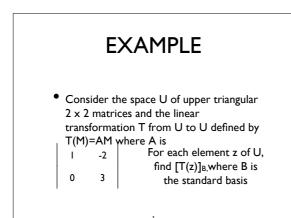
MAT211 Lecture 13

The matrix of a linear transformation

- The B-matrix of a linear transformation
- The columns of the B-matrix of a linear transformation
- ◆Change of basis matrix
 ◆Change of basis in a subspace of Rⁿ
- Change of basis for the matrix of a linear transformation

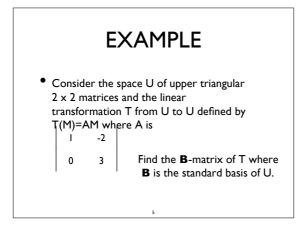
Overview A and B basis of linear space V, T a linear transformation from V to V.

- Coordinate Transformation from V to $R^n L(f) = [f]_B$
- **B**-matrix of T is $L_B \circ T \circ L_{B^{-1}}$
- Change of basis from B to A, SB->A=LA o (LB)⁻¹
- If B is **B**-matrix of T and A is **A**-matrix of T, S the change of basis from B to A, AS=S B



Definition

- Consider a linear transformation T from V to V where V is an n-dimensional linear space. Let B denote a basis of V.
- The matrix B of the transformation from Rⁿ to Rⁿ defined by L_B o T o L_B⁻¹ is called the <u>B-matrix of T.</u>



Theorem

- Consider a linear transformation T from V to V. Let B be matrix of T with respect to a basis B=(b₁,b₂,..,b_n)
- Then columns of B are the B-coordinate vectors [T(b₁)]_B, ,T(b₂)]_B,..,T(b_n)]_B

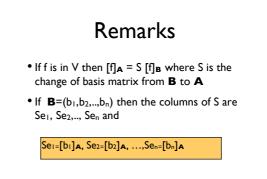
EXAMPLE

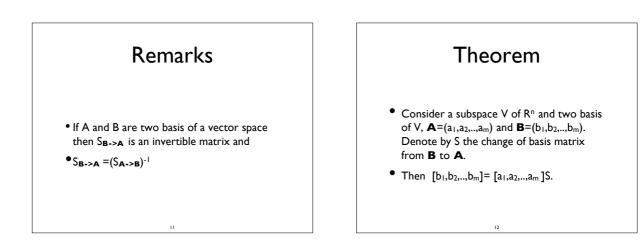
- Give the matrix of the linear transformation T(f)=f''+2f' from P_2 to P_2 with respect to the basis $(1,t,t^2)$.
- Find basis of the kernel and the image and compute rank and nullity of T.

Definition

- Consider two basis **A** and **B** of an ndimensional vector space V.
- The matrix S = S_{B->A} of the linear transformation
- LA o (LB)-1
- from Rⁿ to Rⁿ is called the <u>change of basis</u> <u>matrix from B to A.</u>

EXAMPLE • Find the change of basis matrix S from the standard basis **B** to basis **A** of U^{2x2} where **A** is 1 0 0 0 1 0 0 0 0 1 standard basis **B** to basis **A** of U^{2x2} where **A** is 1 1 0 1 0 0 0 0 1 basis **B** of 0 0 1 0 Find the change of basis matrix from **A** to **B**.



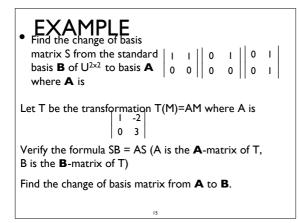


EXAMPLE (4.3-60)

- In the plane V defined by the equation 2x+y-2z=0 consider the basis
- A=((1,2,2),(2,-2,1)) and B=((1,2,2),(3,0,3))
- I. Find the change of basis matrix from ${\sf B}$ to ${\sf A}$
- 2. Find the change of basis matrix from A to ${\sf B}$
- 3. Write an equation relating the matrices $[a_1,a_2]$ and $[b_1,b_2]$ where A= (a_1,a_2) and B= (b_1,b_2)

Theorem

- Consider a linear space V and two basis of V, A=(a₁,a₂,...,a_n) and B=(b₁,b₂,...,b_n). Let T be a linear transformation from V to V, and let A and B be the A-matrix and the B-matrix of T, respectively. Then A is similar to B and
- AS=SB
- where S is the change of basis matrix from **B** to **A**.



A and **B** basis of linear space V, T a linear transformation from V to V.

- Coordinate Transformation from V to $R^n \; L(f) {=} \; [f]_B$
- **B**-matrix of T is L_B o T o L_B⁻¹
- Change of basis from B to A, $S_{B->A}=L_A \circ (L_B)^{-1}$
- If B is **B**-matrix of T and A is **A**-matrix of T, $S_{B->A} A=S_{A->B} B$