2x) + \frac{1}{2}\sin(2x) &= 0 \\ \sin(2x)\left(\sin(2x) + \frac{1}{2}\right) &= 0 \end{aligned}$$

Setting the first factor to zero gives the equation $\sin(2x) = 0$, which has solutions $2x = 0$ and $2x = \pi$ in the interval $[0, 2\pi)$. Thus all solutions to this equation are given by $2x = 0 + 2\pi k$ and $2x = \pi + 2\pi k$. Dividing by 2 gives

$$x = \pi k, \frac{\pi}{2} + \pi k$$

Setting the second factor equal to zero gives the equation $\sin(2x) = -1/2$, which has the solutions $2x = \frac{7\pi}{6}$ and $2x = \frac{11\pi}{6}$ on the interval $[0, 2\pi)$. Thus all solutions to this equation are given by $2x = \frac{7\pi}{6} + 2\pi k$ and $2x = \frac{11\pi}{6} + 2\pi k$. Dividing by 2 gives

$$x = \frac{7\pi}{12} + \pi k, \frac{11\pi}{12} + \pi k$$

Collecting all of the solutions, we have

$$\left.\begin{array}{l} \pi k \\ \frac{\pi}{2} + \pi k \\ \frac{7\pi}{12} + \pi k \\ \frac{11\pi}{12} + \pi k \end{array}\right\} \text{ for } k = \ldots, -2, -1, 0, 1, 2, \ldots$$

Question 3: *(11 pts)* Simplify each of the following trigonometric expressions. Your answers should not include any trigonometric functions.

(a) *(2 pts)* $\tan\left(\frac{11\pi}{6}\right)$

This angle is in the fourth quadrant, where tangent is negative, and has a reference angle of $\pi/6$. Therefore

$$\tan\left(\frac{11\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = \frac{-1}{\sqrt{3}}$$

(b) *(2 pts)* $\cos\left(\frac{-3\pi}{4}\right)$

This angle is in the third quadrant, where cosine is negative, and has a reference angle of $\pi/4$. Therefore

$$\cos\left(\frac{-3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$$

(c) *(2 pts)* $\arcsin(\sin(2\pi))$

$$\arcsin(\sin(2\pi)) = \arcsin(0) = 0$$

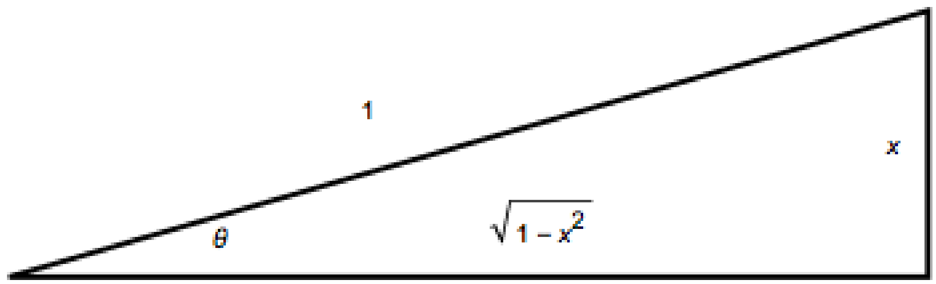
Note that the answer cannot be 2π, as 2π is not in the range of the inverse sine function.

(d) *(2 pts)* $\arccos\left(\frac{1}{2}\right) - \arcsin(-1)$

$$\arccos\left(\frac{1}{2}\right) - \arcsin(-1) = \frac{\pi}{3} - \left(\frac{-\pi}{2}\right) = \frac{5\pi}{6}$$

(e) *(3 pts)* $\tan(\arcsin(x))$.

Let $\theta = \arcsin(x)$. This means that $\sin\theta = x$, so we draw a right triangle with angle θ such that the side opposite the angle has length x and the hypotenuse has length 1. (Recall that the sine is the ratio of opposite side over hypotenuse). We then fill in the length of the adjacent side using the Pythagorean theorem. This gives the following triangle.



We then wish to evaluate $\tan(\arcsin(x)) = \tan(\theta)$. Since the tangent is the ratio of the opposite side to the adjacent side, we have

$$\tan(\arcsin(x)) = \tan(\theta) = \frac{x}{\sqrt{1-x^2}}$$

Question 4: *(6 pts)* Let $f(x)$ be a function such that $f(0) = 20$ and $f(2) = 10$.

(a) *(3 pts)* Assume that $f(x)$ is a linear function. Write a formula for $f(x)$.

The slope of a line that passes through the points $(0, 20)$ and $(2, 10)$ is

$$m = \frac{10-20}{2-0} = -5$$

Noticing that the point $(0, 20)$ is the y-intercept, we have

$$f(x) = -5x + 20$$

(b) *(3 pts)* Now assume that $f(x)$ is an exponential function. Write a formula for $f(x)$.

