If Ψ_ε is of type I, then

$$\chi(\mathcal{D}_+) \equiv B(q_\Delta) + \tau - 1 \mod 4 \qquad (27)$$

§ 4. APPLICATIONS

4.1. Sufficient conditions for applicability.

As in Theorem (3.B), let A be a reduced real plane projective curve of even degree without non-real singular points and let Δ be a singular point diagram of A . Let us consider the following condition

(4.A) For each irreducible component of the non-singular model of A the images of all real path components except at most one do not contain a singular point of A .

Sometimes (4.A) makes Theorem (3.B) to work:

(4.B) Under (4.A), if each singular point of A has no non-real branch then q_Δ vanishes on B_Δ .

(4.B) is generalized below. The generalization is not applied in this paper. We present it for the sake of completeness only.

(4.C) Under (4.A), if at each singular point of A each non-real branch β has an even intersection number with the union of all branches different from β and $\mathrm{conj}\,\beta$, then q_Δ vanishes on B_Δ .

Now as in Theorem (3.D), let Ψ_ε be a smoothing of Φ . Let us consider the condition

(4.A') The $(\mathbb{C}\Psi_\varepsilon \cap \partial\mathcal{D})$ united with all components of $\mathbb{R}\Psi_\varepsilon \cap \mathcal{D}$ homeomorphic to I is connected. It is a substitute for (4.A):

(4.C') Under (4.A'), if each non-real branch β of $\mathbb{C}\Phi \cap \mathcal{D}$ has even intersection number with the union of all branches different

from β and conj β , then q_Δ vanishes on B_Δ .

   The following criteria (4.E) and (4.F) require the condition (4.D) instead of (4.A') $\big[$(4.E) is slightly less general than (4.F)$\big]$. Let $\mathcal{D}^* = \mathcal{D}/conj$, $\mathcal{Y}_\varepsilon = \mathbb{C}\,\Psi_\varepsilon\,/conj$, $\Sigma_\varepsilon = \mathcal{Y}_\varepsilon \cup \mathcal{D}_+$ (cf.3.2).

   (4.D) Each boundary component of Σ_ε has even linking number with the union of the others boundary components of Σ_ε .

   (4.E) Under (4.D), if each component of $\mathcal{D}_+$ contains only one non-closed component of $\mathbb{R}\,\Psi_\varepsilon$, then q_Δ vanishes on B_Δ .

   (4.F) Under (4.D), if the subgroup $in_* \, H_1\,(\partial\,\mathcal{Y}_\varepsilon\,;\,\mathbb{Z}/2\,)$ of $H_1\,(\Sigma\,;\,\mathbb{Z}/2\,)$ is contained in $in_* \, H_1\,(\partial\Sigma\,;\,\mathbb{Z}/2\,) + in_* \, H_1\,(\mathcal{D}_+\,;\,\mathbb{Z}/2\,)$, then q_Δ vanishes on B_Δ .

   Proofs are given in 6.6.

4.2. Korchagin's curves.

Let A be a real plane projective curve of degree 7 with only one singular point and let there be 4 branches at the singular point, all branches be real, and one of them be transversal to the others, which are ordinarily tangent to each other (such a singular point is denoted by Z_{15} in Arnold's notations). In some (perhaps non-linear) coordinates x, y in some neighbourhood of the point the curve is defined by an equation

$$ x\,(y - a\,x^2)(y - b\,x^2)(y - c\,x^2) = 0 \quad (\,a \neq b\,,\,b \neq c\,,\,c \neq a\,). $$

Real schemes of such curves were treated by Korchagin [11] . He conjectured some congruences. They constitute a part of the following propositions which does not involve type of the curve (purely real point of view).

   (4.G) Let A has a real scheme outlined in the fig. 3 ($\langle\alpha\rangle$ designates a set of α ovals each lying outside others)

a)   β)

$\langle \beta \rangle$ $\langle \beta \rangle$
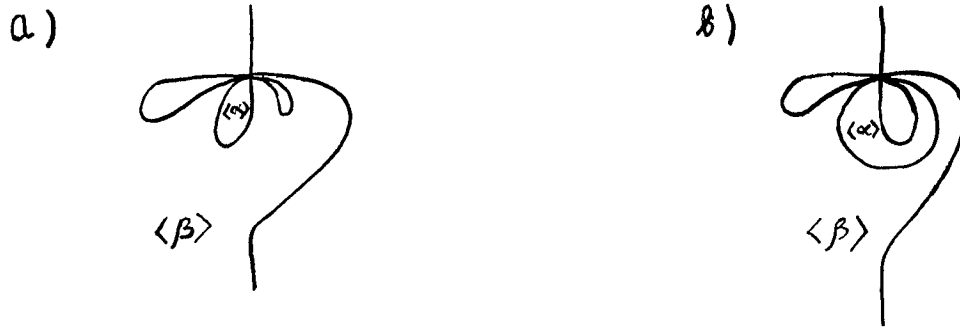
Fig. 3.

If $\alpha + \beta = 6$ then $\alpha - \beta \equiv 4 \mod 8$, if $\alpha + \beta = 5$ then $\alpha - \beta \equiv 4 \pm 1 \mod 8$, if $\alpha + \beta = 4$ and A is of type II then $\alpha - \beta \not\equiv 0 \mod 8$, if A is of type I then $\alpha - \beta \equiv 0 \mod 4$.

(4.H) Let A has a real scheme represented on the fig. 4

$\langle \beta \rangle$

Fig. 4.

If $\alpha + \beta = 6$ then $\alpha - \beta \equiv -2 \mod 8$, if $\alpha + \beta = 5$ then $\alpha - \beta \equiv -2 \pm 1 \mod 8$, if $\alpha + \beta = 4$ and A is of type II then $\alpha - \beta \equiv 2 \mod 8$, if A is of type I then $\alpha - \beta \equiv 2 \mod 4$

The condition $\alpha + \beta = 6 - i$ means that A is an $(M-i)$-curve.

To prove (4.G) and (4.H) it is enough to add the straight line tangent at the singular point to three (pairwise tangent) branches of A and to apply the theorem (3.B) to the reducible curve $A \cup \ell$

of degree 8. The theorem is applicable due to (4.A). We have a possibility not to do a straightforward calculation of the invariant $B(q_\Delta)$. Really, the theorem implies that for each scheme from fig. 3 and 4 there are a congruence of the form $\alpha - \beta \equiv u \bmod 8$ in M-case, a congruence of the form $\alpha - \beta \equiv u \pm 1 \bmod 8$ in $(M-1)$-case etc; the true values of u can be taken from examples constructed by Korchagin [11].

4.3. The case of non-degenerate double points.

Let A be a real plane projective curve of degree $m = 2k$ and let real non-degenerate double points with real tangents be the only singularities of A. Let $\mathbb{R}P^2_+$ be a half of $\mathbb{R}P^2$ bounded by $\mathbb{R}A$. As in 2.4, denote by Γ_A the union of components $\Gamma^1_1, \ldots, \Gamma^z$ of $\mathbb{R}A$ containing singular points. Denote by $C^i_1, \ldots, C^i_{z(i)}$ $(1 \leq i \leq z)$ components of $\mathbb{R}P^2 \setminus (\Gamma_A \cup \Theta)$ which lie on the other side of Γ^i than $\mathbb{R}P^2_+$ and by L some simple loop in Γ_A non-contractible in $\mathbb{R}P^2$ (if such a loop exists).

We present a straightforward independent description of $(V_\Delta, \circ, q_\Delta)$ and B_Δ constructed in 2.4.

