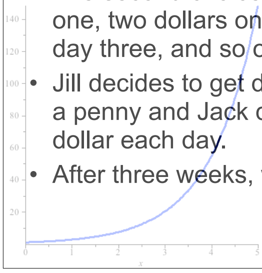
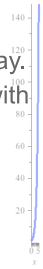
Exponential growth and decay

- Jack and Jill's parents ask them how they would like to get their allowance.
- The first choice is to get a penny on day one, two pennies on day two, four pennies on day three, and so on, doubling every day.
- The second choice is to get a dollar on day one, two dollars on day two, three dollars on day three, and so on, adding a dollar each day.
- Jill decides to get double each day starting with a penny and Jack decides to get an extra dollar each day.
- After three weeks, who has more money?

EXPONENTIAL VS. LINEAR GROWTH





REVIEW: A model of population growth

Suppose that the number of individuals in this population grows at a rate proportional to the size of this population.

P(t) = number of individuals at the time t.

Then

$$dP/dt = k \cdot P$$

where k is a constant that depends on the population (k is called the **relative growth rate**).

The solution to the intial value problem $dP/dt = k \cdot P$, $P(0)=P_0$ is $P(t)=P_0e^{kt}$

Exponential Change: $y = y_0e^{kt}$

If the constant k is positive then the equation represents growth. If k is negative then the equation represents decay.

Exponential growth and decay Some examples

t=time
be careful with units

Newton Law of cooling	T(t)= find it!	$dT/dt=k(T - T_s)$
Half life	$y(t)=y_0\ e^{-kt}$	
Population growth	$P(t)= P_0\ e^{kt}$	$dP/dt = k\ P$
Continuously compound interest	$A(t)=A_0\ e^{rt}$	

EXAMPLE (7.4.4): A bacteria culture grows with constant relative growth rate. After two hours, there are 600 bacteria and after 8 hours the count is 75,000

a. Find the initial population
b. Find an expression for the population after t hours.
c. Find the number of cells after 5 hours.
d. Find the rate of growth after 5 hours
e. When will the population reach 200,000?

Example (7.4.10)

- A sample of tritium-3 decay to 94.5% of its original amount after a year.
- a. What is the half-life of tritium 3?
- b. How long would it take the sample to decay 20% of its original amount?

EXAMPLE (7.4.14)

- A thermometer is taken from a room where the temperature is 20°C to the outdoors, where the temperature is 5°C. After one minute the thermometer reads 12°C.
  1. What will the reading on the thermometer be after one more minute?
  2. When will the thermometer read 6°C?

$\boxed{dT/dt=k(T - T_s)}$

Continuously compounded interest

- If an initial amount of money A_0 is invested at r% interest, is continually compounded, and the amount at time t is denoted by A(t) then

$$A(t) = A_0 e^{rt}, so \qquad \frac{dA}{dt} = rA(t)$$

EXAMPLE(7.4.19)

If $3000 is invested at 5% interest, find the value of the investment at the end of 5 years if the interest is compounded

1. Annually
2. Semiannually
3. Monthly
4. Weekly
5. Daily
6. Continuously

- A cup of coffee is initially 170 degrees Fahrenheit and is left in a room with ambient temperature 70 degrees Fahrenheit. Suppose that when the coffee is first placed in the room, it is cooling at a rate of 20 degrees per minute. Assuming Newton's law of cooling applies, how long does it take for the coffee to cool to 110 degrees?

Newton’s law of cooling

The rate of cooling an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large.

$$\frac{dT}{dt} = k(T - T_s)$$

where T(t) is the temperature of the object at the time t, k is a constant, T is the temperature of the surroundings

The table gives estimates of the world population in millions from 1750 to 2000.

a. Use the exponential model and the population figures for 1750 and 1800 to predict the world population in 1900 and 1950. Compare with the actual figures
b. Use the exponential model and population figures for 1850 and 1900 to predict the world population in 1950 and 2000. Compare with the actual population

Year	Population
1750	790
1800	980
1850	1260
1900	1650
1950	2560
2000	6080