This is an exponential function, with "initial value" 20, and that halves for every change of 2 in the x-direction. Therefore the function is

$$f(x) = 20 \cdot \left(\frac{1}{2}\right)^{x/2}$$

Question 5: *(8 pts)* Consider the function

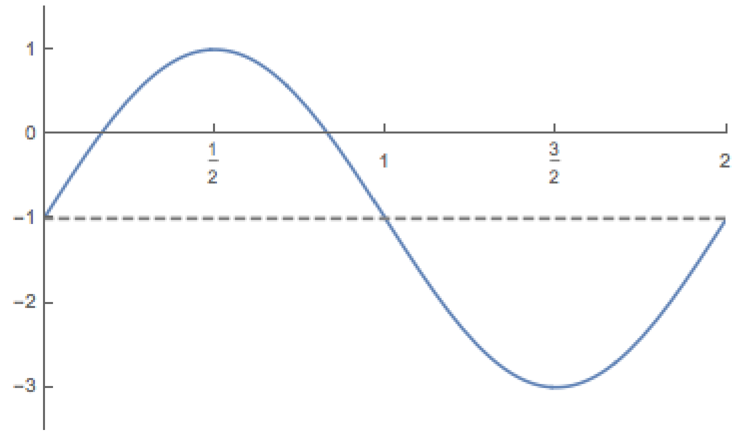
$$g(x) = 2\sin(\pi x) - 1$$

(a) (*4 pts*) Identify the period, amplitude, phase shift, and center line of this function.

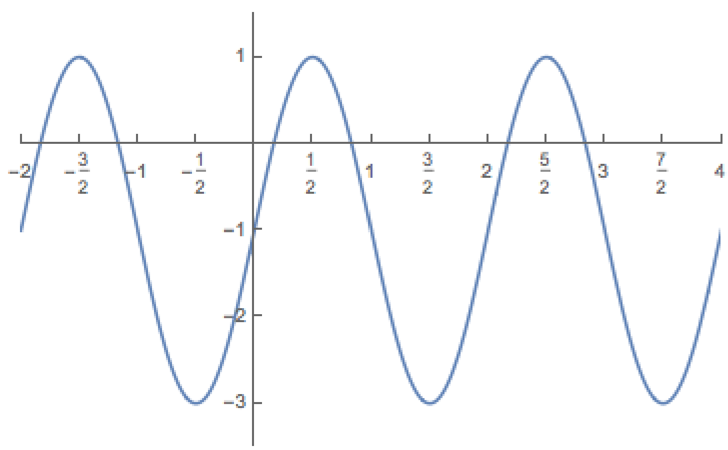
Using the form $f(x) = a\sin(b(x+d)) + c$, the period is $\frac{2\pi}{\pi} = 2$, the amplitude is 2, there is no phase shift ($d = 0$), and the center line is at $y = -1$.

(b) (*4 pts*) Using your information from part (a), draw a graph of the function.

Two graphs are shown below. The first shows only one period. The x-axis is labeled starting at 0, because there is no phase shift. Then each subsequent marking on the x-axis occurs every $2/4 = 1/4$ units. The center line is plotted in gray. The graph has the shape of the sine function, so it begins at the center line, then increases to the maximum value. The amplitude is 2.



This second graph shows multiple periods of the graph.



Question 6: (*8 pts*) A scientist is studying a sample of a radioactive material. At time $t = 0$, the mass of the radioactive material is 480 grams. After 150 days, the mass of the sample is 60 grams. Assume that the radioactive sample undergoes exponential decay.

(a) (*3 pts*) Write a formula for the function $a(t)$ that gives the amount a of radioactive material present in the sample after t days.

After 150 days, the mass is 60 grams, or $\frac{60}{480} = \frac{1}{8}$ of the original amount. Thus the function will be an exponential function that starts with a value of 480 and decreases by 1/8 every 150 days, which is

$$a(t) = 480 \cdot \left(\frac{1}{8}\right)^{t/150}$$

(b) (*3 pts*) Find the half-life of the radioactive material.

Half of the starting amount is 240 grams. So we solve

$$\begin{aligned} 240 &= 480 \cdot \left(\frac{1}{8}\right)^{t/150} \\ \frac{1}{2} &= \left(\frac{1}{8}\right)^{t/150} \\ \frac{1}{2} &= \left((\frac{1}{2})^3\right)^{t/150} \\ \frac{1}{2} &= \left(\frac{1}{2}\right)^{3t/150} \\ 1 &= \frac{t}{50} \\ t &= 50 \end{aligned}$$

So the half-life is 50 days.

(c) (*2 pts*) Find the time at which 7.5 grams of material remain.

After 150 days, the amount is 60 grams. After 50 more days (one more half life) the amount is 30 grams. After another half life, the amount is 15 grams. After one more half life, the amount is 7.5 grams. Thus after $150 + 150 = 300$ days, the amount will be 7.5 grams.

Question 7: (*16 pts*) Consider the function

$$f(x) = \frac{2x^2 - 3x + 1}{x^2 - x - 2}$$

(a) (*3 pts*) What is the domain of this function? Write your answer in interval notation.

The domain of a rational function is all numbers except where the denominator is equal to zero. Setting the denominator equal to zero and solving by factoring, we have

$$0 = x^2 - x - 2 = (x - 2)(x + 1)$$

So the domain is all real numbers except $x = -1, 2$. In interval notation, this is

$$(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$$

(b) (*3 pts*) Identify the zeroes of the function.