(4.I) Let ∂^i_j be an element of $H_1(\Gamma_A; \mathbb{Z}/2)$ realized by the boundary of C^i_j and let ℓ be an element of $H_1(\Gamma_A; \mathbb{Z}/2)$ realized by L. Then V_Δ is a subspace of $H_1(\Gamma_A; \mathbb{Z}/2)$ generated by ℓ, ∂^i_j $(1 \leq i \leq z, 1 \leq j \leq z(i))$. The bilinear form $\circ$ and the quadratic function q_Δ are determined by the following:

(i) $q_\Delta(\partial^i_j)$ is equal modulo 4 to a number of singular points through which the boundary of C^i_j passes only once; $q_\Delta(\ell)$ is equal modulo 4 to $(-1)^k$ plus the number of singular points through which L passes as in fig. 5 (not as in fig. 6);
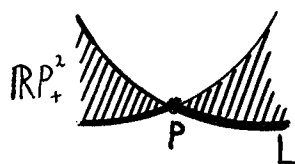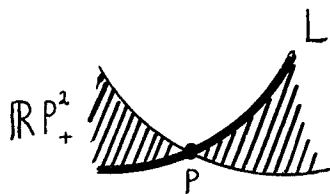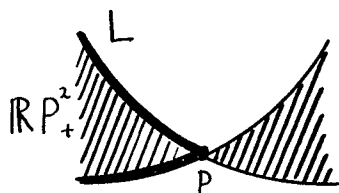
Fig. 5.

Fig. 6.

(ii) $\partial_j^i \circ \partial_n^k$ $((i,j) \neq (k,n))$ is equal modulo 2 to the number of singular points common to the boundaries of C_j^i and C_n^k

(iii) $\partial_j^i \circ \ell$ is equal modulo 2 to the number of singular points through which the boundary of C_i passes only once and L passes as in fig. 5.

The space B_Δ is generated by elements realized by smoothly immersed circles.

We have described all ingradients of Theorem (3.B). As a result, we obtain that in the situation considered in this section the theorem works iff the following condition is satisfied.

(4.J) Each real branch of A (i.e. smoothly immersed circle) contractible in $\mathbb{R}P^2$ passes through $n \equiv 0 \mod 4$ singular points and each real branch of A non-contractible in $\mathbb{R}P^2$ passes through $n \equiv (-1)^{k+1} \mod 4$ singular points.

To prove the equivalence of (4.I) and the definitions from 2.3, 2.4 it is sufficient to observe the following: for each non-degenerate

double point P with real tangents the surfaces R_p, $\mathcal{Y}_p$ are homeomorphic to a disk, $\sum_p$ to a Möbius band and further, $V_p =$

$$= \mathbb{Z}_{/2} , W_p = 0 \quad \text{and} \quad q_{/p}(v) = -1 \quad \text{for} \quad v \in V_p , v \neq 0 .$$

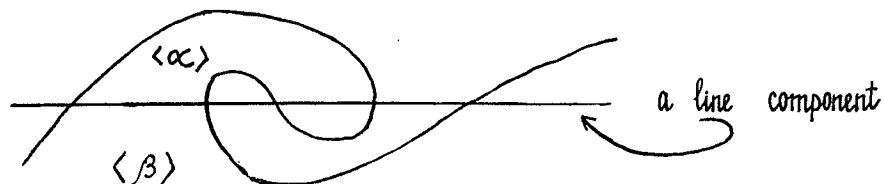Let us apply the criterion (4.J) to curves of degree 6 represented by figures 7-11 (the first four curves are supposed to be reducible)
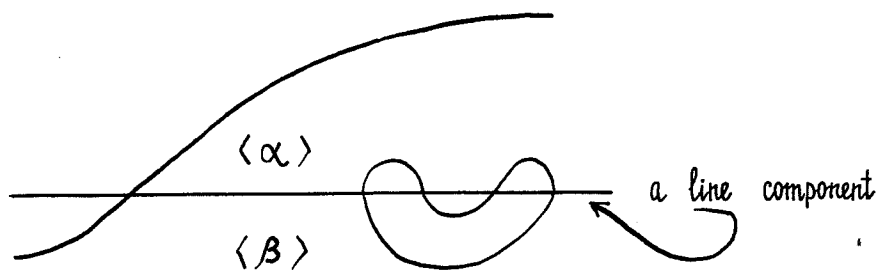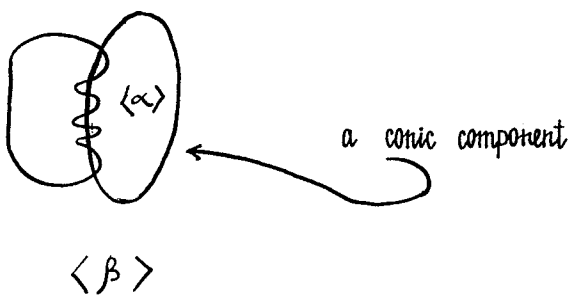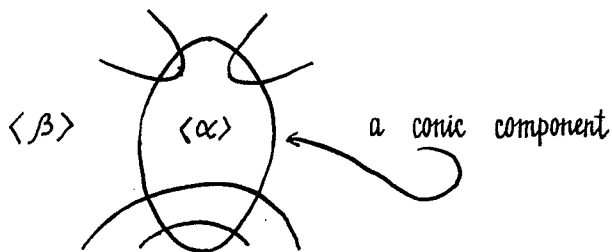
Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.

$\langle \alpha \rangle$

$\langle \beta \rangle$

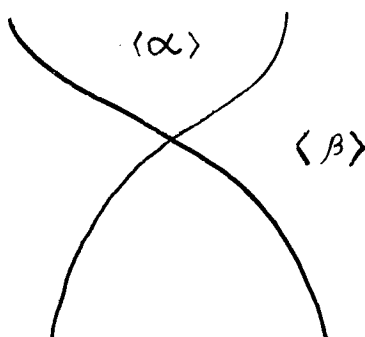Fig. 11.

According to (4.J) Theorem (3.B) is aplicable to all these situations. As in 4.2, the Theorem immediately implies that the appropriate congruences for $\alpha - \beta$ are to be satisfied. The concrete form of congruences may be obtained by computation of $B(q_\Delta!)$ or by known examples (cf. 4.2)

	Assumption	Assertion
fig.7	$\alpha + \beta = 6$	$\alpha - \beta \equiv 6 \mod 8$
	$\alpha + \beta = 5$	$\alpha - \beta \equiv 5 \text{ or } 7 \mod 8$
	$\alpha + \beta = 4$ and A is of type II	$\alpha - \beta \not\equiv 2 \mod 8$
	A is of type I	$\alpha - \beta \equiv 2 \mod 4$
fig.8	$\alpha + \beta = 5$	$\alpha - \beta \equiv 5 \mod 8$
	$\alpha + \beta = 4$	$\alpha - \beta \equiv 4 \text{ or } 6 \mod 8$
	$\alpha + \beta = 3$ and A is of type II	$\alpha - \beta \not\equiv 1 \mod 8$
	A is of type I	$\alpha - \beta \equiv 1 \mod 4$
fig.9	$\alpha + \beta = 3$	$\alpha - \beta \equiv 5 \mod 8$ (i.e. $\alpha=2, \beta=1$)
	$\alpha + \beta = 2$	$\alpha - \beta \equiv 0 \text{ or } 2 \mod 8$
	A is of type I	$\alpha - \beta \equiv 1 \mod 4$

	Assumption	Assertion
fig.10	$\alpha + \beta = 3$ $\alpha + \beta = 2$ $\alpha + \beta = 1$ and A is of type II A is of type I	$\alpha - \beta \equiv 3 \mod 8$ (i.e. $\alpha = 3, \beta = 0$) $\alpha - \beta \equiv 2$ or $4 \mod 8$ (i.e. $\alpha = 2, \beta = 0$) $\alpha - \beta \equiv -1 \mod 8$ (i.e. $\alpha = 1, \beta = 0$) $\alpha - \beta \equiv -1 \mod 4$
fig.11	$\alpha + \beta = 8$ $\alpha + \beta = 7$ $\alpha + \beta = 6$ and A is of type II A is of type I	$\alpha - \beta \equiv 4 \mod 8$ $\alpha - \beta \equiv 3$ or $5 \mod 8$ $\alpha - \beta \not\equiv 0 \mod 8$ $\alpha - \beta \equiv 0 \mod 4$

Table 1.

Purely real part of first four blocks (results not refering to the type of the curve) was originally obtained by G.M.Polotovskij [14] (via a different approach). In [14] he considered curves of degree 6 decomposed into curves being non-singular and transversal each other. One can find there a big stock of situations in which (3.B) works.

