MAT211 Lecture 10

Coordinates

- Definition
- Linearity of coordinates
- The matrix of a linear transformation
- Standard matrix - B-matrix
- Similar matrices

Review

- Linear combination
- Subspace of R^n
- Span
- Basis

- Consider a basis $\boldsymbol{B}=(v_1, v_2,..,v_m)$ of a subspace V of R^n and a vector x of V.
- We know that x can be written as $x= c_1 v_1+ c_2 v_2+...+ c_n v_n$
- Moreover, the coefficients $c_1, c_2,..,c_n$ are unique.
- The scalars $c_1, c_2,..,c_n$ are the coordinates of x.
- The vector $(c_1, c_2,..,c_n)$ is the B-coordinate of x.
- The B-coordinate of x is denoted by $[x]_B$

Example 3.4-10

If B=(-1,0,1),(-2,10) and V is the span of B, find the B-coordinates of (1,-2,-2).

Remark

Since

$x= c_1 v_1+ c_2 v_2+...+ c_n v_n$,

$x = S [x]_B$, where S is the n x m matrix with columns $v_1, v_2,..,v_m$.

Theorem

If B is a basis of a subspace V or R^n , x and y are vectors in V and k is a scalar then

- $[x+y]_B =[x]_B +[y]_B$
- $[k.x]_B =k.[x]_B$

Definition

Consider a linear tranformation $T: R^n \to R^n$ and a basis $\boldsymbol{B}=(v_1, v_2,..,v_n)$ of R^n.

The n x n matrix with columns

$[T(v_1)]_B$, $[T(v_2)]_B$, ...,$[T(v_n)]_B$

is called the B-matrix of T.

- Observe that the B-matrix of T transforms $[x]_B$ in $[T(x)]_B$ for every x in R^n

Example 3.4-22

Find the matrix B of the linear transformation T(x)=Ax with respect to the basis B=(1,2),(-2,1), where A is

$$\begin{vmatrix} -3 & 4 \\ 4 & 3 \end{vmatrix}$$

Theorem

Consider a linear tranformation $T: R^n \to R^n$ and a basis $\boldsymbol{B}=(v_1, v_2,..,v_n)$ of R^n.

Denote by B the $\boldsymbol{B}$-matrix of T.

Denote A the standard matrix of T.

Denote by S the matrix with columns $v_1, v_2,..,v_n$.

Then AS=SB, $B=S^{-1}AS$ and $A=SBS^{-1}$.

Example 3.4-22

Find the matrix B of the linear transformation T(x)=Ax with respect to the basis B=(1,2),(-2,1), where A is

$$\begin{vmatrix} -3 & 4 \\ 4 & 3 \end{vmatrix}$$

Solve it in three different ways

1. Use the formula $B=S^{-1}AS$
2. Use a commutative diagra
3. Construct B column by column

Definition

Consider to n x n matrices A and B.

We say that A is similar to B if there exists an invertible matrix S such that AS =SB.

Example 3.4-61

Find a basis B of R^2 such that the B-matrix of the linear transformation given by the first matrix is the second matrix.

$$\begin{vmatrix} -5 & 9 \\ 4 & 7 \end{vmatrix} \qquad \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$