The zeroes of a rational function are the same as the zeroes of the numerator, assuming those zeroes are in the domain of the function. Setting the numerator to zero and solving by factoring, we have

$$0 = 2x^2 - 3x + 1 = (2x - 1)(x - 1),$$

so the zeroes are at $x = 1/2, 1$. Note that both of these are in the domain.

(c) (*4 pts*) Describe the end behavior of the function. Write your answer using limit notation.

Using the limit method we learned in the last lecture,

$$\begin{aligned}
\lim_{x\to\pm\infty} \frac{2x^2-3x+1}{x^2-x-2} &= \lim_{x\to\pm\infty} \frac{2x^2-3x+1}{x^2-x-2} \cdot \frac{1/x^2}{1/x^2} \\
&= \lim_{x\to\pm\infty} \frac{2-\frac{3}{x}+\frac{1}{x^2}}{1-\frac{1}{x}-\frac{2}{x}} \\
&= \frac{2}{1} = 2
\end{aligned}$$

So

$$\lim_{x\to\infty} f(x) = 2, \qquad \lim_{x\to\infty} f(x) = 2$$

Note that this means that the function will have a horizontal asymptote at $y = 2$.

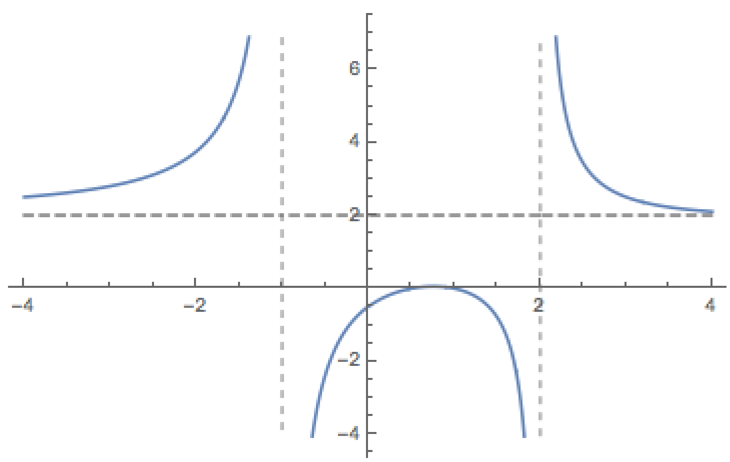
(d) (*1 pt*) Identify the y-intercept of the function.

The y-intercept is $f(0) = -1/2$.

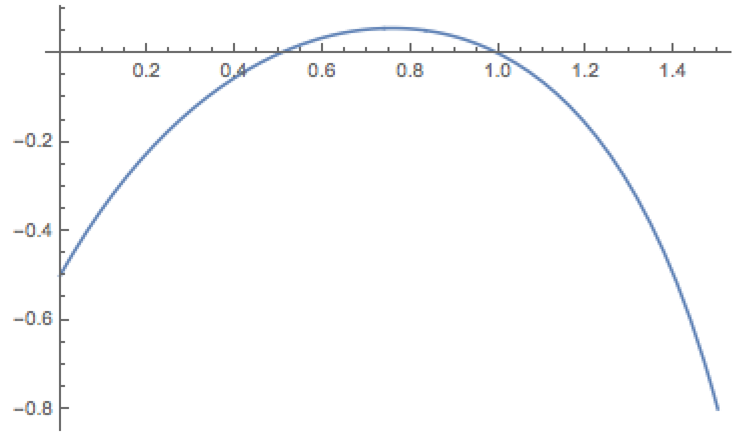
(e) (*5 pts*) Using the information from the previous parts of this question, draw a graph of the function. Your graph should have the proper domain, asymptotes, zeroes, and y-intercept, and should pass through the following points

$$f(-2) = \frac{15}{4} \qquad f(3/2) = \frac{-4}{5} \qquad f(3) = \frac{5}{2}$$

Two graphs are shown below. The first graph shows the large-scale behavior of the function. Note that it has vertical asymptotes at $x = -1, 2$, the two points that are not in the domain of the function, and that there is a horizontal asymptote at $y = 2$.



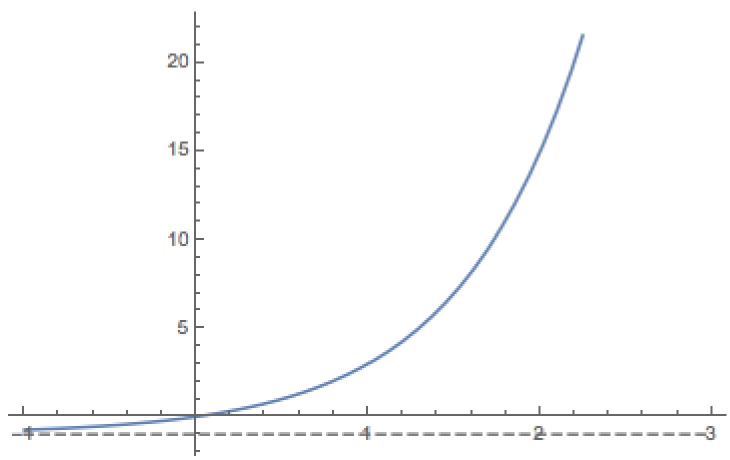
This second graph zooms in to the function around $x = 1/2$, so that the zeroes can be seen. Note that it has zeroes at $x = 1/2$ and 1.