The first row of the last block was conjectures by I.V.Itenberg when classifying curves of degree 6 with one non-degenerated double point.

Numerous examples prove necessity of (4.J), see e.g. [14]. In particular, there exist reducible curves of degree 6 shown in fig. 12
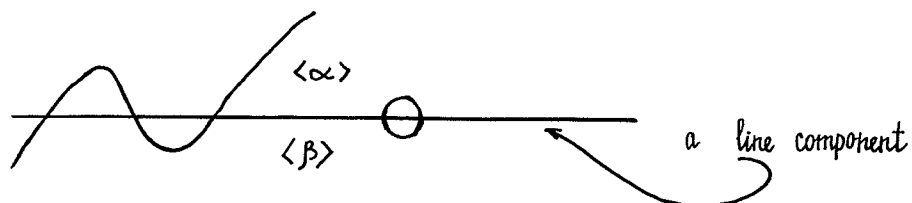
any α, β with $\alpha + \beta \leq 5$

Fig. 12.

4.4. Smoothings of some plane curve singularities.

Here we consider cases when in some (perhaps nonlinear) coordinates x, y in some neighbourhood of the singular point the curve is defined by a real equation.

$$(y - a x^2)(y - b x^2)(y - c x^2) = 0$$

(a, b, c are distinct numbers not necessarily real)

(J_{10} in Arnold's notations; three nonsingular branches ordinarily tangent to each other) or by a real equation

$$(y - a x^2)(y - b x^2)(y - c x^2)(y - d x^2) = 0$$

(a, b, c, d are distinct numbers not necessarily real)

(X_{21} ; four nonsingular branches ordinarily tangent to each other). We have chose namely these singularities because of their applications in constructing curves with presecribed topological properties, see [21] . For N_{16} (five nonsingular branches transversal to each other), which is the other singularity involved there, our congruences could be applied too. We have omitted these applications since, as it was shown by E.J. Shustin [20] , smoothings of N_{16} are essentially affine nonsingular plane curves of degree 5 with 5 different asymptotes . Such curves are considered above in section 4.3. For J_{10} as for N_{16} , the classification is completed (see [22], [20]), for X_{21} it is closed to completion (see [23] , [20]). In these classification achievements, congruences of the sort considered in our paper play important role.

Let us apply criteria (4.C') and (4.E) to smoothings outlined in the figures 13 - 17

$\langle \alpha \rangle$

$\langle \beta \rangle$

y_{10}

Fig. 13.

$\langle \alpha \rangle$

$\langle \beta \rangle$

y_{10}

Fig. 14.

$\langle \alpha \rangle$

$\langle \beta \rangle$

y_{10}

$\left(\begin{array}{l} \text{singular point has two} \\ \text{conjugate imaginary branches} \end{array} \right)$

Fig. 15.

$\langle \alpha \rangle$

$\langle \beta \rangle$

$\langle \gamma \rangle$

X_{21}

Fig. 16.

$\langle \alpha \rangle$

$\langle \beta \rangle$

$\langle \gamma \rangle$

X_{21}
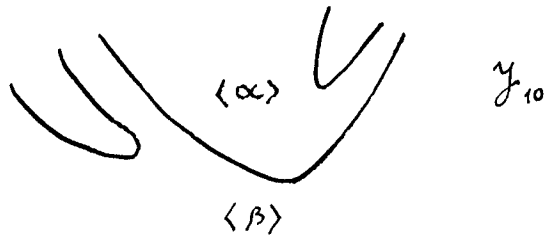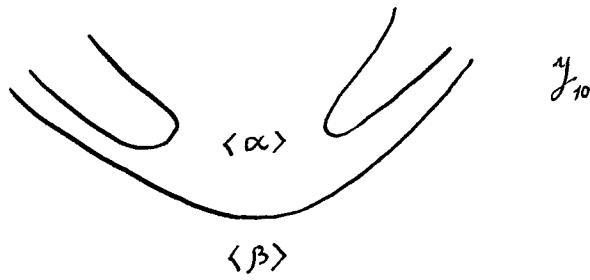
Fig. 17.

According to (4.C') theorem (3.D) is applicable to situations of figu-

res 13, 15-17 and according to (4.E) it is applicable to one of fig. 14. As usually, the concrete form of congruences may be obtained by a computation of $B(q_\Delta)$ or by known examples (see the latters in [22] and [23])
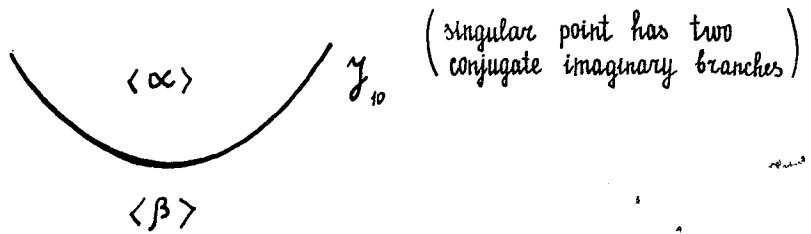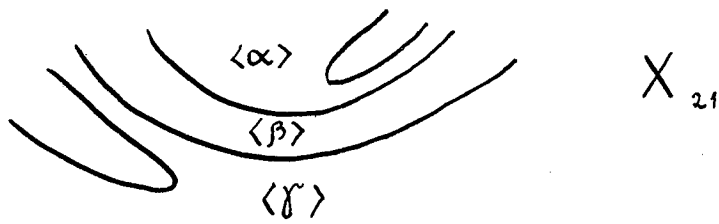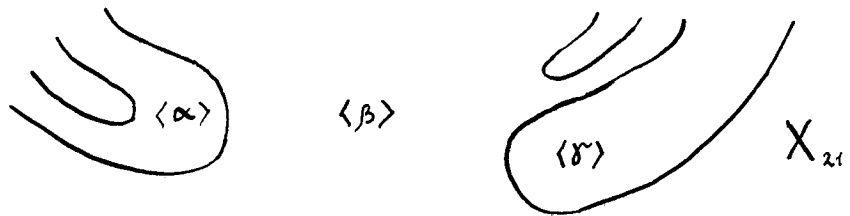
	Assumption	Assertion
fig.13 and fig.15	$\alpha + \beta = 4$ $\alpha + \beta = 3$ $\alpha + \beta = 2$ and the smoothing is of type II the smoothing is of type I	$\alpha \equiv 0 \quad \mathrm{mod}\ 4$ $\alpha = 0$ or 3 $\alpha = 0$ or 2 $\alpha - \beta \equiv 0 \quad \mathrm{mod}\ 4$
fig.14	$\alpha + \beta = 3$ $\alpha + \beta = 2$ $\alpha + \beta = 1$ and the smoothing is of type II the smoothing is of type I	$\alpha = 3$ $\alpha = 2$ $\alpha = 1$ $\alpha - \beta \equiv -1 \quad \mathrm{mod}\ 4$
fig.16	$\alpha + \beta + \gamma = 9$ $\alpha + \beta + \gamma = 8$ $\alpha + \beta + \gamma = 7$ and the smoothing is of type II the smoothing is of type I	$\beta \equiv 1 \quad \mathrm{mod}\ 4$ $\beta \equiv 0$ or $2 \quad \mathrm{mod}\ 4$ $\beta \not\equiv 2 \quad \mathrm{mod}\ 4$ $\alpha - \beta + \gamma \equiv -1 \quad \mathrm{mod}\ 4$
fig.17	$\alpha + \beta + \gamma = 9$ $\alpha + \beta + \gamma = 8$ $\alpha + \beta + \gamma = 7$ and the smoothing is of type II the smoothing is of type I	$\beta \equiv 0 \quad \mathrm{mod}\ 4$ $\beta \equiv -1$ or $1 \quad \mathrm{mod}\ 4$ $\beta \not\equiv 1 \quad \mathrm{mod}\ 4$ $\alpha - \beta + \gamma \equiv 1 \quad \mathrm{mod}\ 8$

In [20] one can find some more isotopy types of smoothing of X_{21} and appropriate congruences which can be derived from our theorems. They can be supplemented by congruences refering to the type of smoothing.

