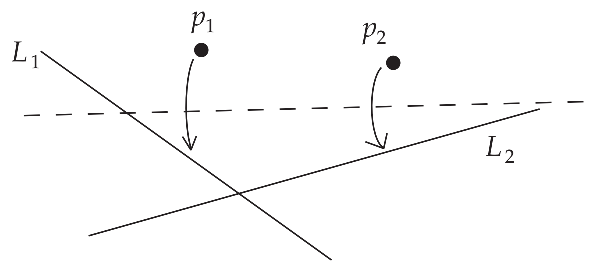
HANDOUT

A More Complicated Fold

The origami angle trisection method is able to do what it does by using a rather complex origami move:

> Given two points p_1 and p_2 and two lines L_1 and L_2, we can make a crease that simultaneously places p_1 onto L_1 and p_2 onto L_2.

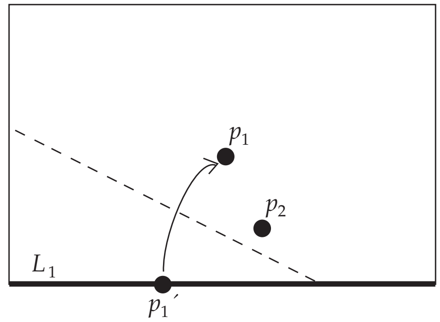


Question 1: Will this operation always be possible to do, no matter what the choice of the points and lines are?

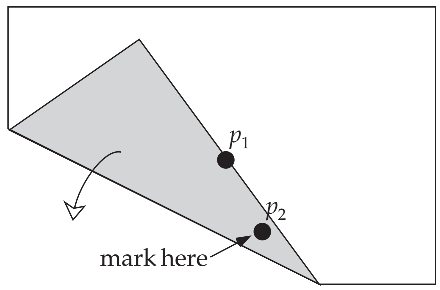
Question 2: Remember that when we fold a point p to a line L over and over again, we can interpret the creases as being tangent to a parabola with focus p and directrix L. What does this tell us about this more complex folding operation? How can we interpret it geometrically? Draw a picture of this.

Activity: Let's explore what this operation is doing in a different way. Take a sheet of paper and mark a point p_1 (somewhere near the center is usually best) and let the bottom edge be the line L_1.

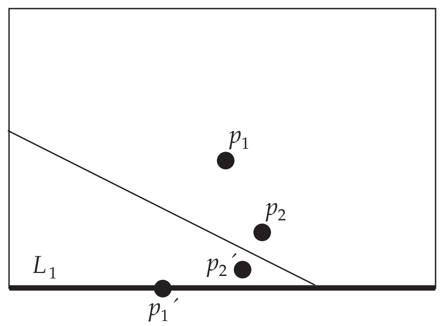
Pick a second point p_2 to be anywhere else on the paper. Our objective is to see where p_2 goes as we fold p_1 onto L_1 over and over again.



So pick a spot on L_1 (call it p_1') and fold it up to p_1. Using a marker or pen, draw a point where the folded part of the paper touches p_2. (If no other parts of the paper touch p_2, try a different choice of p_1'.) Then unfold. You should see a dot (which we could call p_2') that represents where p_2 went as we make the fold.



Now choose a different p_1' and do this over and over again. Make enough p_2' points so that you can connect the dots and see what kind of curve you get.



Question 3: What does this curve look like? Look at other people's work in the class. Do their curves look like yours? Do you know what kind of equation would generate such a curve?

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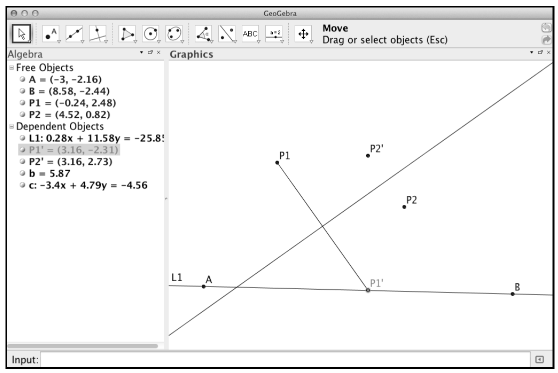
Simulating This Curve with Software

We're still considering this unusual origami maneuver:

> Given two points p_1 and p_2 and two lines L_1 and L_2, we can make a crease that simultaneously places p_1 onto L_1 and p_2 onto L_2.

So that you don't have to keep folding paper over and over again, let's model our folding activity using geometry software, like Geogebra. This will allow us to look at many examples of the curve this operation generates and do so very quickly.



Here's how to set it up:

(1) Make the line L_1 and the point p_1.

(2) Make a point p'_1 on L_1 and construct a line segment from p_1 to p'_1.

(3) Construct the perpendicular bisector of $\overline{p_1 p'_1}$. This makes the crease line.

(4) Now make a new point, p_2.

(5) Reflect the point p_2 about the crease line made in step (3). In Geogebra, this is done using the **Reflect Object about Line** tool. The new point should be labeled p'_2.

Then when you move p'_1 back and forth along L_1, the software will trace out how p'_2 changes. You can either draw this curve by turning on the **Trace** of p'_2 (CTRL-click or right-click on p'_2 to turn this on in Geogebra) or use a **Locus** tool to plot the locus of p'_2 as p_1 changes.

Activity: Move p_2 to different places on the screen and see how the curve changes. How many different basic shapes can this curve take on? Describe them in words.

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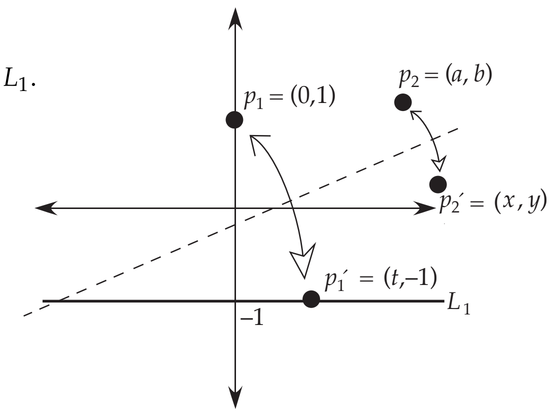
What Kind of Curve Is It?

To see what type of curve this operation is giving us, make a model of the fold.

Let $p_1 = (0, 1)$.
Let L_1 be the line $y = -1$.
We'll fold p_1 to $p_1' = (t, -1)$ on L_1.
Let $p_2 = (a, b)$ be fixed.
Then, we want to find the coordinates of $p_2' = (x, y)$, the image of p_2 under the folding. This will give us an equation in terms of x and y that should describe the curve that we got in our folding activity.



Instructions: Find the equation of the crease line that we get when folding p_1 onto p_1'. Use this and the geometry of the fold to get equations involving x and y. Combine these to get a single equation in terms of x and y (with the constants a and b in it as well, but no t variables). What kind of equation is this?