Math 200 Lecture 2 (Mon/ Wed) Exam II Spring 2007 Scott Simon

Axioms and Definitions for Geometry

UNDEFINED TERMS . The plane is our universe of discourse, and points and lines are subsets of the plane. A line is a set of points with properties as defined by the axioms. The distance between any two points A and B is a number denoted by $|AB|$, again with properties as specified by the axioms.

INCIDENCE AXIOM.

1. For any two distinct points, there is a unique line that contains these two points.
2. Every line contains at least two distinct points.
3. For any line, there exists a point not on this line.

DEFINITION. Two lines l and m are said to be transverse if they are distinct ($l = m$) and have at least one point in common. Two lines are parallel if they are not transverse.

THE PARALLEL AXIOM. For any line l and a point P not on l, there exists a unique line containing P and parallel to l.

THE RULER AXIOM. Let l be any line. Then there is a bijection $f : l \to R$ such that, for any two points A, B on l, the distance between A and B, $|AB|$, is given by $|f(A) - f(B)|$. This bijection f is called a coordinate system on l.

DEFINITION. If A, B, and C are points on a line l, we say B is between A and C if there is a coordinate system f on l for which$f(A) < f(B) < f(C)$. The set of all points on l that are between between A and C is called the line segment $\overline{AB}$.

DEFINITION. Let A, B and C be three distinct points on a line l. We say that A and C are on opposite sides of B if B is between A and C . If A and B are not on opposite sides of C , we say A and B are on the same side of C .

DEFINITION. If l is a line and V and A are distinct points on l, we define the ray $\overrightarrow{VA}$ to be all of the points on l that are on the same side of V as A.
BINOMIAL THEOREM:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$$

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You may write answers in terms of factorials and exponents.

1. Suppose I have 10 buckets (each one is a different color) and 6 marbles (each one is a different color).

(a) (5 pts) How many different ways are there to put the marbles in the buckets (assuming I don't care what order I do it in)?

(b) (5 pts) How many different ways are there to put the marbles in if the order still doesn't matter, but now at most one marble can go into any bucket?

2. Suppose that there are 7 lunchboxes, 7 sandwiches, 5 apples, and 4 thermoses of milk. We wish to put together 7 lunches so that none contain more than one sandwich, apple, or thermos.

(a) (5 pts) How many different possible ways are there to pack the lunchboxes if each lunchbox has a child's name on it? That is, the outcome is different if the same set of lunches is put into different lunchboxes.

(b) (5 pts) How many different possible ways are there to pack the lunches if all of the lunchboxes are identical (i.e. it doesn't matter who gets which lunchbox)?

3. (5 pts) Write the number $0.123123123\ldots$ as a fraction.

4. (a) (5 pts) Find a bijection between the set of all positive integers and the set of all numbers of the form $n/2^k$ for some integers n and k.

(b) (5 pts) Find a bijection from $[0, 1]$ to a proper subset of itself. You may use part (a) even if you do not solve it.

5. (5 pts) Let P, Q, R be distinct points such that P lies on the line $\overleftrightarrow{QR}$. Show that R lies on the line $\overleftrightarrow{PQ}$.

6. For each of the interpretations of the terms point, line, and distance given below, determine if they are consistent with the indicated axioms.

(a) Points are elements of the set $\mathbb{R}^2$, and lines are the usual lines or circles. The distance is the usual Euclidean distance.

i. (3 pts) Prove that the incidence axiom is satisfied or show that it isn't.

ii. (3 pts) Prove that the parallel axiom is satisfied or show that it isn't.

(b) Points are elements of the set $\{(x, y) \in \mathbb{R}^2 | x^2 + y^2 < 1\}$ and lines are line segments whose endpoints lie on the unit circle. The distance between (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{(1 - x_1^2 - y_1^2)(1 - x_2^2 - y_2^2)}}.$$

i. (3 pts) Prove that the incidence axiom is satisfied or show that it isn't.

ii. (3 pts) Prove that the parallel axiom is satisfied or show that it isn't.