§ 5. PREREQUISITE FOR PROOFS

5.1. Rohlin-Guillou-Marin congruence . (see [7]).

Let X be an oriented smooth closed four-dimensional manifold and let F be its smooth closed two-dimensional submanifold realizing in $H_2(X ; \mathbb{Z}_{/2})$ the class which is the Poincaré dual to the Stiefel-Whitney class $w_2(X)$. Then

$$2 B(q) \equiv \sigma(X) - F \circ F \mod 16 \qquad (28)$$

where q is the Rohlin-Guillou-Marin form of the pair (X, F), $\sigma(X)$ is the signature of X and $F \circ F$ is the natural Euler number of F in X (self-intersection number).

5.2. Informative subspaces.

Let $(V, \circ, q)$ be a nonsingular $\mathbb{Z}_{/4}$-quadratic space. Its subspaces are defined to be quadratic spaces $(\mathcal{U}, \circ|, q|)$, where $\mathcal{U}$ is a vector subspace of V, $q| = q|_{\mathcal{U}}$ and $\circ| = \circ|_{\mathcal{U} \times \mathcal{U}}$ We say that a subspace is informative if $\mathcal{U}$ contains its own orthogonal (with respect to $\circ$) complement $\mathcal{U}^\perp$ and q vanishes on $\mathcal{U}^\perp$. This is conformed to the definition in 2.1: every informative subspace is an informative space (but not vice versa!)

5. A. A nonsingular $\mathbb{Z}_{/4}$-quadratic space is cobordant to any

its <u>informative subspace</u>.

In various equivalent forms this fact is well known. For the proof it is enough to check that the graph $H \subset \mathcal{U} \oplus V$ os the inclusion $\mathcal{U} \hookrightarrow V$ provides (according to the definition of the Witt group, see 2.1) vanishing of $(\mathcal{U}, \circ |, q|) \oplus (V, \circ, -q)$ in the Witt group.

[The definition of an informative subspace may be transformed with evident alterations from nonsingular to informative ambient spaces. An informative space accurs to be cobordant to any its informative subspace .]

5.3. Additivity for B .

Let F be a closed subspace and let $(V, \circ, q)$ be a $\mathbb{Z}_{/4}$-quadratic space with $V = H_1(F; \mathbb{Z}_{/2})$ and $\circ$ = the intersection form. Given a decomposition of F into two compact subsurfaces F_1 and F_2 (perhaps non-connected) with common boundary $\partial = \partial F_1 = \partial F_2$, there naturally arises subspaces $\mathcal{U}_1 = in_* H_1(F_1; \mathbb{Z}_{/2})$,

$\mathcal{U}_2 = in_* H_1(F_2; \mathbb{Z}_{/2})$, $\mathcal{U} = \mathcal{U}_1 + \mathcal{U}_2$ of V. It is evident that $\mathcal{U}^\perp = in_* H_1(\partial; \mathbb{Z}_{/2})$.

So, according to 5.2, if q vanishes on $in_* H_1(\partial;$ $\mathbb{Z}_{/2})$ then $(\mathcal{U}, \circ |, q|)$ is an informative subspace of $(V, \circ, q)$, $(\mathcal{U}_1, \circ |, q|)$ and $(\mathcal{U}_2, \circ |, q|)$ are informative spaces and $B(q) =$
$= B(q|_{\mathcal{U}_1}) + B(q|_{\mathcal{U}_2})$. In fact, q vanishes on $in_* H_1(\partial; \mathbb{Z}_{/2})$ iff $(\mathcal{U}, \circ |, q|)$ is an informative subspace iff $(\mathcal{U}_1, \circ |, q|)$ and $(\mathcal{U}_2, \circ |, q|)$ are informative spaces.

The $\mathbb{Z}_{/4}$-quadratic space $(H_1(F; \mathbb{Z}_{/2}), \circ, q)$ induces (by in_*) $\mathbb{Z}_{/4}$-quadratic spaces $(H_1(F_j; \mathbb{Z}_{/2}), \circ, q_j)$, where $\circ$ = the intersection form and $q_j = q \circ in_*$. These spaces are informative if (and only if) $(\mathcal{U}_1, \circ |, q|)$ and $(\mathcal{U}_2, \circ |, q|)$ are informative spaces and then they have the same associated nonsingular spaces. Thus <u>if</u> q <u>vanishes</u> <u>on</u> $in_* H_1(\partial;$

$\mathbb{Z}/2$) then

$$B(q) = B(q_1) + B(q_2) \qquad (29)$$

5.4. Low-dimensional $\mathbb{Z}/4$-quadratic spaces. (see, e.g. [2]).

Any $\mathbb{Z}/4$-quadratic space can be decomposed in an orthogonal sum of one and two-dimensional quadratic spaces. Besides if $\mathbb{Z}/4$-quadratic space $(V, \circ, q)$ is odd (i.e. there exists $v \in V$ with $v \circ v \neq 0$) then it can be decomposed in an orthogonal sum of one-dimensional spaces only. There are only two different (up to isomorphism) one dimensional spaces:

$$(\mathbb{Z}/2, \circ, q_+) \quad \text{with} \quad a \circ a = 1, q_+(a) = 1 \quad \text{for} \quad a \neq 0 ;$$
$$(\mathbb{Z}/2, \circ, q_-) \quad \text{with} \quad a \circ a = 1, q_-(a) = -1 \quad \text{for} \quad a \neq 0.$$

Both spaces are odd. It is clear that $B(q_+) = 1$ and $B(q_-) = -1$. Two-dimensional spaces are even iff they are indecomposable, we have no need of their precise form and remark only that in this case values of B are 0 and 4. Since a two-dimensional odd space $(V, \circ, q)$ is a sum of two one-dimensional spaces, it has $B(q) = 0$ or ± 2.

§ 6. PROOF OF THE MAIN THEOREMS

6.1. Prototype. Marin's proof for nonsingular curves.

The factor-space $\mathbb{C}P^2/conj$ carries a natural smooth structure (as always when the fixed point set of a smooth involution has (real) codimension 2). It is well known that this manifold is diffeomorphic to S^4. The complex point set $\mathbb{C}A$ of the (real) curve

A is invariant under the complex conjugation. Its image $\mathbb{C}A/\text{conj}$ is a compact two-dimensional submanifold of $\mathbb{C}P^2/\text{conj} = S^4$ with boundary $\mathbb{R}A$. It is clear that $\mathbb{C}A/\text{conj}$ is orientable iff the curve is of type I.

Apply the congruence (28) to $F = \mathbb{C}A/\text{conj} \cup \mathbb{R}P_+^2$. Straightforward calculations show that

$$F \circ F = 2k^2 - 2\chi(\mathbb{R}P_+^2)$$

(see, e.g. [16]) and so (28) turns into

$$\chi(\mathbb{R}P_+^2) - k^2 \equiv B(q) \mod 8 .$$

It remains to calculate $B(q)$.

Now apply 5.3 to $F_1 = \mathbb{R}P_+^2$, $F_2 = \mathbb{C}A/\text{conj}$ and $\partial = \mathbb{R}A$. Some straightforward calculations more show that q vanishes on $in_* H_1(F_1;$ $\mathbb{Z}/2)$ and thus on $in_* H_1(\partial; \mathbb{Z}/2)$ (see [12]). So we get $B(q) = B(q_2)$, see (29).

If A is an M-curve then the genus of F_2 is zero. Consequently, $q_2 = 0$ and $B(q_2) = 0$. So we get (1.A). If A is an $(M-1)$-curve then F_2 is homeomorphic to the projective plane with several holes. So in this case the associated nonsingular space has dimension one and we get $B(q_2) = \pm 1$ (see 5.4) and (1.B). If A is an $(M-2)$-curve of type II then F_2 is homeomorphic to the Klein bottle with several holes. In this case the associated nonsingular space has dimension 2 and it is odd, so that we get $B(q_2) = 0$ or 2 (see 5.4 and (1.C)). If A is of type I, then q_2 is even and $B(q_2) \equiv 0 \mod 4$. This gives (1.D).