Question 8: (*12 pts*) Sketch the graph of each of the following functions. Make sure your graphs include any necessary asymptotes and have the correct domain. Label at least 3 points on each graph.

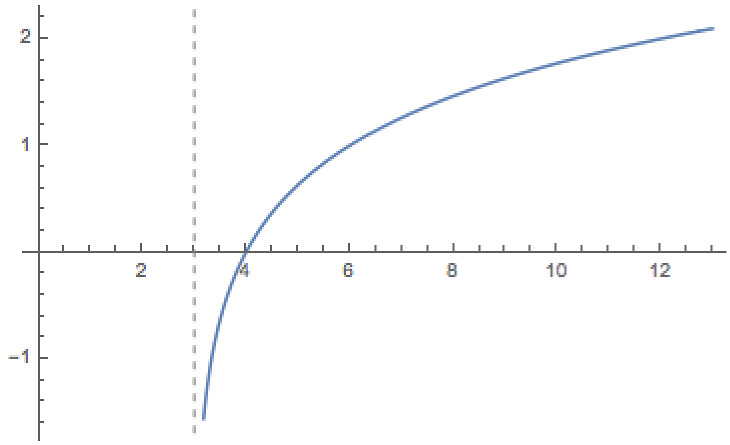
(a) (*4 pts*) $f(x) = 4^x - 1$

This is a graph of a standard exponential function, with $b = 4 > 1$, shifted down one unit. The graph will have a horizontal asymptote at $y = -1$. The easiest points to plot at $(0, 0), (1, 3)$, and $(2, 15)$. The graph is shown below.



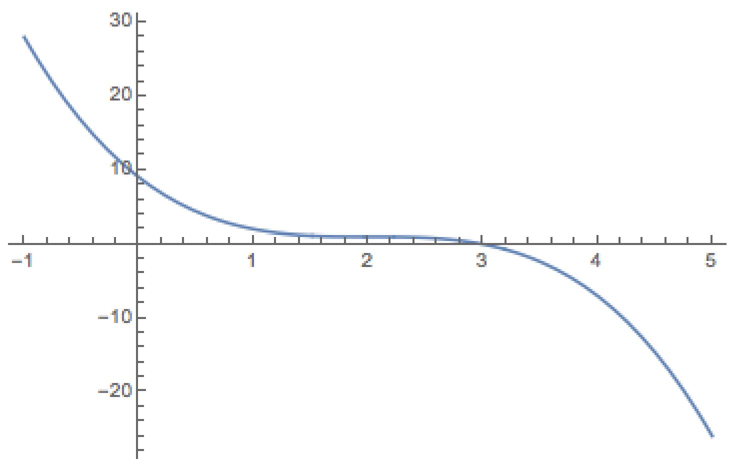
(b) (*4 pts*) $g(x) = \log_3(x - 3)$

This is a graph of a standard logarithmic function, with $b = 3 > 1$, shifted right three units. The graph will have a vertical asymptote at $x = 3$, and a domain of $(3, \infty)$. The easiest points to graph plot are $(4, 0), (6, 1)$ and $(12, 2)$. The graph is shown below.



(c) (*4 pts*) $h(x) = -(x-2)^3 + 1$

This graph will be the graph of x^3, shifted right 2 units, reflected over the x-axis, then shifted up 1 unit. The easiest points to plot are $(1,2), (2,1)$, and $(3,0)$. The graph is shown below.



Question 9: (*10 pts*) Let

$$f(x) = \frac{1}{x-1} \qquad g(x) = 2x^2 - 2 \qquad h(x) = \frac{2x}{x^2+1}$$

Compute each of the following. Simplify your answer as much as possible

(a) (*3 pts*) $f(f(3))$ We have

$$f(3) = \frac{1}{3-1} = \frac{1}{2}$$

therefore

$$\begin{aligned} f(f(3)) &= f\left(\frac{1}{2}\right) \\ &= \frac{1}{\frac{1}{2} - 1} = \frac{1}{\frac{-1}{2}} = -2 \end{aligned}$$

(b) (*3 pts*) $(f \circ h)(x)$

$$\begin{aligned}(f \circ h)(x) &= f\left(\frac{2x}{x^2+1}\right) \\ &= \frac{1}{\frac{2x}{x^2+1} - 1} \\ &= \frac{x^2+1}{2x-(x^2+1)} \\ &= \frac{x^2+1}{-x^2+2x-1}\end{aligned}$$

(c) (*4 pts*) $\dfrac{g(x+1)-g(1)}{x}$, assuming $x \neq 0$.

