# MAT 211 - Introduction to linear algebra, Final Review

(1) (1) Compute the **determinant** of the matrix

(2) Determine the all the values of $k$ for which the matrix is invertible.

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & k \end{bmatrix}$$

(2) Find an orthogonal matrix $A$ such that if $T$ is a linear transformation on $\mathbb{R}^3$ defined by $T(x) = AX$ then $T(1/\sqrt{2}, 1/\sqrt{2}, 0) = (1, 0, 0)$.

(3) Consider the vectors

$$\begin{aligned} \vec{v}_1 &= (1,2,3,4) \\ \vec{v}_2 &= (5,6,7,8) \\ \vec{v}_3 &= (4,3,2,1) \\ \vec{v}_4 &= (1,1,1,1) \end{aligned}$$

in $\mathbb{R}^4$. Compute the **dimension** of $\text{span}\{v_1, v_2, v_3, v_4\}$.

(4) Find a basis of the subspace of all $3 \times 3$ matrices formed by all $3 \times 3$ symmetric matrices. Give the coordinates of the identity and of $\mathbf{E} = \begin{pmatrix} 1 & 10 & 20 \\ 10 & 0 & 30 \\ 20 & 30 & 0 \end{pmatrix}$, with respect to that basis.

(5) The three transformations below are defined the linear space formed by all $2 \times 2$ matrices. For each of them, determine whether it linear. If it is, find kernel, image, nullity and rank.

(a) $T(A) = A^{-1}$.
(b) $T(A) = tr(A).A$
(c) $T(A) = -A$.

(6) Consider the transformations from $P_2$ to $P_2$ defined below. Determine whether each of them is linear and if it is, find the correspoinding matrix with respect to the standard basis. (EC find the matrix of $T$ with respect to the basis $(1+t, t+t^2, 1+t^2)$)

(a) $T(f(t)) = f(1)t$.
(b) $T(f(t)) = f(0) + 3f(1)t + f(2)t^2$.

(7) Compute the area of the parallelogram in $\mathbb{R}^3$ whose vertices are $(1,1,1)$, $(1,3,5)$, $(2,4,6)$, and $(2,6,10)$.

(8) The matrices

$$A = \begin{bmatrix} 2 & 1 & -1 & 0 & 2 \\ 0 & 1 & -1 & 3 & 7 \\ 0 & 0 & 7 & 8 & 9 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 2 & 2 & -2 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 3 & -3 & -3 & -3 & 0 \\ 5 & 5 & -5 & 5 & 0 \\ 7 & 7 & 7 & 7 & 7 \end{bmatrix}$$

are both invertible. Find $\det(BAB^{-1})$.

(9) Find the eigenvalues of the matrix

$$\begin{bmatrix} 5 & -4 \\ 4 & -5 \end{bmatrix}.$$

Then find an example of a non-zero eigenvector corresponding to each of these eigenvalues.

(10) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

Then find the eigenvalues of the matrix $A^3$.

(11) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix}.$$

and find the algebraic multiplicity of each eigenvalue.

(12) Denote by $A$ the matrix of the projection onto the subspace $W$ of $R^2$ defined by $\{(x, y, z, w)/x+y+z = 0 \text{ and } y+z+w = 0\}$. Find $A$ and $A^2$, $A^{100}$.