3126. A Prime Number Conjecture
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we have

$$a = \tfrac{1}{2}\{p - c - \sqrt{(-3c^2 + 2cp + p^2 - 4q)}\},$$

$$b = \tfrac{1}{2}\{p - c + \sqrt{(-3c^2 + 2cp + p^2 - 4q)}\}.$$

For these values to be real, it is necessary and sufficient that

$$3c^2 - 2cp - p^2 + 4q \leq 0,$$

so that $D \geq 0$ *and*

$$c \in [\tfrac{1}{3}(p - 2\sqrt{D}), \tfrac{1}{3}(p + 2\sqrt{D})]. \tag{iv}$$

Noting that a, b, c appear symmetrically in (iii), we deduce that

$$a, b, c \in [\tfrac{1}{3}(p - 2\sqrt{D}), \tfrac{1}{3}(p + 2\sqrt{D})]. \tag{v}$$

The cubic equation with roots a, b, c has the form

$$t^3 - pt^2 + qt - abc = 0.$$

The roots of the equation

$$\frac{d}{dt}(t^3 - pt^2 + qt - abc) \equiv 3t^2 - 2pt + q = 0$$

are

$$t_1 = \tfrac{1}{3}(p - \sqrt{D}), \qquad t_2 = \tfrac{1}{3}(p + \sqrt{D}).$$

From Rolle's theorem, the inequalities $a < b < c$, and the relation (v) we obtain the required inequalities (ii).

This method may also be applied to more general cases.

Professor R. L. Goodstein, in his book *The uniform calculus and its applications* (Oxford, 1948, p. 356) has considered the system (i) when $p = 2$, $q = 1$. Goodstein's procedure is more complicated than that used here.

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3126. A prime number conjecture

Let P_n denote the nth prime, and let a number N be said to be of type n if N is expressible as the sum or difference of two relatively prime numbers A_1, A_2 such that

(1) P_n is the largest prime which divides A_1 or A_2

(2) every prime P_r, $1 \leq r \leq n$, is a factor of the product $A_1 \,.\, A_2$.

I conjecture that every prime is of type n for some n. The following table exhibits some numerical evidence which supports the conjecture. (The number 1 is counted as prime.)

Table giving types of Primes up to type 5.

Prime Numbers.	$P_2 = 3$	$P_3 = 5$	$P_4 = 7$	$P_5 = 11$
3	$1 + 2$			
5	$2 + 3$			
7	$2^2 + 3$	$3.5 - 2^3$		
11	$2^3 + 3$	$3.5 - 2^2$	$3.7 - 2.5$	
13	$2^2 + 3^2$	$3.5 - 2$	$2^2.7 - 3.5$	$5.11 - 2.3.7.$
17	$2^3 + 3^2$	$2 + 3.5$	$5.7 - 2.3^2$	$7.11 - 2^2.3.5$
19	$2^4 + 3$	$2^2 + 3.5$	$2^3.5 - 3.7$	$2.3.7^2 - 5^2.11$
23	$3^3 - 2^2$	$2^3 + 3.5$	$5.7 - 2^2.3$	$2.3^2.11 - 5^2.7$
29	$3^3 + 2$	$3^2.5 - 2^4$	$5.7 - 2.3$	$2^2.3.7 - 5.11$
31	$2^2 + 3^3$	$2^4 + 3.5$	$2.5 + 3.7$	$2.3.11 - 5.7$
37	$2^2.3^2 + 1$	$3^2.5 - 2^3$	$2.3.5 + 7$	$2.5.7. -3.11$
41	$2^5 + 3^2$	$3^2.5 - 2^2$	$2^2.5 + 3.7$	$3.5^2.7 - 2^2.11^2$
43	$2^4 + 3^3$	$3^2.5 - 2$	$3.5 + 2^2.7$	$2^3.3.5 - 7.11$
47	$2^4.3 - 1$	$2 + 3^2.5$	$2^2.3. + 5.7$	$7.11 - 2.3.5$

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3127. On Implication

$p, q, \ldots$ are propositions, and $\varphi(p, q)$ is a proposition about p and q, order being significant, i.e., $\varphi(p, q)$ is not necessarily the same as $\varphi(q, p)$. As p and q together give four true-false arrangements (TT, TF, FT, FF), and if we identify φ by *its* set of truth values for these four arrangements, then there are 16 possible functions φ. One of these is the implication

$$\varphi(p, q) = (p \Rightarrow q)$$

for which the accepted truth table is

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The purpose of this note is to show that no other table is possible, despite the misgivings suffered by some people regarding the last two entries. For convenience we shall regard the arrangement of the p and q columns as fixed and write the entries in the third