

Example (3.3-27)

Determine whether the following vectors form a basis of \mathbb{R}^4

$(1,1,1,1), (1,-1,1,-1), (1,2,4,8), (1,-2,4,-8)$

Exercise 1.2-30

Find the polynomial of degree 3 whose graph passes through the points $(0,1), (1,0), (-1,0), (2,-15)$

$$-2x^3 - x^2 + 2x + 1$$

Find the inverse of the rotation matrix.

$$\begin{vmatrix} \cos(a) & -\sin(a) \\ \sin(a) & \cos(a) \end{vmatrix}$$

Let T be a clockwise rotation in \mathbb{R}^2 by $\pi/2$ followed by an orthogonal projection onto the y axis.

1. Find the matrix of T .
2. Determine whether T is invertible
3. Find $\text{im}(T)$ and $\text{ker}(T)$

Find the inverse of the matrix. Interpret your result geometrically.

$$\begin{vmatrix} a & b \\ b & -a \end{vmatrix}$$

For the matrix A below, find all the 2×2 matrices X that satisfy the equation $A \cdot X = I_2$.

$$\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix}$$

3.1-23 Describe the image and kernel of the reflexion about the line $y=x/3$ in \mathbb{R}^2 .
 Compute the dimensions of the kernel and the image.

(2.4-31) For which values of the constants a , b and c is the following matrix invertible?

$$\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

3.2-46 Find a basis of the kernel and image of the matrix.
 Determine the dimensions of the kernel and image.
 Determine the rank.
 Justify your answers.

$$\begin{vmatrix} 1 & 2 & 0 & 3 & 5 \\ 0 & 0 & 1 & 4 & 6 \end{vmatrix}$$

Example (3.3-31)

Let V be the subspace of \mathbb{R}^4 defined by the equation $x_1 - x_2 + 2x_3 + 4x_4 = 0$

Find a linear transformation T from \mathbb{R}^4 to \mathbb{R}^4 such that $\ker(T) = \{0\}$, $\text{im}(T) = V$.

Describe T by its matrix.

Give an example of a 5×4 matrix A with $\dim(\ker A) = 3$.
 Compute $\dim(\text{im } A)$.

Consider the vectors of \mathbb{R}^5 , $(1, 1, 0, 0, 0)$, $(0, 0, 0, 2, 2)$, $(1, 1, 0, 1, 1)$, $(0, 0, 1, 0, 0)$.

Compute the dimension of the subspace V of \mathbb{R}^5 spanned by those vectors.

Are they linearly independent?

Is $(1, 2, 0, 0, 0)$ a linear combination of those vectors?