Note that

$$g(1) = 2 \cdot 1^2 - 2 = 0$$

so

$$\begin{aligned}\frac{g(x+1)-g(1)}{x} &= \frac{2(x+1)^2 - 2 - 0}{x} \\ &= \frac{2x^2+4x+2-2}{x} \\ &= \frac{2x^2+4x}{x} = \frac{x(2x+4)}{x} \\ &= 2x+4\end{aligned}$$

Question 10: (*13 pts*) Assume that θ is an angle with $0 < \theta < \pi/2$ such that $\sin\theta = \dfrac{15}{17}$. Evaluate the following trigonometric functions. It may be helpful for you to know that $15^2 = 225$ and $17^2 = 289$.

(a) *(3 pts)* $\cos\theta$

We're told in the problem that θ is in the first quadrant, so $\cos\theta$ is positive. Therefore we can use the Pythagorean identity to find that

$$\begin{aligned}\cos\theta &= \sqrt{1-\sin^2\theta} \\ &= \sqrt{1-\frac{15^2}{17^2}} \\ &= \sqrt{\frac{289}{289} - \frac{225}{289}} \\ &= \sqrt{\frac{64}{289}} \\ &= \frac{8}{17}\end{aligned}$$

(b) *(3 pts)* $\cos(2\theta)$

We use one of the double-angle formulas for cosine.

$$\begin{aligned}
\cos(2\theta) &= 2\cos^2\theta - 1 \\
&= 2 \cdot \left(\frac{8}{17}\right)^2 - 1 \\
&= 2 \cdot \frac{64}{289} - 1 \\
&= \frac{128}{289} - \frac{289}{289} \\
&= \frac{-161}{289}
\end{aligned}$$

(c) *(4 pts)* $\sin(\theta + \pi/6)$

Using the sum formula for sine,

$$\begin{aligned}
\sin(\theta + \pi/6) &= \sin\theta\cos(\pi/6) + \cos\theta\sin(\pi/6) \\
&= \frac{15}{17} \cdot \frac{\sqrt{3}}{2} + \frac{8}{17} \cdot \frac{1}{2} \\
&= \frac{8 + 15\sqrt{3}}{34}
\end{aligned}$$

(d) *(3 pts)* $\sin(\theta/2)$

Because θ is in the first quadrant, $\theta/2$ is as well, so $\sin(\theta/2)$ is positive. Using the half-angle identity,

$$\begin{aligned}
\sin(\theta/2) &= \sqrt{\frac{1 - \cos(\theta)}{2}} \\
&= \sqrt{\frac{1 - \frac{8}{17}}{2}} \\
&= \sqrt{\left(\frac{17}{17} - \frac{8}{17}\right) \cdot \frac{1}{2}} \\
&= \sqrt{\frac{9}{34}} \\
&= \frac{3}{\sqrt{34}}
\end{aligned}$$

Question 11: (*12 pts*) Evaluate each of the following limits.

(a) (*3 pts*) $\lim\limits_{x \to 7} \dfrac{x^2 - 5x - 14}{x^2 - 6x - 7}$

If we just try to plug in $x = 7$, we'll get $\frac{0}{0}$, which tells us that we should be able to factor an

$(x-7)$ term from the top and bottom, which we'll then cancel.

$$\begin{aligned}\lim_{x\to 7}\frac{x^2-5x-14}{x^2-6x-7} &= \lim_{x\to 7}\frac{(x-7)(x+2)}{(x-7)(x+1)}\\ &= \lim_{x\to 7}\frac{(x+2)}{(x+1)}\\ &= \frac{7+2}{7+1}=\frac{9}{8}\end{aligned}$$

(b) (*2 pts*) $\lim\limits_{x\to 4}\dfrac{x^2-5x-14}{x^2-6x-7}$

In solving the problem above we saw that the domain of the function is all real numbers except $x=-1,7$. Thus 4 is in the domain. Since rational functions are continuous at all points in their domain, we have that the value of the limit at 4 will be the value of the function when we plug in 4. This gives

$$\lim_{x\to 4}\frac{x^2-5x-14}{x^2-6x-7}=\lim_{x\to 7}\frac{(x-7)(x+2)}{(x-7)(x+1)}=\frac{(4-7)(4+2)}{(4-7)(4+1)}=\frac{6}{5}$$

(c) (*3 pts*) $\lim\limits_{x\to-\infty}\dfrac{x^3-3x^2+2}{4x^3+2x-7}$

Using our method for evaluating infinite limits of rational functions,

$$\begin{aligned}\lim_{x\to-\infty}\frac{x^3-3x^2+2}{4x^3+2x-7} &= \lim_{x\to-\infty}\frac{x^3-3x^2+2}{4x^3+2x-7}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\\ &= \lim_{x\to-\infty}\frac{1-\frac{3}{x}+\frac{2}{x^3}}{4+\frac{2}{x^2}-\frac{7}{x^3}}\\ &= \frac{1-0+0}{4+0-0}=\frac{1}{4}\end{aligned}$$

(d) (*4 pts*) $\lim\limits_{x\to 0}\dfrac{\cos(2x)-1}{\cos x-1}$ (*Hint*: Use an identity)

