# MAT 211, Spring 2012
# Solutions to Homework Assignment 1

**Maximal grade for HW1: 100 points**

**Section 1.1** Find all solutions of the linear system.
2. (10 points)

$$\begin{cases} 4x + 3y = & 2 \\ 7x + 5y = & 3 \end{cases}$$

**Answer:** $x = -1, y = 2$.

**Solution:** Let us multiply the first equation by 2 and subtract from the second, we get

$$\begin{cases} 4x + 3y & = 2 \\ -x - y & = -1 \end{cases} \Leftrightarrow \begin{cases} 4x + 3y & = 2 \\ x + y & = 1 \end{cases}$$

Let us multiply the second equation by 3 and subtract from the first one, we get

$$\begin{cases} x & = -1 \\ x + y & = 1 \end{cases} \Leftrightarrow \begin{cases} x & = -1 \\ y & = 2 \end{cases}$$

4. (10 points)

$$\begin{cases} 2x + 4y = & 2 \\ 3x + 6y = & 3 \end{cases}$$

**Answer:** $x$ is arbitrary, $y = 1/2 - x/2$.

**Solution:** Let us divide the first equation by 2 and divide the second one by 3, we get

$$\begin{cases} x + 2y & = 1 \\ x + 2y & = 1 \end{cases}$$

This means that $x$ can be arbitrary, and $y$ can be found from the equation $x + 2y = 1 \Leftrightarrow y = 1/2 - x/2$.

10. (20 points)

$$\begin{cases} x + 2y + 3z = & 1 \\ 2x + 4y + 7z = & 2 \\ 3x + 7y + 11z = & 8 \end{cases}$$

**Answer:** $x = -9, y = 5, z = 0$.

**Solution:** The augmented matrix of the system has a form

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 4 & 7 & 2 \\ 3 & 7 & 11 & 8 \end{array}\right)$$

Let us subtract from the second row the first multiplied by 1 and from the third row – the first multiplied by 3:

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 5 \end{array}\right)$$

Swap the second row and the third:

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

Subtract from the first row the third multiplied by 3 and from the second – the third multiplied by 2:

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

Subtract from the first row the second multiplied by 2:

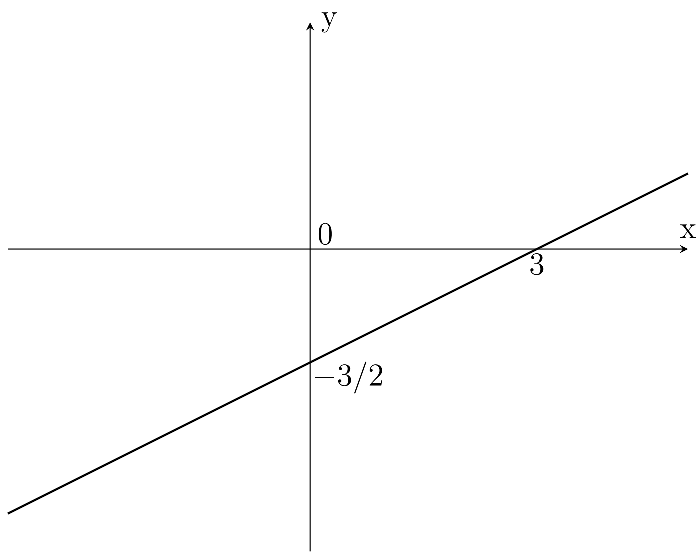
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

Finally, $x = -9, y = 5, z = 0$.

12. (14 points) Find all solutions and represent your solutions graphically:

$$\begin{cases} x - 2y = & 3 \\ 2x - 4y = & 6 \end{cases}$$

**Solution:** Since the second equation is just the first one multiplied by 2, there are infinitely many solutions parameterized by the graph of a function $y = x/2 - 3/2$:




34.(20 points) Find all the polynomials $f(t)$ of degree $\leq 2$ whose graph run through the points $(1, 1)$ and $(3, 3)$, such that $f'(2) = 3$.

**Answer:** No solutions.

**Solution:** Let $f(t) = at^2 + bt + c$, then $f'(t) = 2a + b$. We get a system of three linear equations on $a, b, c$:

$$\begin{cases} a + b + c & = 1 \\ 9a + 3b + c & = 3 \\ 4a + b & = 3 \end{cases}$$

If we multiply the third equation by 2 and add to the first equation, we get $9a + 3b + c = 7$, what contradicts the second equation. Therefore this system has no solutions.

**Section 1.2** 18. (16 points) Determine which of the matrices below are in reduced row-echelon form:

a)

$$\begin{pmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

**Solution:** No: leasing 1's in second and third row are above each other.

b)

$$\begin{pmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**Solution:** Yes.

c)

$$\begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

**Solution:** No: every row above a leading 1 should contain leading 1 to the left.

d)

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 \end{pmatrix}$$

**Solution:** Yes.

20. (10 points) How many types of $2 \times 2$ matrices in reduced row-echelon form are there?

**Solution:** There are: a single rank 0 matrix

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

two rank 1 types

$$\begin{pmatrix} 1 & * \\ 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$

and a single rank 2 matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$