6.2. An auxiliary surface and its decomposition.

For every singular point p of the curve A let us fix a suffi-

ciently small ball $\mathcal{D}_p \subset \mathbb{CP}^2$ with center in p . Introduce an auxiliary curve A_ε defined by an equation $a = -\varepsilon(x_0^{2k} + x_1^{2k} + x_2^{2k})$ where a is the polynomial chosen as in 2.4 up to a positive constant factor and ε is a positive number such that for any $t \in (0, \varepsilon]$ the curve defined by the equation $a = -t(x_0^{2k} + x_1^{2k} + x_2^{2k})$ has no singular point and is transversal to $\partial \mathcal{D}_p$ for each p , cf. 2.3.

Let us factorize $\mathbb{CP}^2$ by $conj$ and take in $S^4 = \mathbb{CP}^2/conj$ surface $F = \mathbb{C}A_\varepsilon/conj \cup \mathbb{RP}_\varepsilon^2$ where $\mathbb{RP}_\varepsilon^2 \subset \mathbb{RP}^2$ is defined by the inequality $a \geqslant -\varepsilon(x_0^{2k} + x_1^{2k} + x_2^{2k})$. This auxiliary surface has a corner $\mathbb{R}A_\varepsilon$. The intersection of F with $\mathcal{D}_p^* = \mathcal{D}_p/conj$ is nothing but the surface $\sum_p$ constructed in 2.4. Let us set $\sum = \cup \sum_p$ and $\mathcal{Y} = \cup \mathcal{Y}_p$

We decompose F into three pieces Π, Π' and Π'' where Π' coincides with $\mathbb{RP}_\varepsilon^2$, Π is a union of $\mathcal{Y}$ with a small color C of $\mathbb{R}A_\varepsilon \setminus \cup \operatorname{Int} \mathcal{D}_p^*$ in $\mathbb{C}A_\varepsilon/conj \setminus \cup \operatorname{Int} \mathcal{D}_p^*$ and $\Pi'' = \mathcal{Cl}(F \setminus (\Pi' \cup \Pi))$. Each piece is a compact surface. Boundaries ∂' and ∂'' of Π' and Π'' form together the boundary ∂ of Π . An interior of Π'' is homeomorphic to $(\mathbb{C}A \setminus \mathbb{R}A)/conj$.

The surface Π plays first fiddle and we need to know some its details. Each oval of $\mathbb{R}A_\varepsilon$ begotten by an oval of $\mathbb{R}A$ gives rise to a component of Π homeomorphic to an annulus. Let us remove these components and denote the remainder by Π_Δ . A boundary of Π_Δ consists of real (contained in ∂') and imaginary (contained in ∂'') circles. There exist natural isomorphisms

$$\varphi : H_1(\Pi_\Delta \cup \sum ; \mathbb{Z}/_2) \longrightarrow \tilde{V}_\Delta \qquad (30)$$

$$H_1(\Pi_\Delta ; \mathbb{Z}/_2) \longrightarrow V'_\Delta$$

$$H_1(\partial' \cap \Pi_\Delta ; \mathbb{Z}/_2) \longrightarrow R_\Delta$$

$$H_1(\partial'' \cap \Pi_\Delta ; \mathbb{Z}_{/2}) \longrightarrow B'_\Delta$$

(31)

$$H_1(\Pi_\Delta ; \mathbb{Z}_{/2}) / in_* H_1(\partial' \cap \Pi_\Delta ; \mathbb{Z}_{/2}) \longrightarrow V_\Delta$$

$$H_1(\Pi_\Delta ; \mathbb{Z}_{/2}) / in_* H_1(\partial \Pi_\Delta ; \mathbb{Z}_{/2}) \longrightarrow V_\Delta / B_\Delta$$

($\tilde{V}_\Delta, V'_\Delta, V_\Delta, R_\Delta, B'_\Delta, B_\Delta$ are as in 2.4) such that the diagram

is commutative.

6.3. Computation of the $\mathbb{Z}_{/4}$-quadratic form.

In this section we compute a form $q^*: H_1(\Pi_\Delta \cup \Sigma ; \mathbb{Z}_{/2}) \longrightarrow \mathbb{Z}_{/4}$ induced by inclusion $\Pi_\Delta \cup \Sigma \hookrightarrow F$ from the Rohlin-Guillou-Marin form q of (S^4, F). More precisely we prove that the isomorphism ψ (see (30)) identifies q^* with $\tilde{q}_\Delta$.

Recall that $\tilde{V}_\Delta = H_1(\Gamma_A ; \mathbb{Z}_{/2}) \oplus H_1(\Sigma ; \mathbb{Z}_{/2})$ and remark that ψ^{-1} coincides with a sum of composite homomorphism

$$H_1(\Gamma_A ; \mathbb{Z}_{/2}) \xrightarrow{in_*} H_1(W ; \mathbb{Z}_{/2}) \xrightarrow{in_*^{-1}} H_1(\Pi_\Delta \cup \Sigma ; \mathbb{Z}_{/2})$$

(here Γ_A is as in 2.4 and W is a regular neighbourhood of $\Pi_\Delta \cup \cup \Sigma$ containing Γ_A) and $in_* : H_1(\Sigma ; \mathbb{Z}_{/2}) \longrightarrow H_1(\Pi_\Delta \cup \Sigma ; \mathbb{Z}_{/2})$.

Now it is clear that to prove the coincidence of the quadratic forms
it is sufficient to check that $q^* \circ \varphi^{-1}$ satisfies condition (iii)
from 2.4, which determines $\tilde{q}_\Delta$. Moreover only formula (10)
requires to be verified since the others are immediate consequences
of the definition of the Rohlin-Guillou-Marin form.

In fact (10) is nothing but a special case of the general rule: if
$\xi \in H_1(\mathbb{RP}^2_\varepsilon ; \mathbb{Z}_{/2})$ then

$$q(in^F_* \xi) = \begin{cases} (-1)^k , & \text{if } in^P_*(\xi) \neq 0 \\ \\ 0 , & \text{if } in^P_*(\xi) = 0 \end{cases} \tag{32}$$

(here in^F is the inclusion $\mathbb{RP}^2_\varepsilon \hookrightarrow F$ and in^P is
the inclusion $\mathbb{RP}^2_\varepsilon \hookrightarrow \mathbb{RP}^2$).

In the case where $in^P_*(\xi) = 0$ the equality turns into
$q(in^F_*(\xi)) = 0$ and was proved by Marin [12] (cf. 6.1). It remains
to show that $q(in^F_*(\xi)) = (-1)^k$ if $in^P_*(\xi) \neq 0$. It may be done
as in the previous case using a special membrane. We like to do it
in another way.

Let us suppose that $in^P_* H_1(\mathbb{RP}^2_\varepsilon ; \mathbb{Z}_{/2}) \neq 0$ and denote by
$q_{/y}$ and q_R the forms induced by $in_* : H_1(\mathbb{C}A_\varepsilon /conj ; \mathbb{Z}_{/2}) \longrightarrow$
$\longrightarrow H_1(F; \mathbb{Z}_{/2})$ and $in_* : H_1(\mathbb{RP}^2_\varepsilon ; \mathbb{Z}_{/2}) \longrightarrow H_1(F; \mathbb{Z}_{/2})$ from q.
As it follows from Marin's result there exists $\tau \in \{-1,1\}$ such
that

$$q(in^F_*(\xi)) = \begin{cases} \tau & \text{for any } \xi \text{ with } in^P_*(\xi) \neq 0 \\ \\ 0 & \text{for any } \xi \text{ with } in^P_*(\xi) = 0 \end{cases}$$

