MAT 211, Spring 2012
Practice problems for the Midterm 2

1. Determine if the following matrices are invertible. If they are invertible, find their inverses:

$$(a)\quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad (b)\quad \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}; \quad (c)\quad \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix};$$

$$(d)\quad \begin{pmatrix} 5 & 7 \\ 10 & 14 \end{pmatrix}; \quad (e)\quad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}; \quad (f)\quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{pmatrix};$$

$$(g)\quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{pmatrix}; \quad (h)\quad \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}; \quad (i)\quad \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

2. For given matrices of linear transformations $T : \mathbb{R}^n \to \mathbb{R}^m$ compute:

(1) m and n – the dimensions of the source and the target spaces

(2) The dimensions of the kernel and the image. Check the Rank-Nullity Theorem.

(3) The bases in the kernel and the image.

$$(a)\quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad (b)\quad \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}; \quad (c)\quad \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix};$$

$$(d)\quad \begin{pmatrix} 5 & 7 & 12 & 0 \\ 10 & 14 & 24 & 0 \end{pmatrix}; \quad (e)\quad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}; \quad (f)\quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$(g) \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{pmatrix}; \quad (h) \quad \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}; \quad (i) \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

3. a) Is there a linear transformation from $\mathbb{R}^7$ to $\mathbb{R}^3$ with two-dimensional kernel? Give an example or prove that it does not exist.

b) Is there a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^7$ with two-dimensional kernel? Give an example or prove that it does not exist.

c) a) Is there a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ with two-dimensional kernel? Give an example or prove that it does not exist.

d) Is there a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^7$ with two-dimensional image? Give an example or prove that it does not exist.

4. Determine if the following subset is a linear subspace, if yes, find its dimension:

(a) A circle on the plane passing through 0;

(b) An intersection of two distinct planes in $\mathbb{R}^3$ passing through 0;

(c) A union of two distinct planes in $\mathbb{R}^3$ passing through 0;

(d) The span of vectors $(1, 2, 3, 4)$ and $(3, 5, 7, 9)$;

5. For the vectors $v_1 = (1, 0, -1), v_2 = (2, 3, 4), v_3 = (0, 1, 0), v_4 = (1, 1, 1)$ find the dimension of their span by identifying redundant vectors.

6. Find a basis in a subspace in $\mathbb{R}^3$ defined by a system of equations:

$$(a) \quad 5x - 3y + 7z = 0; \quad (b) \quad \begin{cases} 5x - 3y + 7z = 0 \\ 2x - 3y + z = 0 \end{cases};$$

$$(c) \quad \begin{cases} 5x - 3y + 7z = 0 \\ 2x - 3y + z = 0 \\ 3x + 6z = 0 \end{cases}.$$

7. Show that the vectors

$$v_1 = (0, 1, 2, 3), v_2 = (0, 0, 0, 1), v_3 = (-1, -2, -3, -4), v_4 = (0, 0, 1, 2)$$

form a basis in $\mathbb{R}^4$. Compute the coordinates of a vector $(7, 8, 9, 10)$ in this basis.

8. Let $f(x) = a + bx + cx^2$.

(a) Show that $T(a, b, c) = (f(0), f(1), f(2))$ defines a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$. Find its matrix.

(b) Check if T is invertible. If yes, find the inverse matrix.