MAT 132

8.6 Representation of functions as power series

Recall

A power series defines a function whose domain is the interval of convergence of the power series.

Example: The geometric series with a=1, and r=x

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 \ldots = 1/(1-x)$$

if |x|<1

Examples

1.Express $1/(1+2x^3)$ as a power series and find the interval of convergence.

2.Find a power series representation for $x^2/(x+5)$

2 Theorem If the power series $\Sigma\, c_n(x-a)^n$ has radius of convergence $R > 0$, then the function f defined by

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots = \sum_{n=0}^{\infty} c_n(x-a)^n$$

is differentiable (and therefore continuous) on the interval $(a-R, a+R)$ and

(i) $f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \cdots = \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$

(ii) $\int f(x)\,dx = C + c_0(x-a) + c_1\frac{(x-a)^2}{2} + c_2\frac{(x-a)^3}{3} + \cdots$

$$= C + \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1}$$

The radii of convergence of the power series in Equations (i) and (ii) are both R.

.Consider the power series $\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)^2}$

1. Find the radius of convergence.
2. Verify the theorem (i).
3. If f is the function defined by the power series, find the domain of f, and the domain of f'. Do f and f' have the same radius of convergence? Do they have the same interval of convergence?

Examples

1. Express $1/(1+x)^2$ as a power series.

What is the radius of convergence.

2. Find a power series representation of f(x) $=\tan^{-1}(x)$
3. Find a power series representation for f(x) $= e^x$ and find the radius of convergence
4. Find a power series representation for f(x) $= e^{-x}$ and find the radius of convergence
5. Use 4. to find the value of e^{-1} correct to five decimal places.