Hence, firstly, by 5.3 invariants $B(q_{/y}), B(q_R)$ are well defined and

$$B(q) = B(q_{/y}) + B(q_R) \tag{33}$$

and, secondly,

$$B(q_R) = \tau \mod 8 \tag{34}$$

Furthermore, as in 6.1 applying of (28) we get

$$\chi(\mathbb{R}P^2_\varepsilon) \equiv k^2 + B(q) \mod 8 \tag{35}$$

Now let us introduce $F' = \mathbb{C}A_\varepsilon/\text{conj} \cup \mathbb{R}P^2_{\geqslant\varepsilon}$ where $\mathbb{R}P^2_{\geqslant\varepsilon} = \mathcal{Cl}(\mathbb{R}P^2 \setminus \mathbb{R}P^2_\varepsilon)$. Repeating previous notations and arguments obviously modified we obtain

$$q'_R = 0$$

$$B(q') = B(q'_y) \tag{36}$$

$$\chi(\mathbb{R}P^2_{\geqslant\varepsilon}) \equiv k^2 + B(q') \mod 8 \tag{37}$$

Both forms q_y, q'_y are defined on $H_1(\mathbb{C}A_\varepsilon/\text{conj}; \mathbb{Z}_{/2})$. For $\xi \in H_1(\mathbb{C}A_\varepsilon/\text{conj}; \mathbb{Z}_{/2})$ the difference $q_y(\xi) - q'_y(\xi)$ coincides with the linking number of ξ with $\mathbb{R}P^2$ multiplied by $2 (:\mathbb{Z}_{/2} \to \mathbb{Z}_{/4})$. This linking number is 0 iff $\xi \circ \xi = 0$. So $q_y = -q'_y$ and thus

$$B(q_y) + B(q'_y) = 0 \mod 8 \tag{38}$$

Let us sum (33), (34), (35), (36), (37) and (38). Then taking into account that $\chi(\mathbb{R}P^2_\varepsilon) + \chi(\mathbb{R}P^2_{\geqslant\varepsilon}) = 1$ we get $\tau = 1 - 2k^2 \mod 8$. Since $\tau \in \{1, -1\}$ this implies $\tau = (-1)^k$.

6.4. Proof of the theorem (3.A).

Let $F, \Pi, \Pi'', \Sigma, \mathbb{R}P^2_\varepsilon, \Pi_\Delta, q$ and q^* be as in 6.2 and 6.3. According to 6.3 the $\mathbb{Z}_{/4}$-quadratic space $(H_1(\Pi_\Delta \cup \Sigma; \mathbb{Z}_{/2}), \circ, q^*)$ is isomorphic to $(\widetilde{V}_\Delta, \circ, \widetilde{q}_\Delta)$ and consequently by the hypothesis of the theorem it is informative. So we can apply 5.3 to

the decomposition $F = F_1 \cup F_2$ with $F_1 = \Pi \cup \sum$ and $F_2 = \Pi'' \cup (\mathbb{R}P_\varepsilon^2 \setminus \text{Int} \sum)$. We get

$$B(q) = B(q_1) + B(q_2)$$

where $q_i = q \circ \{ in_* : H_1(F_i ; \mathbb{Z}/_2) \to H_1(F ; \mathbb{Z}/_2)\}$. The q_1 is zero on $in_* H_1(\Pi \setminus \Pi_\Delta ; \mathbb{Z}/_2) \subset H_1(F_1 ; \mathbb{Z}/_2)$ since $\Pi \setminus \Pi_\Delta$ consists of components of Π which are homeomorphic to an annulus and each contains an oval in its boundary. It follows

$$B(q_1) = B(q^*) = B(\tilde{q}_\Delta)$$

The restriction of q_2 to $in_* H_1(\mathbb{R}P_\varepsilon^2 \setminus \text{Int} \sum)$ are defined by (32). Thus

$$B(q_2) = \tilde{b} + B(q'')$$

where $\tilde{b}$ is as in (3.A) and $q'' = q \circ \{in_* : H_1(\Pi'' ; \mathbb{Z}/_2) \to H_1(F ; \mathbb{Z}/_2)\}$ To finish the proof we repeat Marin's arguments reproduced at the end of 6.1. Here we use that the interior of Π'' is homeomorphic to

$(\mathbb{C}A \setminus \mathbb{R}A)/conj$ and that the number $F \circ F$ is determined by the formulae

$$F \circ F = 2k^2 - 2\chi(\mathbb{R}P_\varepsilon^2)$$

$$\chi(\mathbb{R}P_\varepsilon^2) = \chi(\mathbb{R}P_+^2)$$

6.5. Proof of the theorem (3.B).

It is similar to that of (3.A). They differ in the choice of decomposition $F = F_1 \cup F_2$ only. Here we take $F_1 = \Pi$ and $F_2 = \Pi' \cup \Pi''$ By (32) the q is zero on $in_* H_1(\partial' ; \mathbb{Z}/_2)$ and by the hypothesis of the theorem it is zero on $in_* H_1(\partial'' ; \mathbb{Z}/_2)$. So applying 5.3 we get

$$B(q) = B(q_1) + B(q_2)$$

where $q_i = q \circ \{ in_* : H_1(F_i ; \mathbb{Z}/2) \longrightarrow H_1(F ; \mathbb{Z}/2) \}$. Isomorphisms given in 6.3 show that the q_1 factorized from $H_1(F_1 ; \mathbb{Z}/2)$ to $H_1(F_1 ; \mathbb{Z}/2)/in_* H_1(\partial F_1 ; \mathbb{Z}/2)$ is isomorphic to q_Δ . At last $B(q_2) = b + B(q'')$ where b is as in (3.B) and q'' is as an 6.4. So we get

$$B(q) = B(q_\Delta) + b + B(q'')$$

and the end of the proof is fairly the same as in 6.4.

6.6. Proof of (4.B), (4.C) and (4.F).

First we are going to check that under assumptions of (4.B) or (4.C) the q_Δ vanishes on B_Δ . Since there is an isomorphism (31) between B'_Δ and $H_1(\partial'' \cap \Pi_\Delta ; \mathbb{Z}_2)$ transfering $\tilde{q}_\Delta$ to $q \circ in_*$, where q is the Rohlin-Guillou-Marin form of (S^4, F) , it is sufficient to check the vanishing of q on $in_* H_1(\partial'' \cap \Pi_\Delta ; \mathbb{Z}/2)$

Under assumptions of (4.B) each component of Π'' contains no more than one component of $\partial'' \cap \Pi_\Delta$. Consequently the homology class realized in $H_1(F ; \mathbb{Z}/2)$ by that component of $\partial'' \cap \Pi_\Delta$ is equal to the sum of the others boundary components of Π'' and so this class lies in $in_* H_1(\partial' ; \mathbb{Z}/2)$, where q vanishes.

Now let assumptions of (4.C) be fulfilled. Then for each component of Π'' any homology class realized in $H_1(F ; \mathbb{Z}/2)$ by boundary components of that component of Π'' reduces to the sum of an element of $in_* H_1(\partial' ; \mathbb{Z}/2)$ and elements realized by circle components of $\mathcal{Y} \cap \cap (\cup \partial \mathcal{D}_p^*)$ (ones begotten by imaginary branches of the curve A at its singular points). Consider one such component γ of $\mathcal{Y} \cap \cap \partial \mathcal{D}_p^*$, the class $[\gamma] \in H_1(F ; \mathbb{Z}/2)$ realized by γ and the boundary δ of $F \cap \mathcal{D}_p^*$. Then

$$q(\gamma) = 2 \, lk(\gamma, \delta \backslash \gamma) \mod 4 ,$$

where lk is linking number in $\partial \mathcal{D}_p^* = S^3$ and hence

$$q(\gamma) = 2 \, lk \, (\gamma^*, \, pr^{-1}(\delta \setminus \gamma)) \mod 4$$

where $pr : \mathcal{D}_p \longrightarrow \mathcal{D}_p^*$ is the natural projection and γ^* is any component of $pr^{-1}(\gamma)$. The latter linking number is equal to the intersection number of the non-real branch β of A at p which begots γ^* with the union of all branches different from β and $conj \, \beta$. Thus $q(\gamma) = 0$ and we have finished the proof.

To prove that under assumptions of (4.F) the q_Δ vanishes on B_Δ it is sufficient to note that: boundary components of $\mathcal{Y}_\varepsilon$ give generators of B_Δ; for elements of $in_* H_1(\partial \Sigma_\varepsilon; \mathbb{Z}_{/2})$ the values of the Rohlin-Guillou-Marin form coincide with the linking numbers involved in (4.D); the Rohlin-Guillou-Marin form vanishes on $in_* H_1(\mathcal{D}_+; \mathbb{Z}_{/2})$ cf. 6.1.