If we try to just plug in $x=0$, we'll get $\frac{0}{0}$, so similar to our method in part (a), we'll try to cancel out a common factor from the numerator and the denominator. This time, though, we'll first need to use the double angle identity to rewrite the numerator of the fraction. This gives

$$\begin{aligned}\lim_{x\to 0}\frac{\cos(2x)-1}{\cos x-1} &= \lim_{x\to 0}\frac{(2\cos^2 x-1)-1}{\cos x-1}\\ &= \lim_{x\to 0}\frac{2(\cos^2 x-1)}{\cos x-1}\\ &= \lim_{x\to 0}\frac{2(\cos x-1)(\cos x+1)}{\cos x-1}\\ &= \lim_{x\to 0}2(\cos x+1)\\ &= 2(\cos 0+1)\\ &= 4\end{aligned}$$

Question 12: (*11 pts*)

(a) (*4 pts*) Let x and y be numbers such that $\log_2(x) = 3$ and $\log_2(y) = 1/2$. Evaluate $\log_4(xy^3)$. Notice that this is a logarithm with base 4, not base 2. Your answer should be a fraction.

We'll first use the change of base formula to write the logarithm base 4 in terms of logarithms with base 2, then use log rules to expand the expression. This gives

$$\begin{aligned}
\log_4(xy^3) &= \frac{\log_2(xy^3)}{\log_2 4} \\
&= \frac{\log_2 x + \log_2 y^3}{2} \\
&= \frac{\log_2 x + 3\log_2 y}{2} \\
&= \frac{3 + 3\cdot\frac{1}{2}}{2} \\
&= \frac{9}{4}
\end{aligned}$$

(b) (*3 pts*) Using trigonometric identities, simplify the following expression. Your answer should include only one trigonometric function.

$$\frac{(\sin\theta + \tan\theta)^2 + \cos^2\theta - \sec^2\theta}{\tan\theta}$$

$$\begin{aligned}
\frac{(\sin\theta + \tan\theta)^2 + \cos^2\theta - \sec^2\theta}{\tan\theta} &= \frac{\sin^2\theta + 2\sin\theta\tan\theta + \tan^2\theta + \cos^2\theta - \sec^2\theta}{\tan\theta} \\
&= \frac{(\sin^2\theta + \cos^2\theta) + 2\sin\theta\tan\theta + (\tan^2\theta - \sec^2\theta)}{\tan\theta} \\
&= \frac{1 + 2\sin\theta\tan\theta + (-1)}{\tan\theta} \\
&= \frac{2\sin\theta\tan\theta}{\tan\theta} \\
&= 2\sin\theta
\end{aligned}$$

(c) (*4 pts*) Let $f(x) = \log_4(2x-1) - \log_4(x+3)$. This function is invertible. Find $f^{-1}(y)$.

We set $y = f(x)$ and then solve for x.

$$\begin{aligned}
y &= \log_4(2x-1) - \log_4(x+3) \\
y &= \log_4\left(\frac{2x-1}{x+3}\right) \\
4^y &= \frac{2x-1}{x+3} \\
4^y(x+3) &= 2x-1 \\
x\cdot 4^y - 2x &= -1 - 3\cdot 4^y \\
x(4^y - 2) &= -1 - 3\cdot 4^y \\
x &= \frac{-1 - 3\cdot 4^y}{4^y - 2}
\end{aligned}$$

hence

$$f^{-1}(y) = \frac{-1 - 3 \cdot 4^y}{4^y - 2}$$

MAT 123 Sample Final

1 Conceptual Questions.

This section contains some general questions to help you test your understanding of some of the key concepts we've learned since the midterm. As with the sample midterm, these questions are not necessarily representative of the kind of questions that will appear on the final - the true sample final is in the next section. Note that the final exam will be cumulative, but that these conceptual questions only cover the material we've learned since the midterm.

1. What real-world situations are modeled by exponential growth or decay?

2. How is the number e defined? What makes this number special?

3. How are the sine, cosine, and tangent functions defined using the unit circle? How are these functions defined using right triangles?

4. What are reference angles, and how are they used to evaluate trig functions?

5. State as many trig identities as you can.

6. What are the domain and range of the trigonometric functions? What are the domain and range of the inverse trigonometric functions?

7. Why do mathematicians use radians instead of degrees for angle measure? How do you convert between radian and degree measures?

8. What are the period, amplitude, phase shift, and average value of a sinusoidal trigonometric function? How do you find these values using a graph? Using a formula?

2 Sample Final

This sample final is meant to have the same style of questions and the same length and difficulty as the final. However, topics covered on this sample final may not be on the actual final, and there may be topics on the final that do not appear on the sample final. You are responsible for knowing all of the concepts on the topics sheet for the final. There will be no "cheat sheet" or list of formulas included with the final.

You may not be able to complete questions marked with a star ($\star$) until after the lecture on Monday, June 29.

Question 1: Assume that θ is an angle with $\pi/2 < \theta < \pi$ such that $\sin\theta = 4/5$. Evaluate the following trigonometric functions.

