Therefore (from Cor. II. after 3 of this) the side KH will be greater than the side KL ; the side KD greater than the side KH ; and so always proceeding towards the points X.

It holds thirdly that the four angles together of the quadrilateral $KLHK$ will be greater than the four angles together of the quadrilateral $KHDK$: for this in like case has already been demonstrated in XXIV of this.

It holds fourthly that the same is valid likewise of the quadrilateral $KHDK$ in relation to the quadrilateral $KDPK$; and so on always, proceeding to quadrilaterals more remote from this point A.

Since therefore are present (as in XXV of this) as many quadrilaterals described in the aforesaid mode, as there are, except the first LK, perpendiculars let fall from points of AX to the straight AB, it will hold uniformly (if we assume nine perpendiculars of this sort let fall, besides the first) the sum of all the angles which are comprehended by these nine quadrilaterals will exceed 35 right angles ; and therefore the four angles together of the first quadrilateral $KLHK$, which indeed in this regard has been shown the greatest of all, will fall short of four right angles by less than the ninth part of one right angle. Wherefore, these quadrilaterals being multiplied beyond any assignable finite number, proceeding always toward the parts of the points X, it holds in the same way (as in the same already recited theorem) that the four angles together of this stable quadrilateral $KHLK$ will fall short of four right angles less than any assignable little portion of one right angle.

Therefore these four angles together will be either equal to four right angles, or greater.

But then (from XVI of this) is established the hypothesis either of right angle or of obtuse angle ; and therefore (from V and VI of this) is destroyed the hypothesis of acute angle.

So then it holds, that there will be no place for the hypothesis of acute angle, if the straight AX drawn under however small angle from the point A of AB must at length meet (anyhow at an infinite distance) any perpendicular BX, which is supposed erected at any distance from this point A upon this secant AB.

Quod erat etc.

SOME DIVISIBILITY TESTS.

By WM. E. HEAL, Member of the London Mathematical Society, Marion, Indiana.

In the *Educational Times* for March, 1897, Professor Sylvester proposed the following problem : "If the digits r in number of any integer N read from left to right be multiplied repeatedly by the first r terms of the recurring series

$1, 4, 3, -1, -4, -3$; $\dot{1}, \dot{4}, \dot{3}, -\dot{1}, -\dot{4}, -\dot{3}$, show that, if the sum of these products be divisible by 13, so N will be, and not otherwise." The reason for the rule is apparent when we notice that $1, 4, 3, -1, -4, -3$ are the remainders in reverse order of 10^1, 10^2, 10^3, 10^4, 10^5, 10^6 mod. 13 ; or what is the same thing in the development of $\frac{1}{13}$ as a circulating decimal.

Since we may prefix any number of ciphers to any number, it is clear that we may start with any number of the series only being careful to preserve the cyclical order. For example, we might equally as well write the series $3, -1, -4, -3, 1, 4$.

Example. 11140640173 is divisible by 13 because $1(1)+4(1)+3(1)-1(4)-4(0)-3(6)+1(4)+4(0)+3(1)-1(7)-4(3)=-26=-2(13)$.

728 is divisible by (13) because $3(7)-1(2)-4(8)=-13$.

The reason for the rule suggests its extension to any number whatever.

Thus $\frac{1}{3}$ developed in a circulating decimal gives the constant remainder 1 and we have the well known rule that a number is divisible by 3 if the sum of its digits is so. $\frac{1}{7}$ developed in a circulating decimal gives the series $2, 3, 1, -2, -3, -1$. Thus 6028620892 is divisible by 7 because $2(6)+3(0)+1(2)-2(8)-3(6)-1(2)+2(0)+3(8)+1(9)-2(2)=7$.

For 11 the remainders are $1, -1$, and we have the known rule for divisibility by 11. For 13 the rule is as stated by Sylvester. For 17 we find the series $1, -5, 8, -6, -4, 3, 2, 7, -1, 5, -8, 6, 4, -3, -2, -7$. Thus 442 is divisible by 17 because $3(4)+2(4)+7(2)=34=2(17)$.

For 19 we have the series $1, 2, 4, 8, -3, -6, 7, -5, 9, -1, -2, -4, -8, 3, 6, -7, 5, -9$. It is clear that in this way we can find similar tests of divisibility for any number whatever, but it does not seem worth while to push the matter further except in special cases.

A simple rule for divisibility by 37 may be found in this way. The remainders are $1, -11, 10$. Thus 343619 is divisible by 37 because $1(3)-11(4)+10(3)+1(6)-11(1)+10(9)=74=2(37)$.

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INTRODUCTION TO DIFFERENTIATION.

By JOHN MACNIE, A. M., Professor of Mathematics, University of North Dakota.

1. In the identity $\frac{r^n-1}{r-1}=r^{n-1}+r^{n-2}+\ldots\ldots r+1$, ...(1),

Since r may have any value, let $r=\frac{x^{1/m}}{z^{1/m}}$; then, by substituting this value for r in (1), multiplying both members by $z^{\frac{n-1}{m}}$, and simplifying, we obtain