§ 7. ANOTHER APPROACH

7.1. Prototype: Rohlin's proof.

Let C be an antiholomorphic involution of a closed quasicomplex manifold $\mathcal{Y}$ of complex dimension $2n$. Suppose that

$$\dim H_*(Y; \mathbb{Z}_{/2}) = \dim H_*(\mathcal{Y}; \mathbb{Z}_{/2}) \tag{39}$$

where Y is the fixed point set of C.

By the Atiyah-Singer-Hirzebruch formula, it is hold (and this result does not use (39)) the relation

$$\chi(Y) = \sigma(\mathcal{Y}) - 2 \, \sigma_{(-1)^{n+1}} \tag{40}$$

where σ_λ ($\lambda = 1$ or -1) is the signature of the bilinear form b_λ obtained by restriction of the intersection form of Y to $H_\lambda = \text{Ker} \{ 1 + \lambda c_* : H_{2n}(Y) \longrightarrow H_{2n}(Y) \}$. By the Smith theory arguments the assumption (39) implies that forms b_λ are unimodular. The form $b_{(-1)^{n+1}}$ is even and, since the signature of any even unimodular form is divisible by 8, from (40) it follows

$$\chi(Y) \equiv \sigma(Y) \bmod 16 \tag{41}$$

Turning to a real nonsingular plane projective curve A of even degree, one should associate with it the 2-sheeted branched covering space Y of $\mathbb{C}P^2$ with branch locus $\mathbb{C}A$ and the involution c which covers conj and has Y lying over $\mathbb{R}P_-^2$. Such Y, c exist and are unique. The condition (39) holds iff A is an M-curve. The congruence (41) applied to these Y, c reduces to the Gudkov-Rohlin congruence $(\chi(Y) = 2 - 2 \chi(\mathbb{R}P_+^2), \sigma(Y) = 2 - 2k^2)$.

7.2. The Atiyah-Singer-Hirzebruch formula for manifolds with boundary.

Let C be an antiholomorphic involution of a compact quasicomplex manifold Y of complex dimension $2n$ with a boundary ∂Y Let Y denote, as before, the fixed point set of c . The normal bundle of ∂Y in Y is just the oriented one-dimensional bundle. Thus the complex structure in the tangent bundle TY induces the complex structure in the direct sum of $T(\partial Y)$ with the trivialized one-dimensional bundle. Let us denote the complex structure introduced by θ .

(7.A) The number δ satisfying the formula

$$\chi(Y) = \sigma(Y) - 2\sigma_{(-1)^{n+1}} + \delta \tag{42}$$
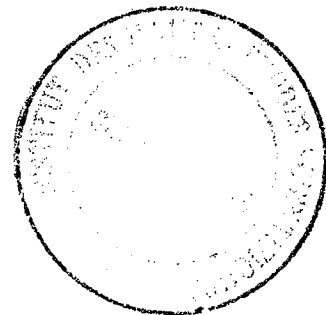
<u>is an invariant of the triple</u> $(\partial Y, c|_{\partial y}, \theta)$.

We have to check that the difference $\delta = \chi(Y) - \sigma(Y) + 2\sigma_{(-1)^{\nu} \nu + 1}$ depends only on $(\partial Y, c|_{\partial y}, \theta)$. Given another pair with the boundary triple isomorphic to $(\partial Y, c|_{\partial y}, \theta)$, let us glue it to (Y, c) along the boundary. By additivity of σ and χ, (40), applied to the closed manifold obtained by the gluing shows that the numbers δ given by the halves coincide.

7.3. $\mathbb{Q}/2\mathbb{Z}$-quadratic spaces.

By $\mathbb{Q}/2\mathbb{Z}$-quadratic space it is called a triple $(V, \circ, q)$ consisting of a finite group V, a symmetric bilinear form $V \times V \longrightarrow \mathbb{Q}/\mathbb{Z}: (x, y) \longmapsto x \circ y$ and a function $q: V \longrightarrow \mathbb{Q}/2\mathbb{Z}$ which is quadratic with respect to that bilinear form, i.e.

$$q(x + y) = q(x) + q(y) + 2x \circ y$$

for $x, y \in V$, where $2: \mathbb{Q}/\mathbb{Z} \longrightarrow \mathbb{Q}/2\mathbb{Z}$ is the canonical isomorphism. The canonical embedding $1/2: \mathbb{Z}/4\mathbb{Z} \longrightarrow \mathbb{Q}/2\mathbb{Z}$ allows to consider $\mathbb{Z}/4$-quadratic spaces as $\mathbb{Q}/2\mathbb{Z}$-quadratic spaces.

A $\mathbb{Q}/2\mathbb{Z}$-quadratic space $S = (V, \circ, q)$ is said to be nonsingular if its bilinear form $\circ$ is nonsingular, i.e. its radical

$$R(S) = \{x \in V \mid \forall y \in V \quad x \circ y = 0\}$$

is the zero-subspace.

Any $\mathbb{Q}/2\mathbb{Z}$-quadratic space can be obtained in the following way. Let L be a finitely generated free abelian group endowed with a non-degenerate even symmetric bilinear form $L \times L \longrightarrow \mathbb{Z}: (x, y) \longmapsto \langle x, y \rangle$. This form has a unique extension to $L \otimes \mathbb{Q}$. By the correlation isomorphism $\phi: L \otimes \mathbb{Q} \longrightarrow L^* = \mathrm{Hom}(L, \mathbb{Q})$ defined by

$$\phi(x) = \langle \ , x \rangle$$

the group $L^{\vee} = \mathrm{Hom}(L, \mathbb{Z})$ can be considered as an intermediate group:

$$L \subset L^{\vee} \subset L \otimes \mathbb{Q}$$

Then we obtain a $\mathbb{Q}/2\mathbb{Z}$-quadratic space $(V, \circ, q)$ taking

$$V = L^{\vee}/L \, ,$$

$$\bar{x} \circ \bar{y} = \langle x, y \rangle \mod \mathbb{Z} \quad \text{for} \quad x \in \bar{x} \in V, \, y \in \bar{y} \in V$$

$$q(\bar{x}) = \langle x, x \rangle \mod 2\mathbb{Z} \quad \text{for} \quad x \in \bar{x} \in V$$

It is easily checked that $\circ$ and q are well defined.

If V has no element of order 2 then evident relations

$$q(v) \mod \mathbb{Z} = v \circ v, \quad \tau^2 q(v) = 0 \qquad \text{if } \tau \text{ is order of } v \quad (43)$$

allow to determine q by $\circ$.

The van der Blij formula [3] states that

$$e^{i\frac{\pi\sigma}{4}} = (\text{card } V)^{-\frac{1}{2}} \sum_{v \in V} e^{2\pi i q} \qquad \qquad (44)$$

where σ is the signature of the form $\langle, \rangle$

7.4. The case where 2-torsion in homology of boundary vanishes.

Let c be an antiholomorphic involution of a compact quasicomplex manifold y of complex dimension $2n$. Let Y denote the fixed point set of c. Endow the group $L = H_{2n}(y)/\text{Tors} + i n_* H_{2n}(\partial y)$ with the form $\langle, \rangle$, induced by the intersection form of y. Consider the $\mathbb{Q}/2\mathbb{Z}$-quadratic space $(V, \circ, q)$ associated with $(L, \langle, \rangle)$. Then V is nothing but $\text{Tors } H_{2n-1}(\partial y) \cap \partial H_{2n}(y, \partial y)$ and $\circ$ is the linking form. Thus we obtain (see 7.3)

(7.B) If $H_{2n-1}(\partial y)$ has no element of order 2 the space $(V, \circ, q)$ is determined by ∂y, namely,

$$V = \text{Tors } H_{2n-1}(\partial Y) \cap \partial H_{2n}(Y, \partial Y)$$

$\circ$ is linking form ,

q is defined via $\circ$ by (43)

