MAT 132

8.4 Power Series

A power series is in this form:

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots + c_n x^n + \cdots$$

or

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots + c_n(x-a)^n + \cdots$$

The coefficients $c_0, c_1, c_2 \ldots$ are constants.

The center "a" is also a constant.

(The first series would be centered at the origin if you graphed it. The second series would be shifted left or right. "a" is the new center.)

®

EXAMPLES

$$\sum_{n=0}^{\infty} 3x^n \qquad \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{n\,3^n}$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \sum_{n=0}^{\infty} n!\,x^n$$

A power series in x is a series that can be expressed as

$$c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n \cdots$$

The coefficients $c_0, c_1, c_2 \ldots$ are constants.

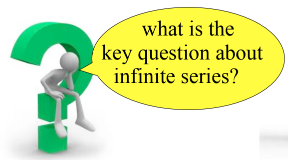
We also use the notation below

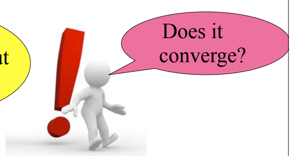
$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n \cdots$$

®

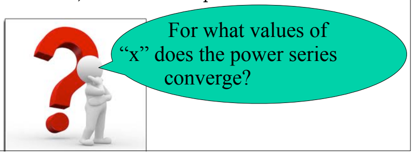
$$c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n \cdots$$

- When we studied infinite series (of constant terms), the key question was..





- If we replace "x" by a number, a powers series becomes an infinite series. Thus, in the case of power series we ask:



EXAMPLE: Find the values of x for which each of the power series is convergent

$$\sum_{n=0}^{\infty} 3x^n \qquad \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{n\,3^n}$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \sum_{n=0}^{\infty} n!\,x^n$$

A power series in (x-a) is a series that can be expressed as

$$\sum_{n=0}^{\infty} c_n(x-a)x^n = c_0 + c_1(x-a)x + c_2(x-a)x^2 + \cdots + c_n(x-a)^n \cdots$$

The coefficients $c_0, c_1, c_2\ldots$ are constants.

"a" is also a constant

®

---

3 Theorem For a given power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ there are only three possibilities:

(i) The series converges only when $x = a$.

(ii) The series converges for all x.

(iii) There is a positive number R such that the series converges if $|x - a| < R$ and diverges if $|x - a| > R$.

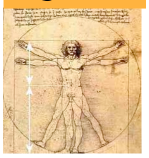
Which of the possibilities of the theorem above hold for each of the power series?

$$\sum_{n=0}^{\infty} 3x^n \qquad \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{n\,3^n}$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \sum_{n=0}^{\infty} n!\,x^n$$

EXAMPLE: Find the values of x for which each of the power series is convergent

Ratio test



$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^n (x+2)^n}{n\,3^n}$$

$$\sum_{n=0}^{\infty} 3(x-1)^n$$

---

3 Theorem For a given power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ there are only three possibilities:

(i) The series converges only when $x = a$.

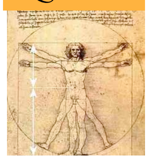
(ii) The series converges for all x.

(iii) There is a positive number R such that the series converges if $|x - a| < R$ and diverges if $|x - a| > R$.

- The number R in (iii) is called the radius of convergence of the power series.
- By convention, R=0 in (i) and R=∞ in (ii)
- The interval of convergence is the set of all values of x for which the power series converges.
- There are four possibilities for a power series centered at a.
  - (a-R,a+R)
  - [a-R,a+R)
  - (a-R,a+R]
  - [a-R,a+R]

EXAMPLE: Find the radius of convergence and the interval of convergence

Ratio test



$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^n (x+2)^n}{n\,3^n}$$

$$\sum_{n=0}^{\infty} 3(x-1)^n$$