(a) $\cos\theta$

(b) $\sin(\theta/2)$

(c) $\sin(\theta + \pi/6)$

(d) $\cos(2\theta)$

Question 2: Simplify or evaluate each of the following. Your answers should not include any trigonometric expressions or logarithms.

(a) $\cos(\arctan x)$

(b) $2\log_5(7) - \log_5(25 + 220)$

(c) $\cos(\arccos(3/4))$

(d) $\arccos(\cos(3\pi/4))$

(e) $\log_6(24) + 2\log_6(3)$

Question 3: Let

$$f(x) = 2x + 3 \qquad g(x) = \frac{x+1}{x-1}$$

Compute each of the following. Simplify your answer as much as possible

(a) $f(g(2))$

(b) $(g \circ f)(x)$.

(c) Let $h(x) = (g \circ f)(x)$. Is h invertible? If so, find a formula for h^{-1}.

Question 4:

(a) Write the equation of a line that passes through the point $(2, 3)$ and is perpendicular to the line that passes through the points $(-5, 1)$ and $(-7, 8)$

(b) A colony of bacteria has an initial population of 100, but has grown to 400 after 3 hours. Write a function that gives the population of the colony of bacteria as a function of t, in hours.

(c) How long will it take for the population of bacteria in part (b) to reach 3200?

Question 5:

(a) Find the zeroes of the function $f(x) = -3x^3 - 18x^2 - 24x$.

(b) Describe the end behavior of $f(x)$. Write your answer using limit notation.

(c) Draw a graph of $f(x)$. Your graph should have the proper zeroes, the proper end behavior, and the correct number of "turns."

(d) Write the quadratic function $g(x) = 4x^2 - 4x + 3$ in vertex form.

(e) Draw a graph of $g(x)$. Your graph should have the correct vertex and the correct y-intercept.

Question 6: Sketch the graph of each of the following functions. Make sure you graphs include any necessary asymptotes and have the correct domain. Label at least 3 points on each graph.

(a) $f(x) = \left(\frac{1}{2}\right)^x + 2$

(b) $g(x) = \tan(x)$

(c) $h(x) = \ln(5x)$

Question 7: Consider the function

$$g(x) = -\cos(4x - 1) + 2$$

(a) Identify the period, amplitude, phase shift, and center line of this function.

(b) Using your information from part (a), draw a graph of the function.

Question 8: Solve each of the following equations, or explain why there is no solution.

(a) $\cos(x) = \cos(2x)$

(b) $\ln(x + 1) = 2 - \ln(x - 1)$

(c) $2x^2 + 4x + 3 = 0$

Question 9: Consider the function

$$f(x) = \frac{4x^3 - x}{x^2 + x - 12}$$

(a) Identify the zeroes of the function.

(b) What is the domain of this function? Write your answer in interval notation.

(c) Describe the end behavior of the function. Write your answer using limit notation.

(d) Identify the y-intercept of the function.

(e) Using the information from the previous parts of this question, draw a graph of the function. Your graph should have the proper domain, asymptotes, zeroes, and y-intercept. To help you graph the function, here are some values of the function at various points.

$$f(-5) = \frac{-495}{8} \qquad f(-2) = 3 \qquad f(2) = -5 \qquad f(4) = \frac{63}{2}$$

Question 10: A scientist is using carbon-14 dating to find the age of a fossil. She estimates that the fossil contains only 10% of the carbon-14 that was originally in the sample. Carbon-14 has a half life of 5730 years. How old is the fossil? Write your answer using a logarithm.

Question 11: Verify the following trigonometric identity, the triple angle formula for sine -

$$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$$

(*Hint:* Use the sum and double angle formulas after writing $\sin(3\theta) = \sin(\theta + 2\theta)$).

($\star$) **Question 12:** Compute the following limits.

(a) $\displaystyle\lim_{x\to 1} \frac{x^2 + x - 2}{x - 1}$

(b) $\displaystyle\lim_{x\to \infty} \frac{x^2 + x - 2}{x - 1}$

(c) $\displaystyle\lim_{x\to 2} (x^2 - 2x + 1)$

(d) $\displaystyle\lim_{h\to 0} \frac{(3 + h)^2 - 3^2}{h}$

MAT 123 - Topics for Final Exam

The final exam will be held in class on 7/1. The following topics will be covered. An approximate reference to the textbook is given for each topic, although not all material we have discussed is in the textbook.

This first listing of topics is all of the topics that appeared on your midterm exam. You will still be responsible for knowing all material from the midterm exam.