Suppose now that $\dim H_*(Y ; \mathbb{Z}_{/2}) = \dim H_*(y ; \mathbb{Z}_{/2})$. Then by the Smith theory $(L, \langle \ , \rangle)$ is an orthogonal sum of subspaces $(L_+, \langle \ , \rangle)$ and $(L_-, \langle \ , \rangle)$, where

$$L_+ = \text{Ker } \{1 - c_* : L \to L\} ,$$

$$L_- = \text{Ker } \{1 + c_* : L \to L\}$$

This immediately implies

(7.C) If $H_{2n-1}(\partial Y)$ has no element of order 2 and

$$\dim H_*(Y ; \mathbb{Z}_{/2}) = \dim H_*(y; \mathbb{Z}_{/2})$$

then the $\mathbb{Q}/2\mathbb{Z}$-quadratic space $(V_{(-1)^n}, \circ, q)$ associated with $(L_{(-1)^n}, \langle \ , \rangle)$ is determined by $(\partial Y, c|_{\partial y})$, namely,

$$V_{(-1)^n} = \text{Tors } H_{2n-1}(\partial Y) \cap \partial H_{2n}(Y, \partial Y) \cap \text{Ker } (1 + (-1)^{n+1} c_*) ,$$

$\circ$ is the linking form ,

q is defined via $\circ$ by (43)

By (7.C), the Atiyah-Singer-Hirzebruch and the van der Blij formulae imply

(7.D) If $H_{2n-1}(\partial Y)$ has no element of order 2 and

$$\dim H_*(Y ; \mathbb{Z}_{/2}) = \dim H_*(y ; \mathbb{Z}_{/2})$$

then

$$\chi(Y) \equiv \sigma(Y) + \delta - 2\beta \mod 16$$

where $\beta \mod 8$ is defined by

$$e^{i\frac{\pi\beta}{4}} = (card\ V_-)^{-\frac{1}{2}} \sum_{v \in V_-} e^{2\pi i q(v)}$$

with V_-, q from (7.C).

7.5. Application.

Let A be a real plane projective curve of degree $m = 2k$ without non-real singular points and let for every singular point in some (perhaps nonlinear) coordinated x, y in some neighbourhood of the point the curve is defined by an equation

$$x^3 + y^5 = 0$$

(E_8 in Arnold's notations) or by an equation

$$(y - ax^2)(y - bx^2)(y - cx^2) = 0 \qquad (a, b, c \text{ are distinct real}$$
$$\text{numbers)}$$

($\mathcal{Y}_{10}$) . Let $\mathbb{R}P_+^2$ be the half of $\mathbb{R}P^2$ which is not contractible to a point in $\mathbb{R}P^2$ (and, of course, is bounded by $\mathbb{R}A$).

(7.E) If A is an M-curve then

$$\chi(\mathbb{R}P_+^2) \equiv 1 - k^2 - 4e + 3j \mod 8$$

where e is the number of points of type E_8 and j is the number of points of type $\mathcal{Y}_{10}$.

To prove (7.E) it is sufficient to apply (7.D) to the case where $\mathcal{Y}$ is obtained by removal of neighbourhoods of singular points from the two-sheeted cover of $\mathbb{C}P^2$ with branch locus $\mathbb{C}A$. The $\mathbb{C}$

should be choosen to cover conj and to have $Y = Fix\ c$ lying over $\mathbb{R}P_+^2$. Then $H_{2n-1}(\partial Y)$ has no torsion and

$$\dim H_*(Y; \mathbb{Z}_{/2}) = \dim H_*(\mathcal{Y}; \mathbb{Z}_{/2}) \ ,$$

$$\chi(Y) = 2\,\chi(\mathbb{R}P_+^2) - e + j \ ,$$

$$\sigma(\mathcal{Y}) = 2 - 2k^2 - 8e - 8j \ ,$$

$$\delta = -(e + j) \ .$$

The last equality is a special case of the general rule: any quasihomogeneous singular point makes a contribution -1 to δ . This rule is a straightforward consequence of the definition of δ .

REFERENCES

1. Arnold V.I. On the arrangement of the ovals of real plane curves, involutions of 4-simensional smooth manifolds and the arithmetic of integral quadratic forms.- Funkc.Anal i Priloz̆, 5, 1971, N 3, 1-9.

2. Barge J., Lannes J., Latour Fr., Vogel P. Λ -sphéres.- Ann.Sci. Ecole Norm.Super. (4) 7, 1974, 463-505 (1975).

3. van der Blij. An invariant of quadratic forms $\text{mod }8$.- Indag. Math., 21, 1959, 291-293.

4. Fiedler T. Now congruences in topology of real plane algebraic curves.- Dokl.Akad.Nauk SSSR, 270, 1983, N 1, 56-58.

5. Gudkov D.A., Utkin G.A. Topology of curves of 6 order and surfaces of 4 order,-Uchen.Zap.Gor'kov.Univ.,87, 1969.

6. Gudkov D.A., Krachnov A.D. On the periodicity of the Euler characteristic of real algebraic $(M-1)$-manifolds.- Funkc.Anal. i Prilož., 7, 1973, N 2, 15-19.

7. Guillou L., Marin A. Une extension d'un théorem de Rohlin sur la signature.- Compt.Rendus, 285, 1977, N 3, 95-97.

8. Kharlamov V.M. New congruences for the Euler characteristic of real algebraic manifolds.- Funkc.Anal. i Priloz., 7, 1973, N 2, 74-78.

9. Kharlamov V.M., Viro O.Ya. Congruences for real algebraic curves with singularities.- Uspehi Mat.Nauk, 35, 1980, N 4, 154-155.

10. Khovanskij A.G. Fewnomials and Pfaff manifolds.- Proc.Int.Cong. Math 16-24 1983 Warsawa, 549-564, 1984.

11. Korchagin A.B. Isotopic classification of plane seventh degree curves with the only singular point Z_{15}, this volume.

12. Marin A. Quelques remarques sur les courber algébriques planes reelles.- Publ.Math.Univ.Paris VII , 9, 1980, 51-68.

13. Nikulin V.V. Involutions of integral quadratic forms and their applications to real algebraic geometry.- Izv.Akad.Nauk SSSR, ser. matem., 47, 1983, N 1, 109-188.

14. Polotovskij G.M. Cataloque of M-splitting curves of 6 order.- Dokl.Akad.Nauk SSSR, 236, 1977, N 3, 548-551; see also Complete classification of M-splitting curves of 6 order in $\mathbb{R}P^2$.- paper placed in VINITI, 20.04.78, N 349-78DEP.

15. Polotovskij G.M. On the classification of the non-singular curves of degree8.- this volume.

16. Rohlin V.A. Proof of Gudkov's conjecture.- Funkc.Anal. i Priloz. 6, 1972, N 2, 62-64.

17. Rohlin V.A. Congruences modulo 16 in Hilbert's sixteenth problem. - Funkc.Anal. i Priloz, 6, 1972, N 4, 58-64.

18. Rohlin V.A. Congruences modulo 16 in Hilbert's sixteenth problem, II .-

Funkc.Anal. i Priloz., 7, 1973, N 2, 91-92.

19. Rohlin V.A. Complex topological characteristics of real algebraic curves.- Uspehi Mat.Nauk, 33, 1978, N 5, 77-89.

20. Shustin E.I. The Hilbert-Rohn method and smoothings of singular points of real algebraic curves.- Dokl.Akad.Nauk SSSR, 281, 1985, N 1, 33-36.

21. Viro O.Ya. Progress in topology of real algebraic manifolds over the last 6 years.- Uspehi Mat.Nauk, 41, 1986, N 3, 45-67.

22. Viro O.Ya. Gluing of plane real algebraic curves and constructions of curves of degrees 6 and 7.- Lect.N.Math., 1060, 1984, 187-200.

23. Viro O.Ya. Gluing of algebraic hypersurfaces smoothings of singularities and constructions of curves.- Proc.Leningrad Int.Topological Conf., 1983, 149-197.

24. Wilson G. Hilbert's sixteenth problem.- Topology, 17, 1978, 53-73.