Math 313 (Fall '09)

Homework 4

due October 8

The following exercises refer to the textbook (the seventh edition).

- Ch9: 11, 14, 25
- Ch11: 10, 12

Sample Midterm

1. Which of the following sets are subgroups of $\mathrm{GL}(2,\mathbb{R})$:
   i) $H_1 = \{X \in \mathrm{GL}(2,\mathbb{R}) \mid \det X = 1\}$;
   ii) $H_2 = \{X \in \mathrm{GL}(2,\mathbb{R}) \mid \det X = -1\}$;
   iii) $H_3 = \{X \in \mathrm{GL}(2,\mathbb{R}) \mid$ the entries in X belong to $\mathbb{Z}\}$;
   iv) $H_4 = \{X = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \mid a, b \in \mathbb{R}\}$

   Explain!
2. What are the possible order of permutations in S_7. How many permutations of order 8 and 10 respectively are in S_7? What is the number of even permutations of order 6 in S_7?
3. Classify all groups with 8 elements.

   Hint: As a first step you should list all groups with 8 elements that you know. Pay attention to the maximal order that occurs in each case. Then consider the abelian case. Finally, discuss the non-abelian case.
4. Let G be a cyclic group. Prove that
   i) any subgroup H of G is cyclic;
   ii) any factor group G/H is cyclic.

   Additionally, give an example to show that it does not suffice to know that H and G/H are cyclic, to conclude that G is cyclic.
5. Show that a group of order 33 must have an element of order 3.
6. The set $\{1, 9, 16, 22, 29, 53, 74, 79, 81\}$ is a group under multiplication modulo 91. Determine the isomorphism class of this group.