MAT211 Lecture 6-7					
Linear Transformations in Geometry Matrix Products and Inverses					

Name	Formula	Notation	Matrix
Scaling (dilation if k>1; contraction if k<1)	k.x k scalar		
Orthogonal projection onto a line L in R ²	(u.x)u	proj∟(x)	
Reflection about a line L in R ²	2(u.x)u-x	ref∟(x)	
Rotation through a fixed angle θ			$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

L is a line through the origin, u is vector in L, u.u=1 V is a plane through the origin, orthogonal to L				
Name	Formula	Notation	Matrix	
Reflection about a plane V in R ³	x-2(u.x)u	ref _v (x)		
Orthogonal projection onto a line L in R ³	(u.x)u	proj∟(x)		
Orthogonal projection onto a plane V in R ³	x - (u.x)u	proj _V (x)		
Reflection about a line L in R ³	2(u.x)u-x	ref _L (x)		
in R ³	2(u.x)u-x Find the ma			

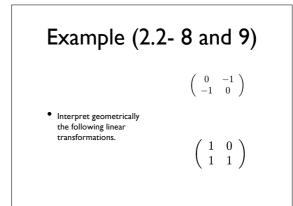
Example

• Interpret geometrically the following linear transformation.

 $T(x,y) = (x - \sqrt{3}y, \sqrt{3}x + y)$

Example (2.2-7) L be the line in R that consists of all scalar

L be the line in R that consists of all scalar multiples of (2,1,2). Find the reflection of the vector (1,1,1) about the line L.



- Consider the transformation from R³ to R² defined by f(x₁,x₂,x₃)= $x_1 \begin{pmatrix} 10 \\ 2 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -2 \end{pmatrix}$
- Determine whether this transformation is linear
- Determine whether this transformation is invertible.

Overview

f(x)=A.x

Function

Linear

Transformations

transformations	Product of matrices
Invertible linear transformations	Invertible matrices
Inverse of a linear transformation	A-1
	I

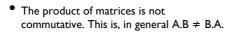
- Let A be an m x n matrix and B a p x q matrix. The product A.B is defined if and only if n=q.
- Let A be an m x p matrix and B a p x q matrix. The product A.B is the matrix of the linear transformation T(x)=A(Bx). The product A.B is an m x q transformation.
- Let B be an n x p matrix and let A be a p x m matrix with columns v₁, v₂,...,v_m. Then the product B.A is equal to the matrix $\begin{pmatrix} | & | & \cdots & | \\ Bv_1 & Bv_2 & \cdots & Bv_m \\ | & | & \cdots & | \end{pmatrix}$

• Let B be an n x p matrix and let A be a p x m matrix with columns v₁, v₂,..,v_m. Then the product i j entry (row i, column j) of the product B.A is given by

 $b_{i1}a_{1j} + b_{i2}a_{2j} + \dots + b_{ip}a_{pj}$

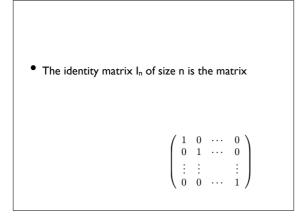
• Compute the products
$$A = \begin{pmatrix} 3 & 2 \\ 1 & 10 \end{pmatrix}$$

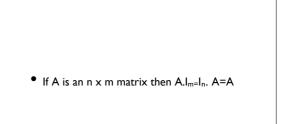
A.B and B.A.
 $B = \begin{pmatrix} 0 & 2 \\ 1 & -1 \end{pmatrix}$



- The product of matrices is associative: (A.B).C=A.(B.C).
- The distributive law holds for matrices A.(B+C)=A.B+A.C

$$B = \left(egin{array}{cc} 0 & 2 \ 1 & -1 \end{array}
ight)$$

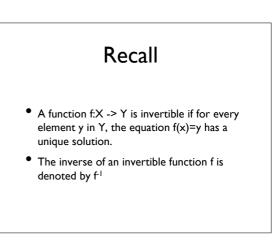




Multiply the matrices, whenever is possible

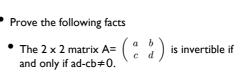
$$A = \left(\begin{array}{rrr} 1 & 4 & 0 \\ 0 & 3 & -1 \end{array}\right) \qquad \qquad B = \left(\begin{array}{rrr} 3 & -1 \\ 5 & 1 \end{array}\right)$$

$$C = \left(\begin{array}{rrr} 1 & 4/5 \\ 3 & -1 \\ 5 & 1 \end{array}\right)$$





- A matrix A is invertible if it is square (that is of size n x n) and the associated linear transformation is invertible.
- If a linear transformation T(x)=Ax is invertible, the associated matrix of the inverse T⁻¹ is denoted by A⁻¹

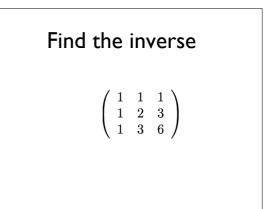


• Find the inverse of A.



- Show that the following matrix is invertible.
- Interpret geometrically the associated linear transformation.

 $\begin{pmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{pmatrix}$





- To find the inverse of a matrix A of size n x n, compute the reduced row echelon form of the matrix $(A \mid I_n)$
 - If $rref(A | I_n)=(I_n|B)$ for some matrix B, then A is invertible and $B=A^{-1}$
 - Otherwise, A is not invertible.

Theorem

- If A is an invertible matrix of size $n \times n$ then $A^{-1}.A=I_n$ and $A.A^{-1}=I_n$
- If A and B are invertible matrix then the matrix A.B is invertible and (A.B) ⁻¹ =B ⁻¹ A⁻¹
- If A and B are two matrices of size n x n such that BA=In then A and B are invertible, A=B⁻¹ and B=A⁻¹.