- Functions - Definition of a function. Different ways to describe a function, including using words, formulas, tables, and graphs. Domain and range. Finding the domain of a function, given a formula. Interval notation for writing domains and ranges. (Section 1.1)

- Graphing Functions - The coordinate plane and plotting ordered pairs. Graph of a function is set of all pairs $(x, f(x))$ for x in the domain of the function. Using a graph to find values of a function. Using a graph to solve equations of the form $f(x) = b$ for b a fixed number. Determining domain and range of a function from a graph. The vertical line test. (Section 1.2)

- Ways to combine functions - addition, subtraction, multiplication, division, and composition. Finding the domain of the resulting functions. (Section 1.4)

- Graphing function transformations - changing a graph of a function via translations, stretching/shrinking, and reflection, both vertically and horizontally. Understand both in terms of graphs (e.g., how to translate a graph) and in terms of formulas (e.g., $g(x) = f(x) + 2$ is the graph of $f(x)$ shifted up two units.). Domain and range of transformed functions. Even and odd functions (Section 1.3)

- Inverse functions - One-to-one functions. Deciding if a function is invertible. Definition of inverse function. Computing inverse functions. Restricting the domain of a function to make it invertible. Domain and range of inverse functions. Composition and inverse functions. (Section 1.5)

- Graphing inverse functions - Obtaining graph of inverse function by reflection. Horizontal line test. Increasing and decreasing functions (Section 1.6)

- Linear functions - Slope formula. Point-slope form for the equation of a line. Slope-intercept form for the equation of a line. Definition of a linear function. Slopes of parallel and perpendicular lines. (Section 2.1)

- Quadratic Functions - Definition of a quadratic function. Completing the square. The quadratic formula to find roots of a quadratic function. The vertex form of a quadratic equation. Graphing quadratic functions using the vertex form and graph transformations. Finding maximum and minimum values of a quadratic function. (Section 2.2)

- Polynomial Functions - Definition of polynomial. Degree of polynomial. Addition, subtraction, multiplication, and division of polynomials. Degree and leading coefficient. Zeroes of a polynomial. Relationship between zeros of a polynomial and factors of the polynomial. End behavior of polynomials. General shape of graphs of polynomials (Section 2.4)

- Rational Functions - Definition of rational functions. Domain of rational functions. End behavior of rational functions. Zeroes of rational functions. Vertical asymptotes. Graphing rational functions. (Section 2.5)
- Exponential Functions - Definition of exponential functions. Graph of exponential functions. Properties of exponents. (Sections 2.3 and 3.1)
- Logarithms - Definition of logarithm as inverse function for exponential. Graph of the logarithm function. (Section 3.1)

The following topics have been covered since the midterm exam.

- Logarithm rules - formulas for sum and difference of logarithms, formula for the multiplication of a logarithm by a constant, change of base formula for logarithms. (Sections 3.2 and 3.3)
- Applications of exponential functions - Exponential growth and decay. Applications to various real-world situations - population growth, radioactive decay, and compounded interest. Writing the equation of an exponential growth function given values of that function at two points. (Sections 3.2 and 3.4)
- Euler's number and the natural logarithm - How Euler's number arises when the number of compounding periods approaches infinity. The natural logarithm. Continuous growth rates. Writing exponential functions $f(t) = Ab^t$ as $f(t) = Ae^{kt}$ for $k = \ln b$. (Section 3.7)
- Measuring angles and the unit circle - The unit circle and its equation. Drawing angles on the unit circle given in degrees. Drawing angles on the unit circle given in radians. Converting between degree and radian measures. Interpreting radian measure of angles using arc length. (Sections 4.1,4.2)
- Trigonometry and the unit circle - Definition of sine and cosine, using the unit circle. Definition of tangent, cosecant, secant, and cotangent. The values of sine, cosine, and tangent at special angles $\pi/6, \pi/4, \pi/3$. Using reference angles to compute the value of any trig function for any angle. The domain, range, and graphs of the sine, cosine, and tangent functions. (Sections 4.3 and 4.4)
- Trigonometry and right triangles - Interpreting trigonometric functions in terms of the ratios of lengths of sides of a right triangle (e.g., $\sin(\theta) = \frac{opp}{hyp}$). The Pythagorean Theorem. (Section 4.5)
- Trigonometric Identities - A list of the trigonometric identities you should know is posted to the website at

  `http://www.math.stonybrook.edu/~jthurman/mat123-sum15/files/homework/trigidentities.pdf`

  You should know all of these identities (or know how to derive them) and be able to use them to compute the values of trig functions. (Sections 4.6, 5.5, and 5.6)

- Inverse trig functions - Restricting domain of trig functions to make them invertible. Definition, domain, and range of the inverse sine, cosine, and tangent functions. Different notations for inverse trig functions (e.g., $\arcsin x$ and $\sin^{-1} x$). Evaluating inverse trig functions using known special angles. Evaluating expressions involving inverse trig functions using right triangles. Identities for composition of trig functions with their inverses (pg 408). (Sections 5.1 and 5.2)

- Graphing trig functions - Using graph transformations to draw the graphs of functions related to the sine and cosine functions. Finding the period, amplitude, phase shift, and center line of trigonometric functions. (Section 5.6)

- Equations with trig - solving equations involving trig functions. Producing infinite lists of solutions by adding and subtracting the period of the function. Using trig identities to rewrite equations into solvable forms. Not in the textbook - studying factoring, trig identities, and values of trig functions at special angles.

The following topics will be covered in class on June 29, and will also be covered on the midterm.

- Basics of limits - Intuitive definition of a limit. Evaluating limits algebraically. Evaluating limits graphically. End behavior of functions and limits.